Data-driven forecast for the Chemostat model

Supervised by

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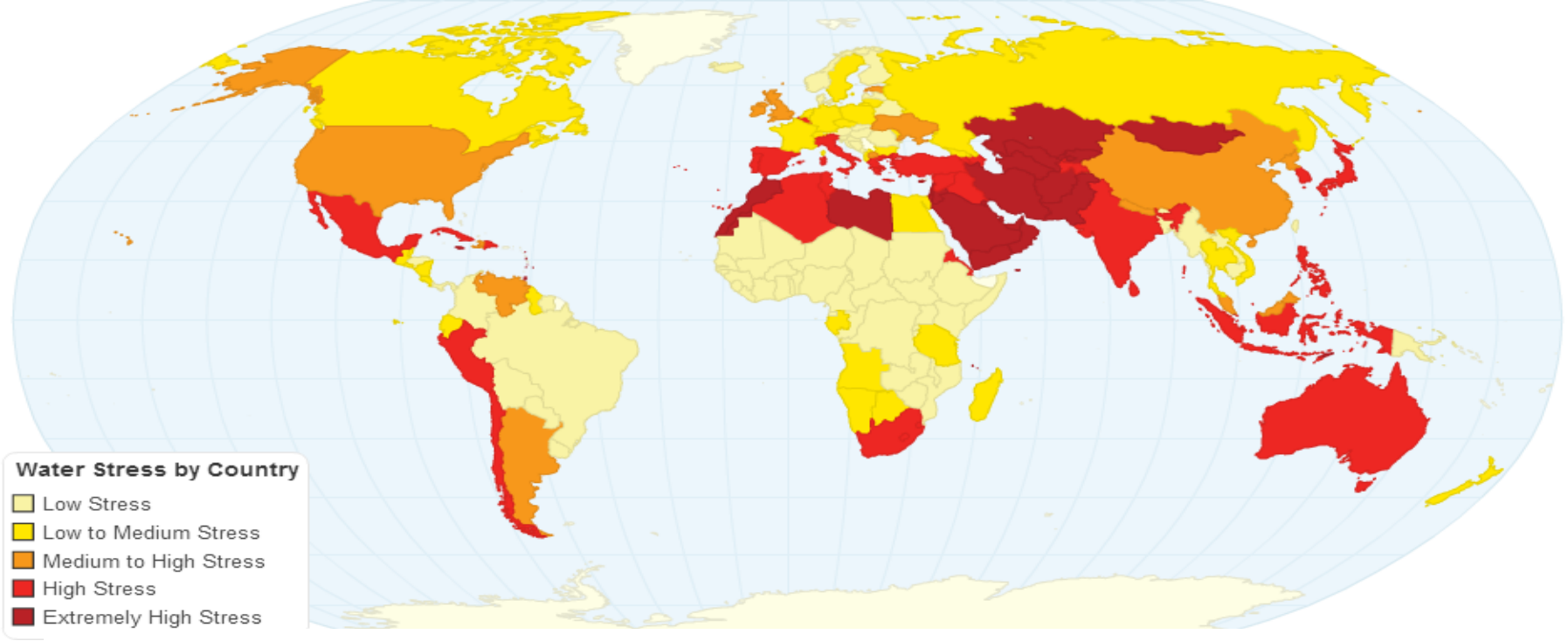
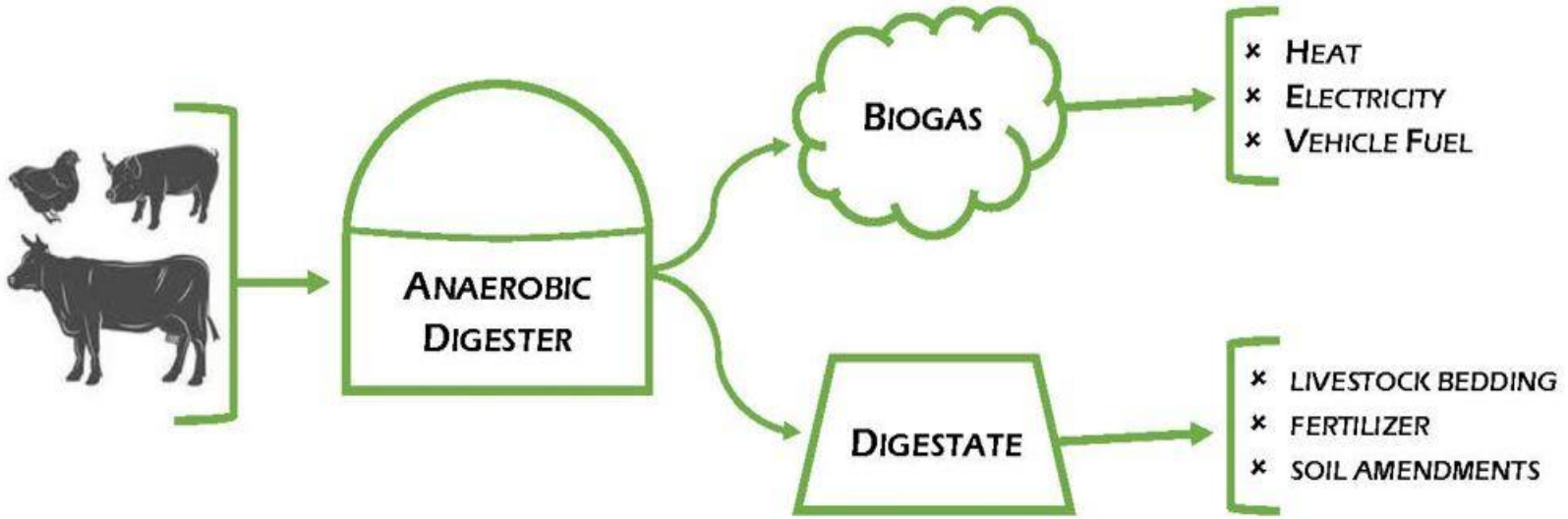
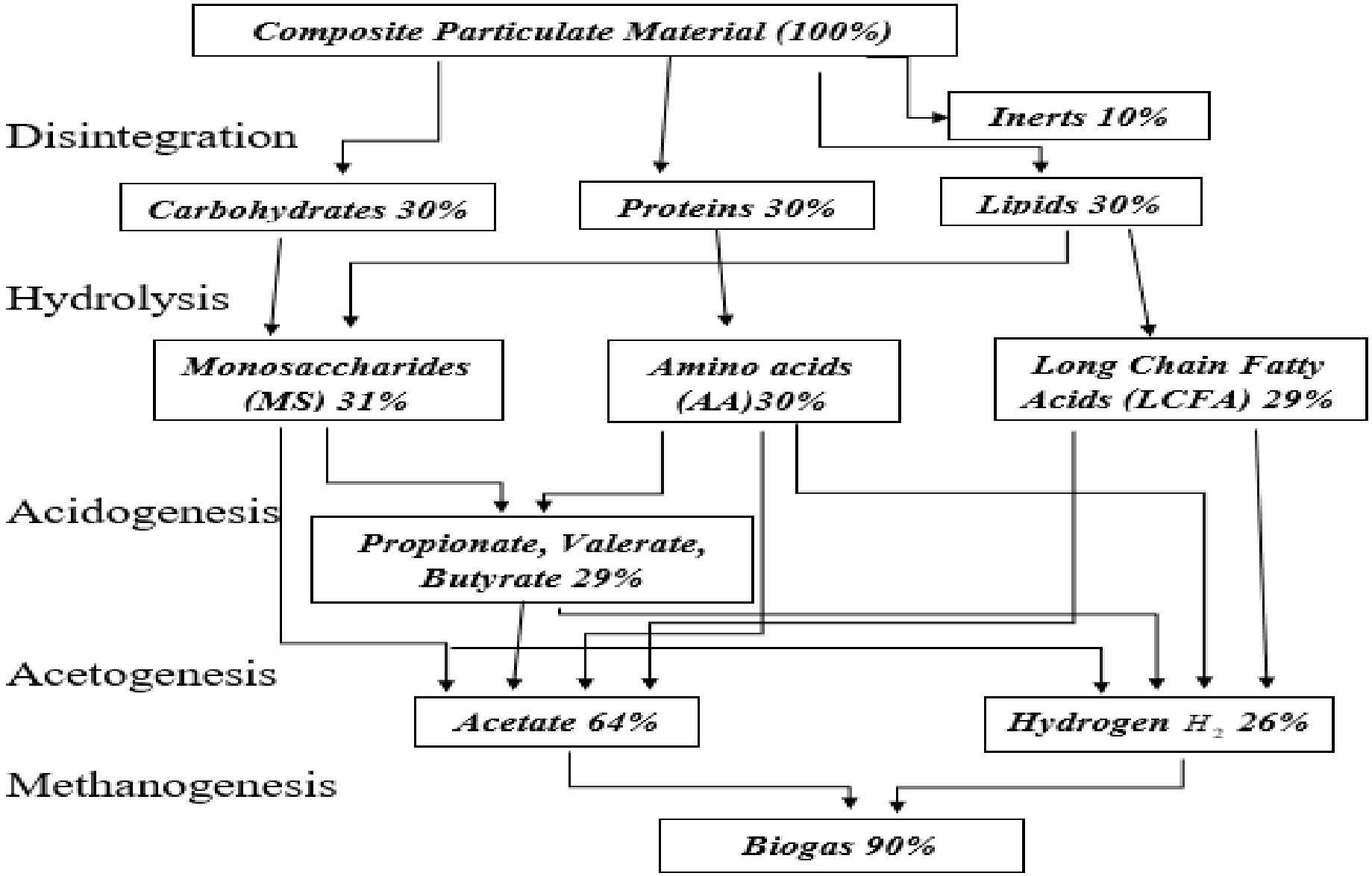

Water Stress by Country
Low Stress
Low to Medium Stress
Medium to High Stress
High Stress
Extremely High Stress


ANAEROBIC DIGESTER
BIOGAS
HEAT
ELECTRICITY
VEHICLE FUEL
DIGESTATE
LIVESTOCK BEDDING
FERTILIZER
SOIL AMENDMENTS

Plan

- Introduction
- ADM1 model
- Dynamic Mode Decomposition - DMD
- Application to the order reduction of ADM1
- The Koopman operator
- Application to the chemostat
- Conclusion

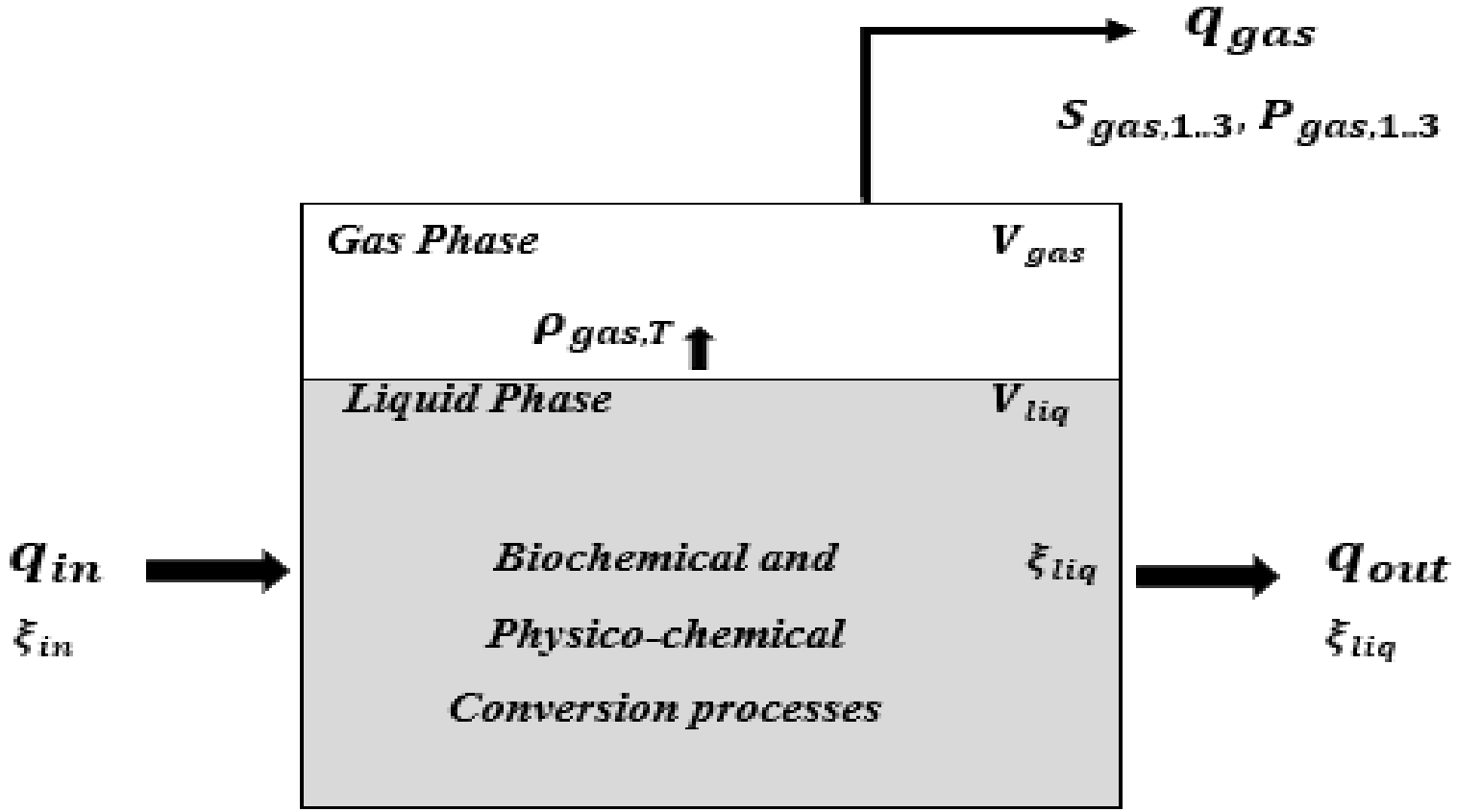
ADM1: Reaction paths and COD flux



BATSTONE et al , "The IWA Anaerobic Digestion Model No 1 (ADM1)", *Water Sci. Technol.*, No. 10, vol. 45, pp. 65-73, May 2002.

Modeling

Dynamical model of the ADM1 is constructed from differential equations



- 35 States variables
- 130 parameters
- 19 Biochemical process rates
- 3 Gas transfer rates
- 6 Acid-base rates

Mass balance reactions :

$$\frac{d\xi_{liq,i}}{dt} = \frac{q}{V}\left(\xi_{in,i} - \xi_{liq,i}\right) + \sum_{j=1}^{Nb} \rho_i v_{ij}$$

v_{ij} Rate coefficient

ρ_i Kinetic rate

Nb :Number of reactions

Dynamic Mode Decomposition- DMD

DMD is a powerful new technique to extract linear reduce model for dynamical systems from high-dimensional data.

DMD is algorithmically a regression of data onto linear dynamics

$$\mathbf{x_{k+1}} = A\mathbf{x_k}$$

$$\mathbf{x_k} \xrightarrow{\quad A \quad} \mathbf{x_{k+1}}$$

Where A is chosen to minimize $\|\mathbf{x_{k+1}} - A\mathbf{x_k}\|_2$

$$\mathbf{X} = \begin{bmatrix} | & | & & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_{m-1} \\ | & | & & | \end{bmatrix},$$

$$\mathbf{X}' = \begin{bmatrix} | & | & & | \\ \mathbf{x}_2 & \mathbf{x}_3 & \cdots & \mathbf{x}_m \\ | & | & & | \end{bmatrix}.$$

$$\mathbf{X}' \approx A\mathbf{X} \cdot \qquad A = \mathbf{X}'\mathbf{X}^{\dagger}$$

The DMD algorithm :

SVD	1.	$\mathbf{X} \approx \boldsymbol{U\Sigma V}^*, \quad \mathbf{X}' \approx \boldsymbol{AU\Sigma V}^* \cdot$
		Singular values matrix
*	2.	$\boldsymbol{U}^*\mathbf{X}'\boldsymbol{V\Sigma}^{-1} = \boldsymbol{U}^*\boldsymbol{AU} = \widetilde{\boldsymbol{A}}$
Eig	3.	$\widetilde{\boldsymbol{A}}\boldsymbol{W} = \boldsymbol{W\Lambda}$
*	4	$\boldsymbol{\Phi} = \mathbf{X}'\boldsymbol{V\Sigma}^{-1}\boldsymbol{W} \cdot$

$$\tilde{X}(\boldsymbol{k\Delta t}) = \boldsymbol{\Phi\Lambda}^{\boldsymbol{t}}\boldsymbol{b_0}$$

Application to the order reduction the ADM1

1- I reshaped all the states variables of ADM1 as :

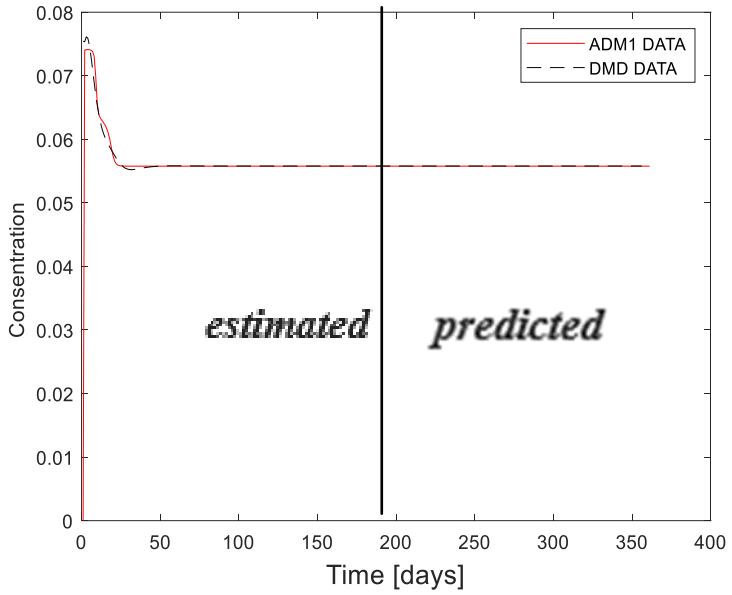
$$\mathbf{x}_k = \begin{bmatrix} x(S_{Su}, t_\kappa) \\ x(S_{aa}, t_\kappa) \\ . \\ . \\ . \\ x(Co_2, t_\kappa) \end{bmatrix},$$

$$\mathbf{X} = \begin{bmatrix} | & | & & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_{m-1} \\ | & | & & | \end{bmatrix},$$

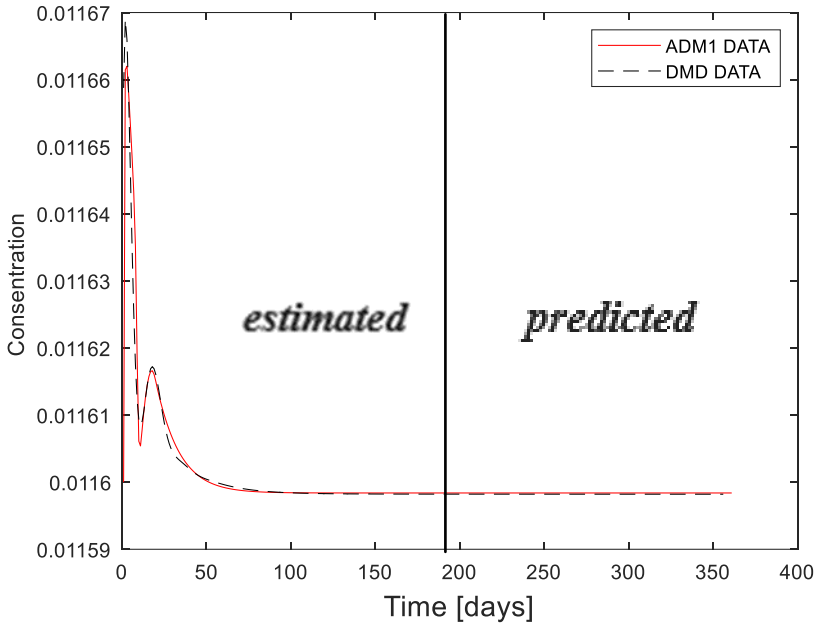
$$\mathbf{X}' = \begin{bmatrix} | & | & & | \\ \mathbf{x}_2 & \mathbf{x}_3 & \cdots & \mathbf{x}_m \\ | & | & & | \end{bmatrix}.$$

m = 180 days

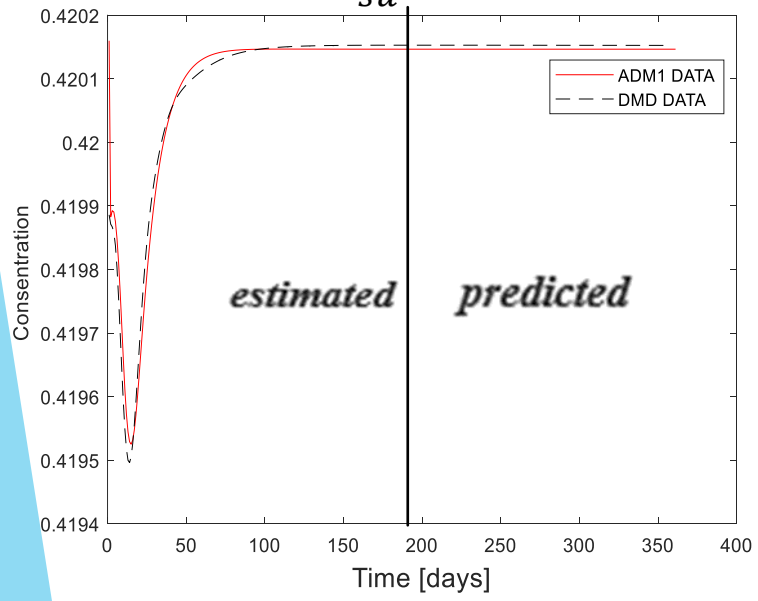
S_{Ch4}



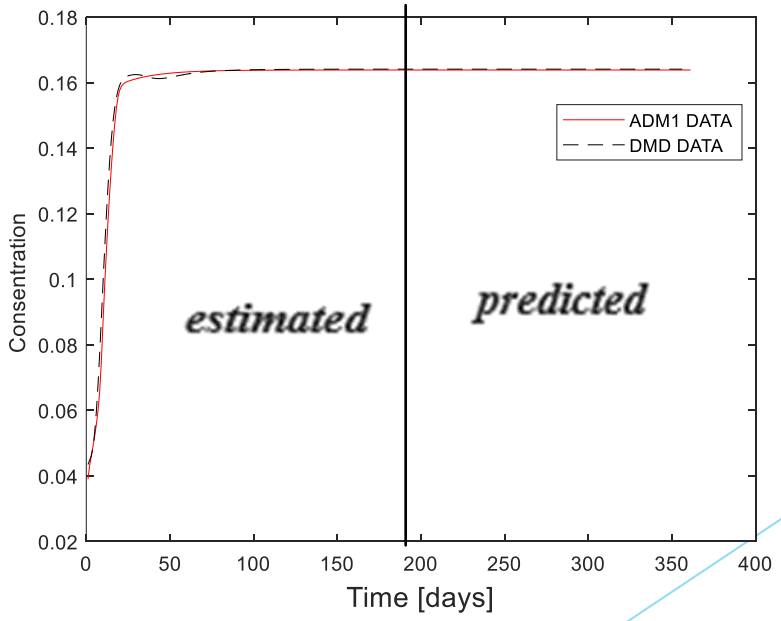
S_{vam}



X_{su}



S_{IC}



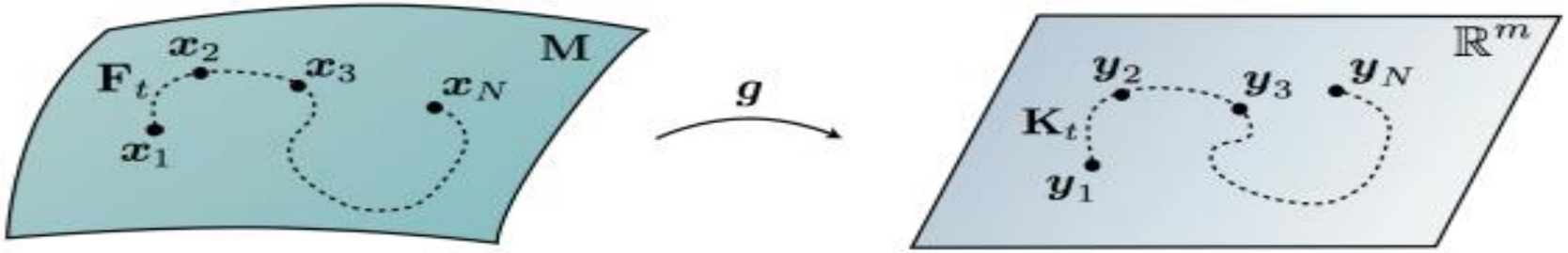
The Koopman operator

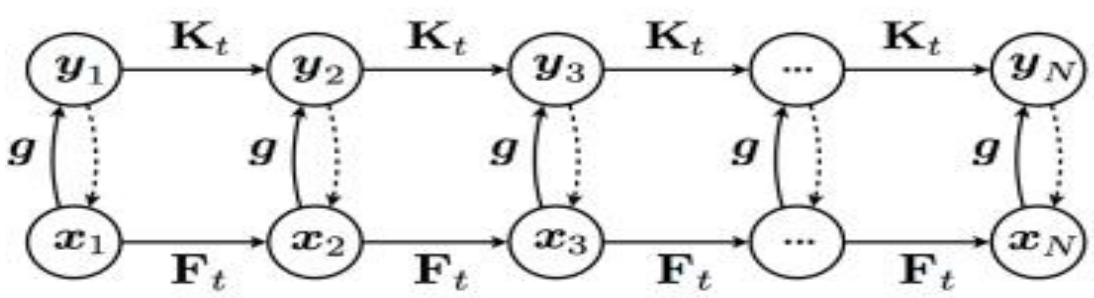
Dynamics

$$\mathbf{x_{k+1}} = \mathbf{F}(\mathbf{x_k}) \cdot$$

Koopman operator

$$\boldsymbol{K_t g}(\boldsymbol{x_k}) = \boldsymbol{g}(\mathbf{F}(\mathbf{x_k})) = \boldsymbol{g}(\mathbf{x_{k+1}})$$



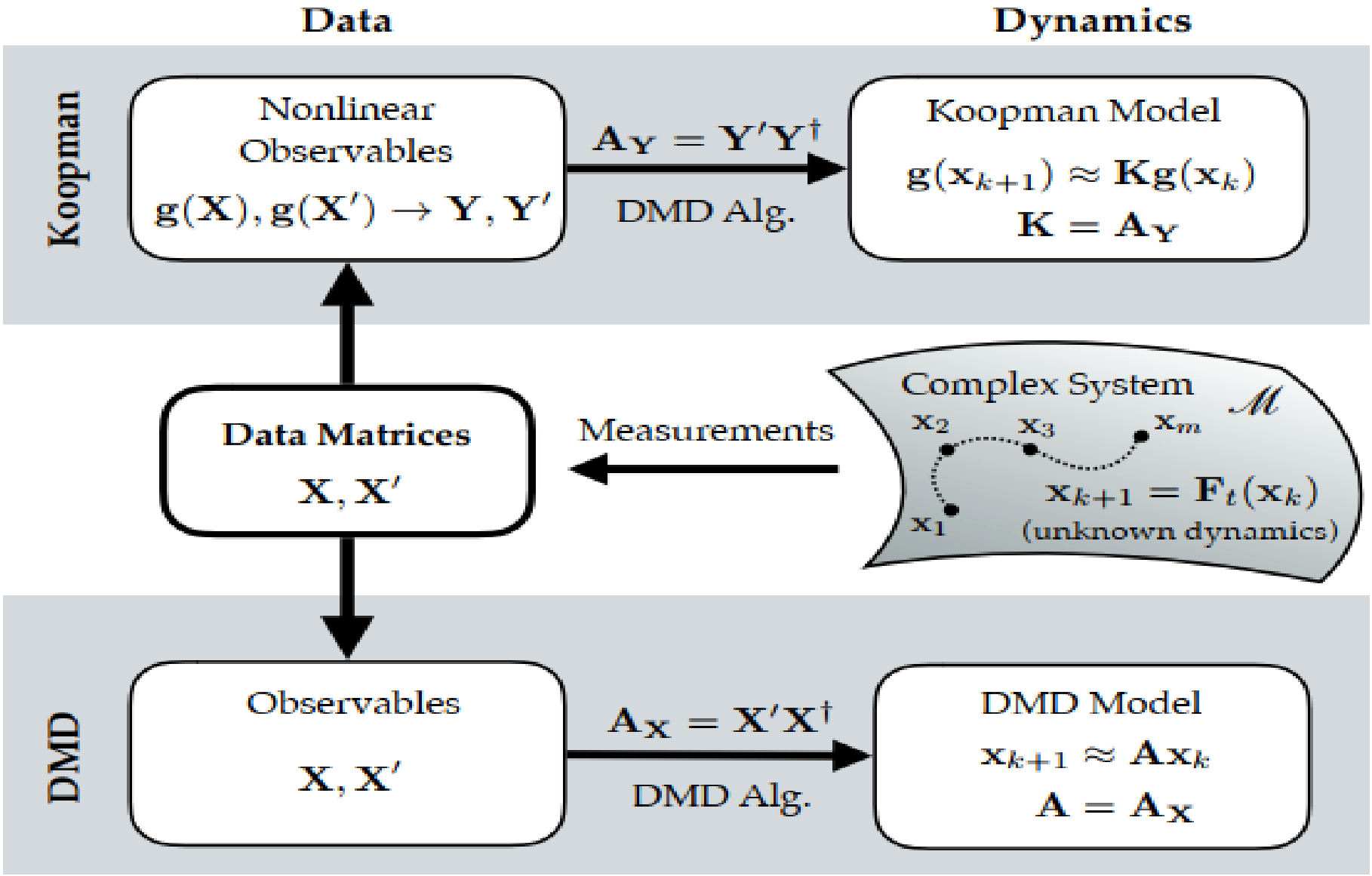


$\mathbf{F}_t : x_k \mapsto x_{k+1}$

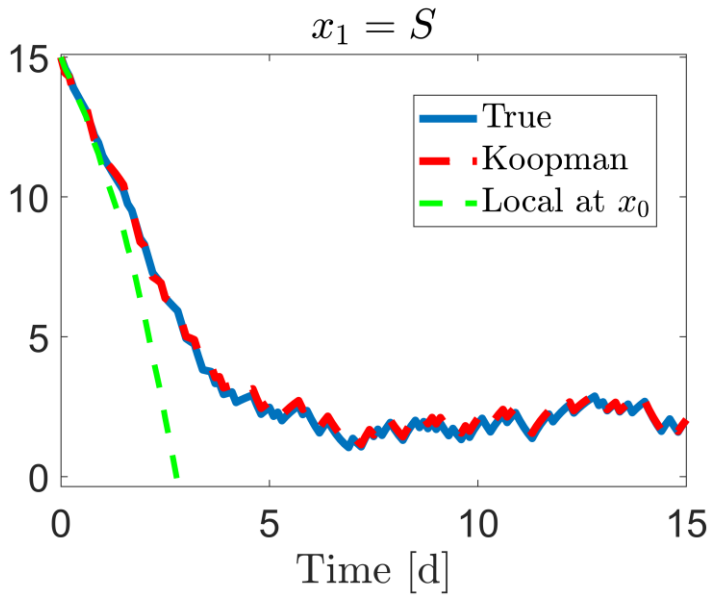
$g : x_k \mapsto y_k$

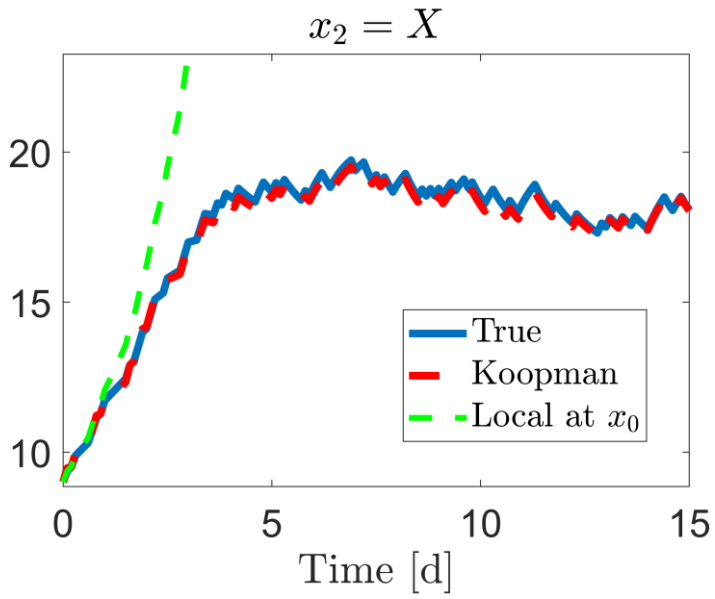
$\mathbf{K}_t : y_k \mapsto y_{k+1}$

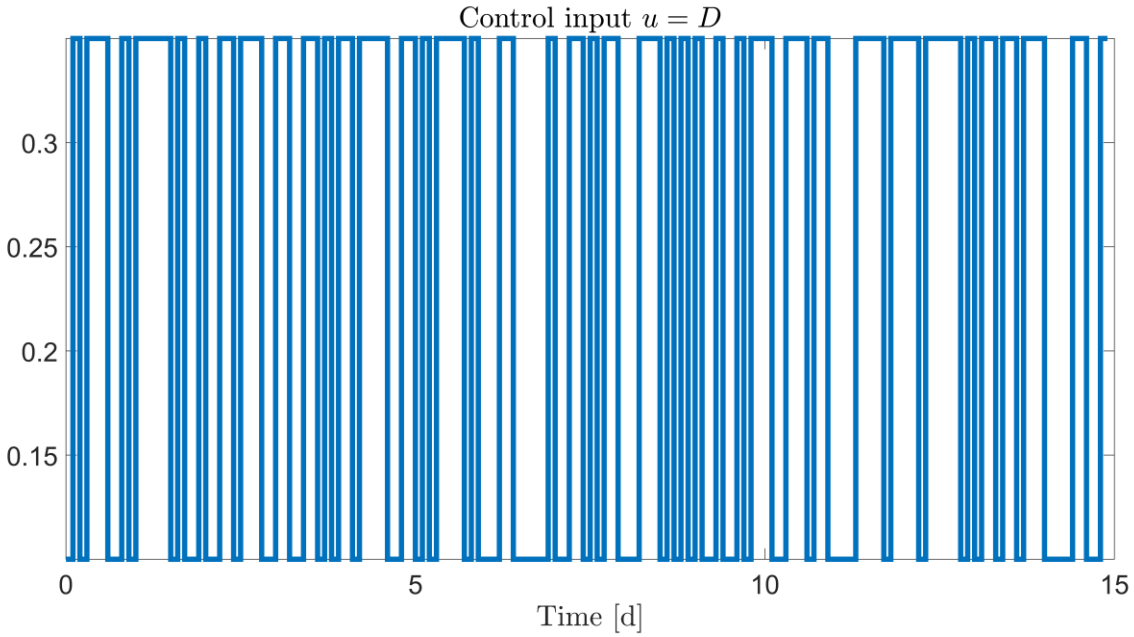
Connections with DMD



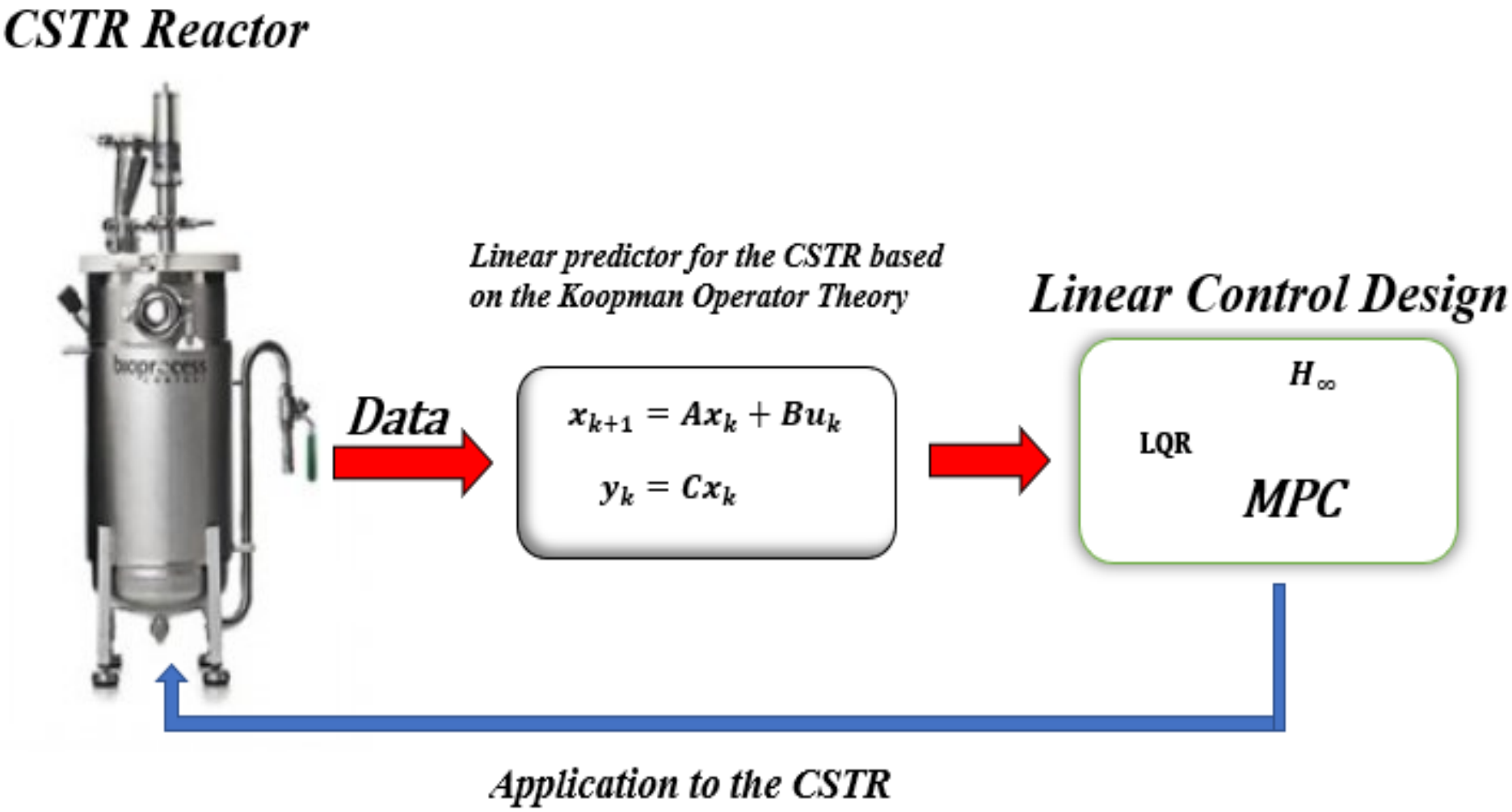
Application to the Chemostat







Future work with chemostat



Conclusion:

- DMD helps us to reduce and linearize the ADM1 from 35 states variables into 7 states.
- DMD gives a validation of the obtention of linear reduced model that can predict the low behavior of the original ADM1.
- We can use DMD for different tasks like state estimation and future state prediction and control.
- We construct a data-driven model of the chemostat based on the Koopman operator framework
- Both the DMD and Koopman approaches are equation-free
- The advantages of this methods are that it is very simple to execute and it makes almost no assumptions about the underlying system.

Thanks for your attention