

Ideal biological reactors  
(*a simplified steady state model  
for the activated sludge unit*)

# Biological systems

In a biological system for wastewater treatment, organic pollutants or nutrients (N and P) are removed from the water streams and used by microorganisms that develop within the system.

Main reactions are biochemical ones.

They are widely applied in environmental engineering:

- Aerobic degradation of organic pollutants
- Biological Nitrogen and Phosphorous removal
- Anaerobic degradation of organic pollutants from wastes and wastewaters (waste sludge digestion, biogas production from agro-wastes, OFMSW stabilization)
- Composting systems
- Biofilters for volatile organic compounds removal from gaseous flows.
- Bioremediations of contaminated soils
- .....

# Biological systems

Many biological systems include a solid/liquid separation unit that allows for the selective retention of microorganisms within the system. As a matter of fact, the higher their concentration, the faster the degradation process will proceed.

Widely employed separators are:

- Gravity settling unit
- Flotation units
- Filters/membranes
- Supports for the development of a microbial biofilm.

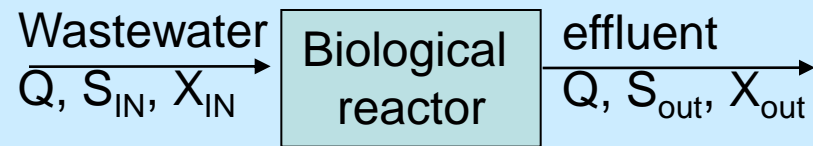
By retaining particulates, the following residence times can be differentiated:

- ***The hydraulic retention time ( $\theta_H$ )*** that is the average residence time of water and of all soluble (unretained) substrates
- ***Biomass retention time ( $\theta_C$ )*** that is the average residence time of all particulate substances that are efficiently retained in the biological system.

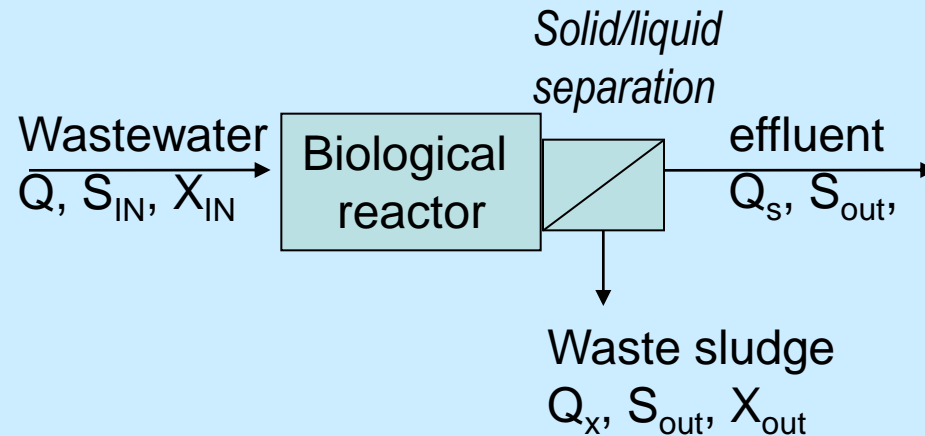
# Biological systems

Systems with/without biomass retention can be described as follows:

No biomass  
retention:  $\theta_C = \theta_H$



biomass retention:  
 $\theta_C > \theta_H$



# Biological systems

Setting **MASS BALANCES** across biological reactors in flow

*Hydrodynamics*: Continuous Flow Stirred Tank Reactor, CFSTR, with constant volume (V)

Lets consider the case of a retained (X) or unretained (S) component, at steady state:

$$V \frac{d[S]}{dt} = Q([S]_0 - [S]) + V \cdot r_S = 0$$

that can be rewritten as :  $\frac{[S]_0 - [S]}{\theta_H} + r_S = 0$

where :  $\theta_H = V / Q$

$$V \cdot \frac{d[X]}{dt} = Q \cdot [X]_0 - Q_X \cdot [X] + V \cdot r_X = 0$$

that can be rewritten as :  $\frac{[X]_0}{\theta_H} - \frac{[X]}{\theta_C} + r_X = 0$

or :  $\frac{\theta_C / \theta_H \cdot [X]_0 - [X]}{\theta_C} + r_X = 0$

where :  $\theta_C = V / Q_X$

$r_i$  = overall reaction rate →  
From Petersen matrix

# Biological systems

- Biomass retention is especially interesting when biomass growth is the kinetically limiting process; biomass wash-out can be easily prevented by adopting adequate  $\theta_c$  without over-sizing the biological reactor
- Since biomass does the job, the higher its concentration the faster is the degradation per unit of volume, the smaller the biological reactor.
- Biomass maximum in-reactor concentration depends on the efficiency of the retention unit or by other operational parameter (e.g. oxygenation)

# Biological systems

Example: Let us considered a biological system for the aerobic degradation of organic pollutants.

Data:

- wastewater characteristics:  $[X_S]_0, [S]_0$
- Reactor hydrodynamics: CFSTR with biomass retention
- Petersen Matrix:

	<b>S</b>	<b>Xs</b>	<b>Xp</b>	<b>XB</b>	<b>rate</b>
<b>p1</b>	-1/Y			1	r1
<b>p2</b>	1	-1			r2
<b>p3</b>		(1-f)	f	-1	r3

$$r_1 = \hat{\mu} \cdot \left( \frac{S}{S + k_S} \right) \cdot X_B \quad r_2 = k_H \cdot \left( \frac{X_S / X_B}{X_S / X_B + k_X} \right) \cdot X_B \quad r_3 = b \cdot X_B$$

# Biological systems

	S	Xs	Xp	XB	rateo
p1	-1/Y			1	r1
p2	1	-1			r2
p3		(1-f)	f	-1	r3

$$r_1 = \hat{\mu} \cdot \left( \frac{S}{S+k_S} \right) \cdot X_B \quad r_2 = k_H \cdot \left( \frac{X_S/X_B}{X_S/X_B+k_X} \right) \cdot X_B \quad r_3 = b \cdot X_B$$

These are the relevant mass balances at steady state:

$$\left\{ \begin{array}{l} \frac{d[S]}{dt} = \frac{([S]_0 - [S])}{\theta_H} + \left[ -\frac{\hat{\mu}}{Y} \cdot \left( \frac{[S]}{[S]+k_S} \right) \cdot [X_B] + k_H \cdot \left( \frac{[X_S]/[X_B]}{[X_S]/[X_B]+k_X} \right) \cdot [X_B] \right] = 0 \\ \frac{d[X_S]}{dt} = \frac{(\theta_C/\theta_H \cdot [X_S]_0 - [X_S])}{\theta_C} + \left[ -k_H \cdot \left( \frac{[X_S]/[X_B]}{[X_S]/[X_B]+k_X} \right) \cdot [X_B] + (1-f) \cdot b \cdot [X_B] \right] = 0 \\ \frac{d[X_B]}{dt} = \frac{(0 - [X_B])}{\theta_C} + \left[ \hat{\mu} \cdot \left( \frac{[S]}{[S]+k_S} \right) \cdot [X_B] - b \cdot [X_B] \right] = 0 \\ \frac{d[X_P]}{dt} = \frac{(0 - [X_P])}{\theta_C} + \left[ f \cdot b \cdot [X_B] \right] = 0 \end{array} \right.$$

This is a non-linear system that can be solved numerically.



# Biological systems

To get an analytical simple solution, a simplified case can be usefully considered by neglecting the hydrolysis process thus considering that the influent wastewater contains biodegradable/soluble organics, unbiodegradable/soluble organics, unbiodegradable/particulate organics.

	S	X <sub>P</sub>	X <sub>B</sub>	rateo
p1	-1/Y		1	r1
p3		f	-1	r3

$$r_1 = \hat{\mu} \cdot \left( \frac{S}{S + k_S} \right) \cdot X_B$$

$$r_3 = b \cdot X_B$$

The system simplifies at steady state:

$$\left\{ \begin{array}{l} \text{eq.1: } \frac{d[S]}{dt} = \frac{([S]_0 - [S])}{\theta_H} + \left[ -\frac{\hat{\mu}}{Y} \cdot \left( \frac{[S]}{[S] + k_S} \right) \cdot [X_B] \right] = 0 \\ \text{eq.2: } \frac{d[X_B]}{dt} = \frac{(0 - [X_B])}{\theta_C} + \left[ \hat{\mu} \cdot \left( \frac{[S]}{[S] + k_S} \right) \cdot [X_B] - b \cdot [X_B] \right] = 0 \\ \text{eq.3: } \frac{d[X_P]}{dt} = \frac{(0 - [X_P])}{\theta_C} + \left[ f \cdot b \cdot [X_B] \right] = 0 \end{array} \right.$$

# Biological systems

This system is easily solved:

$$\text{eq. 2: } \frac{[X_B]}{\theta_C} = \left[ \hat{\mu} \cdot \left( \frac{[S]}{[S] + k_S} \right) \cdot [X_B] - b \cdot [X_B] \right] \quad \text{by simplifying } [X_B]:$$

$$\frac{1}{\theta_C} = \left[ \hat{\mu} \cdot \left( \frac{[S]}{[S] + k_S} \right) - b \right], \quad \text{solving for } [S]:$$

$$[S] = \frac{k_S \cdot (1 + b \cdot \theta_C)}{\theta_C \cdot (\hat{\mu} - b) - 1}$$

$$\text{eq. 1: } \frac{([S]_0 - [S])}{\theta_H} = \left[ \frac{\hat{\mu}}{Y} \cdot \left( \frac{[S]}{[S] + k_S} \right) \cdot [X_B] \right] = 0$$

$$\text{remebering that, from eq. (1): } \hat{\mu} \cdot \left( \frac{[S]}{[S] + k_S} \right) = \frac{1}{\theta_C} - b \quad \text{substituti ng :}$$

$$\frac{([S]_0 - [S])}{\theta_H} = \frac{1}{Y} \cdot \left( \frac{1}{\theta_C} - b \right) \cdot [X_B] \quad \text{solving for } [X_B]$$

$$[X_B] = \frac{\theta_C}{\theta_H} \cdot \frac{Y \cdot ([S]_0 - [S])}{1 + b \cdot \theta_C} = \frac{\theta_C}{1 + b \cdot \theta_C} \cdot \frac{Y \cdot ([S]_0 - [S])}{\theta_H}$$

# Biological systems

$$\text{eq. 3: } \frac{[X_P]}{\theta_C} = f \cdot b \cdot [X_B]$$

$$[X_P] = f \cdot b \cdot \theta_C \cdot [X_B]$$

In summary, the following expressions have been found

$$[S] = \frac{k_S \cdot (1 + b \cdot \theta_C)}{\theta_C \cdot (\hat{\mu} - b) - 1}$$

$$[X_B] = \frac{\theta_C}{\theta_H} \cdot \frac{Y \cdot ([S]_0 - [S])}{1 + b \cdot \theta_C}$$

$$[X_P] = f \cdot b \cdot \theta_C \cdot [X_B]$$

# Biological systems

$$[S] = \frac{k_S \cdot (1 + b \cdot \theta_C)}{\theta_C \cdot (\hat{\mu} - b) - 1}$$

The concentration of degradable substrate that is left at the end of the process is affected by:

- Kinetic parameters of biomass ( $k_S$  and  $b$ )
- The biomass retention time
- It does not depend on  $[S]_0$

$$[X_B] = \frac{\theta_C}{1 + b \cdot \theta_C} \cdot \frac{Y \cdot ([S]_0 - [S])}{\theta_H}$$

The biomass concentration in the system grows :

- with the treated organic load
- with  $\theta_C$ , up to an asymptotic value

$$[X_P] = f \cdot b \cdot \theta_C \cdot [X_B]$$

Biomass debris grows :

- with the decay rate
- with biomass concentration and thus with the treated load
- with  $\theta_C$

# Biological systems

The total amount of soluble and particulate organics in the system includes the inert fractions:  $[S_i]_0$ ,  $[X_i]_0$ . These components do not react, therefore their mass balance is simply:

$$\text{eq. 4: } \frac{d[S_i]}{dt} = \frac{([S_i]_0 - [S_i])}{\theta_H} = 0$$

$$\text{eq. 5: } \frac{d[X_i]}{dt} = \frac{(\theta_C / \theta_H \cdot [X_i]_0 - [X_i])}{\theta_C} = 0$$

therefore :

$$[S_i] = [S_i]_0$$

$$[X_i] = [X_i]_0 \cdot \frac{\theta_C}{\theta_H}$$

# Biological systems

Total soluble and particulate compounds are:

$$[S_T] = \frac{k_s \cdot (1 + b \cdot \theta_C)}{\theta_C \cdot (\hat{\mu} - b) - 1} + [S_i]_0$$

$$[X_T] = [X_B] + [X_P] + [X_i] = \frac{\theta_C}{\theta_H} \cdot \frac{Y \cdot ([S]_0 - [S])}{1 + b \cdot \theta_C} + f \cdot b \cdot \theta_C \cdot [X_B] + [X_i]_0 \cdot \frac{\theta_C}{\theta_H}$$

Note that the fraction of the particulate components that is made of active biomass decreases with increasing  $\theta_C$  for the higher accumulation of  $[X_P]$  e  $[X_i]$ :

$$\frac{[X_B]}{[X_T]} = \frac{\theta_C / \theta_H \cdot Y \cdot ([S]_0 - [S])}{(1 + f \cdot b \cdot \theta_C) \cdot \theta_C / \theta_H \cdot Y \cdot ([S]_0 - [S]) + [X_i]_0 \cdot (1 + b \cdot \theta_C)}$$

# Biological systems

The rate of sludge waste from the biological system, that equals the rate of accumulation of particulate matter, is calculated as the product of the waste flow rate  $Q_x (=V/\theta_c)$  per the particulate matter concentration in the system  $[X_T]$ .

$$F_x = \frac{V}{\theta_c} \cdot [X_T] = Q \cdot Y \cdot ([S]_0 - [S]) \cdot \frac{(1 + f \cdot b \cdot V)}{1 + b \cdot \theta_c} + Q \cdot [X_i]_0$$

The observed specific production of particulate organics referred to the amount of degraded organic matter, (observed yield:  $Y_{obs}$ ) is finally calculated as:

$$Y_{obs} = \frac{F_x}{Q \cdot ([S]_0 + [X_i]_0 - [S])} = \frac{Y \cdot (1 + f \cdot b \cdot \theta_c)}{1 + b \cdot \theta_c} \frac{([S]_0 - [S])}{([S]_0 + [X_i]_0 - [S])} + \frac{[X_i]_0}{([S]_0 + [X_i]_0 - [S])}$$

# Biological systems

$$Y_{obs} = \frac{F_X}{Q \cdot ([S]_0 + [X_i]_0 - [S])} = \frac{Y \cdot (1 + f \cdot b \cdot \theta_C)}{1 + b \cdot \theta_C} \frac{([S]_0 - [S])}{([S]_0 + [X_i]_0 - [S])} + \frac{[X_i]_0}{([S]_0 + [X_i]_0 - [S])}$$

The observed yield depends on:

- metabolic parameters of microorganisms that develop in the system:
  - it decreases with increasing decay rates (b);
  - It increases with increasing Y and f;
- The nature of the treated wastewater: specifically, it increases with decreasing WW degradability (degradable/total organics)
- Operational parameters: it decreases with increasing  $\theta_C$ ,



# Biological systems

## Oxygen demand for organic matter oxidation ( $F_{O_2}$ )

It comprises 2 terms: an endogenous and an exogenous request:

The exogenous oxygen ( $F_{O_2,es}$ ) request is proportional to the organic matter that is degraded (equal to:  $Q(S_0 - S)$ ) multiplied by the fraction of it that is oxidized (equal to  $(1 - Y)$ ).

According to ASM1 process 1:  $\frac{1}{Y_H} S_S + \left(\frac{1 - Y_H}{Y_H}\right) S_O + i_{XB} S_{NH} + \frac{i_{XB}}{14} S_{ALK} \rightarrow X_{BH}$

it follows that:  $\frac{\Delta S_O}{\Delta S_S} = \left(\frac{1 - Y_H}{Y_H}\right) / \frac{1}{Y_H} = (1 - Y_H)$

and therefore:  $F_{O_2,ex} = Q \cdot ([S]_0 - [S]) \cdot (1 - Y_H)$

# Biological systems

## Oxygen demand for organic matter oxidation ( $F_{O_2}$ )

The endogenous term ( $F_{O_2, \text{end}}$ ): it is proportional to the rate of production of soluble organic matter from biomass decay and following hydrolysis of the released  $X_S$ .

$$F_{O_2, \text{end}} = V \cdot \frac{\Delta S_O}{\Delta S_S} \cdot \left( \frac{dS_S}{dt} \right)_{\text{from biomass decay}}$$

let us assume that between biomass decay and hydrolysis, the first one is the slowest  
 $\Rightarrow$  the rate of S release is proportional to the rate of biomass decay:

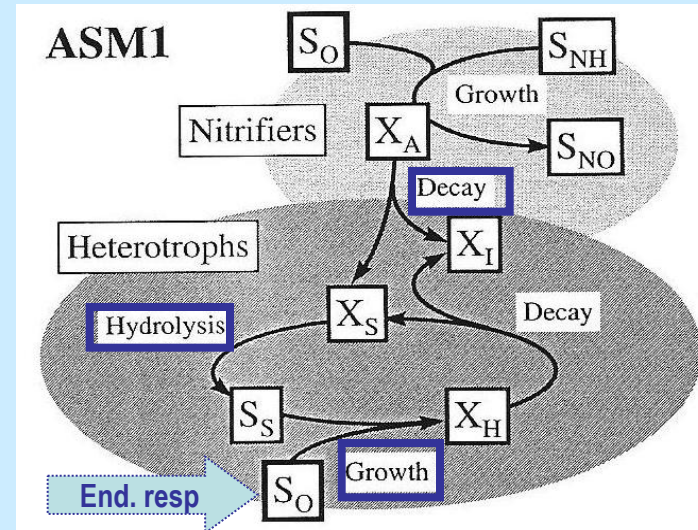
$$\left( \frac{dS_S}{dt} \right)_{\text{from biomass decay}} = \left( \frac{dX_{BH}}{dt} \right)_{\text{decay}} \cdot \left( \frac{\Delta X_S}{\Delta X_{BH}} \right)_{\text{decay}} \cdot \left( \frac{\Delta S_S}{\Delta X_S} \right)_{\text{hydrolysis}}$$

By expressing each term according to the ASM1 model:

$$\Rightarrow \left( \frac{dS_S}{dt} \right)_{\text{from biomass decay}} = b_H \cdot X_{BH} \cdot (1 - f_P) \quad (1)$$

and therefore:  $F_{O_2, \text{end}} = V \cdot (1 - Y_H) \cdot b_H \cdot X_{BH} \cdot (1 - f_P)$

## Endogenous respiration according to ASM1



# Biological systems

Some comments on **biomass retention time**. Its value affects:

- the biomass concentration in the system

$$[X_B] = \frac{\theta_C}{1 + b \cdot \theta_C} \cdot \frac{Y \cdot ([S]_0 - [S])}{\theta_H} \quad \text{if } \theta_C \uparrow \text{ then } [X_B] \uparrow$$

- the un-degraded soluble organic matter [S] that exit with the treated water and therefore the removal efficiency:

$$\eta = 1 - [S]/[S]_0 = 1 - \frac{k_s \cdot (1 - b \cdot \theta_C)}{[S]_0 \cdot \theta_C \cdot (\hat{\mu} - b) - 1} \quad \text{if } \theta_C \uparrow \text{ then } \eta \uparrow$$

- the nature of the waste sludge:

$$\frac{[X_B]}{[X_T]} = \frac{Y \cdot ([S]_0 - [S])}{Y \cdot (1 + f \cdot b \cdot \theta_C) \cdot ([S]_0 - [S]) + [X_i]_0 \cdot (1 + b \cdot \theta_C)} \quad \text{if } \theta_C \uparrow \text{ then } [X_B]/[X_T] \downarrow$$

Since  $[X_B]$  is the most fermentable fraction of the whole sludge components

if  $\theta_C \uparrow \rightarrow$  the sludge is more stabilized

# Biological systems

Some comments on **biomass retention time**. Its value affects:

- The amount of waste sludge

$$Y_{obs} = \frac{F_X}{Q \cdot ([S]_0 + [X_i]_0 - [S])} = \frac{Y \cdot (1 + f \cdot b \cdot \theta_C)}{1 + b \cdot \theta_C} \frac{([S]_0 - [S])}{([S]_0 + [X_i]_0 - [S])} + \frac{[X_i]_0}{([S]_0 + [X_i]_0 - [S])}$$

if  $\theta_C \uparrow$  then  $Y_{obs} \uparrow$

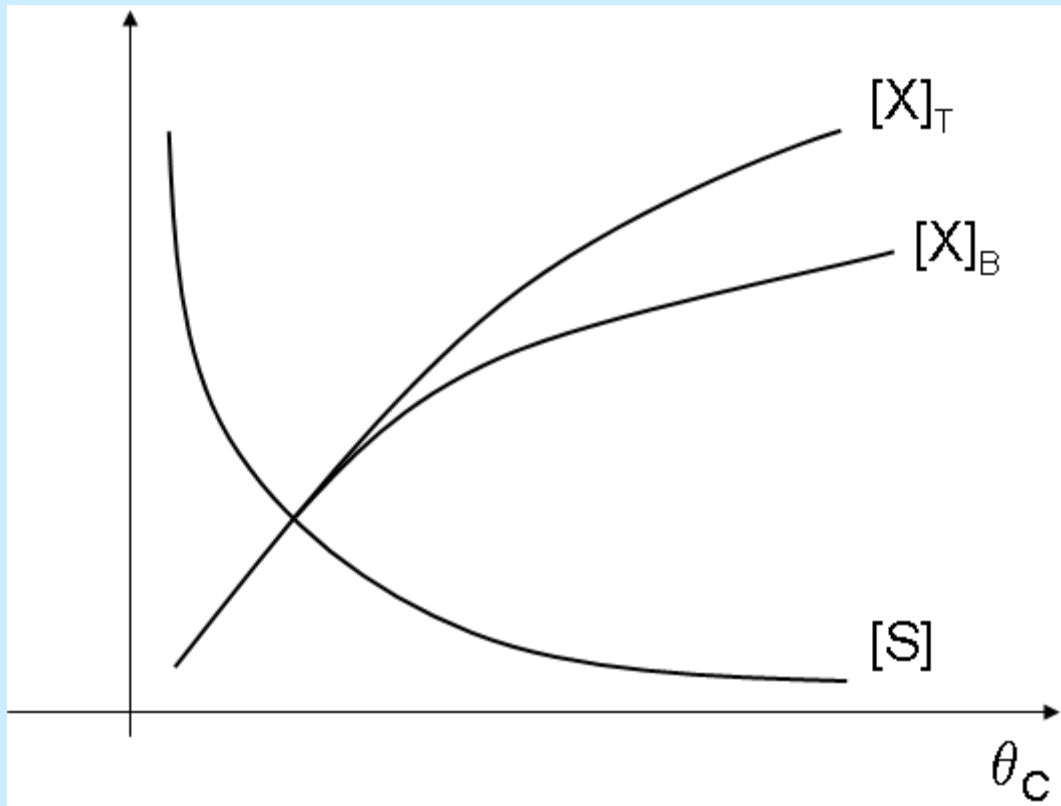
- the oxygen request:

$$F_{O_2} = F_{O_2,end} + F_{O_2,ex} = Q \cdot ([S]_0 - [S]) \cdot \left( -Y_H \right) + V \cdot (1 - Y_H) \cdot b_H \cdot X_{BH} \cdot \left( -f_P \right)$$

if  $\theta_C \uparrow$  then  $F_{O_2} \uparrow$

# Biological systems

Qualitative graph describing the dependence of the main system components from the biomass residence time:



# Biological systems

- The biomass residence time should be sufficiently high to allow for biomass growth in the system. If microorganisms do not remain in the reactor at least for their duplication time, they would leave the system too early to give birth to a new generation → microorganisms wash-out will inexorably occur.
- How long is this **minimum value**  $\theta_{C,\min}$ ?

*wash-out condition: if  $\theta_C \rightarrow \theta_{C,\min} \Rightarrow [X_B] \rightarrow 0 \Rightarrow [S] \rightarrow [S]_0$*

$$\text{From eq.2 } \frac{1}{\theta_C} = \left[ \hat{\mu} \cdot \left( \frac{[S]}{[S] + k_S} \right) - b \right]$$

*but, for  $\theta_C = \theta_{C,\min}$  then  $[S] = [S]_0$*

$$\frac{1}{\theta_{C,\min}} = \left[ \hat{\mu} \cdot \left( \frac{[S]_0}{[S]_0 + k_S} \right) - b \right]$$

*under the assumption that :  $[S]_0 \gg k_S$*

$$\frac{1}{\theta_{C,\min}} = \hat{\mu} - b$$

*The evacuation rate = the net growth rate*

# Biological systems

## Simplified procedure for a basic design of the biological reactor:

Data:

- wastewater characteristics:  $Q$ ,  $[S]_0$ ,  $[X]_0$ ,  $[S_i]_0$ ,  $[X_i]_0$
- design effluent concentrations :  $[S]_T = [S] + [S_i]$
- maximum biomass concentration that is admissible (depending on the solid/separation unit):  $[X]_{Tmax}$ ,

We want to calculate:

- the volume of the biological reactor,
- the amount of waste sludge that is produced and needs further treatment,
- the amount of oxygen that has to be dissolved by the aeration system

1. The minimum  $\theta_C$  that is requested to achieve the desired  $[S]_T$ ,

$$[S]_T = \frac{k_S \cdot (1 + b \cdot \theta_C)}{\theta_C \cdot (\hat{\mu} - b) - 1} + [S_i]_0 \Rightarrow \theta_C = \frac{k_S + ([S]_T - [S_i]_0)}{([S]_T - [S_i]_0) \cdot (\hat{\mu} - b) - k_S \cdot b}$$

# Biological systems

2. The minimum value of  $\theta_H$  that corresponds to the maximum  $[X]_T$  e :

$$[X_T] = [X_B] + [X_P] + [X_i] = \frac{\theta_C}{\theta_H} \cdot \frac{Y \cdot ([S]_0 - [S])}{1 + b \cdot \theta_C} (1 + f \cdot b \cdot \theta_C) + [X_i]_0 \cdot \frac{\theta_C}{\theta_H}$$

$$\theta_H = \frac{\theta_C}{[X]_T} \cdot \left[ \frac{Y \cdot ([S]_0 - ([S]_T - [S_i]_0))}{1 + b \cdot \theta_C} (1 + f \cdot b \cdot \theta_C) + [X_i]_0 \right]$$

3. The reactor minimum reactor volume corresponding to the minimum hydraulic residence time:

$$V = Q \cdot \theta_H$$

4. The waste sludge production:

$$F_X = Q \frac{Y \cdot ([S]_0 - [S])}{1 + b \cdot \theta_C} + f \cdot b \cdot V \cdot [X_B] + Q \cdot [X_i]_0$$

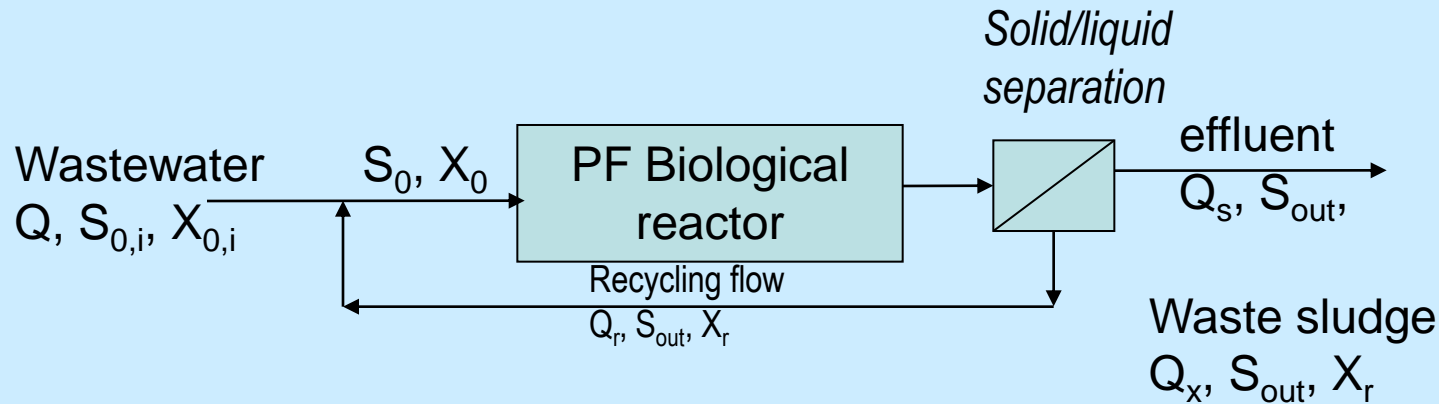
5. The oxygen request:

$$F_{O_2} = Q \cdot ([S]_0 - [S]) \cdot (-Y_H) + V \cdot (1 - Y_H) \cdot b_H \cdot X_{BH} \cdot (-f_P)$$



# Biological systems

What if the reactor hydrodynamics is more similar to a plug flow reactor?



Along the PF reactor:

- soluble degradable organic matter has a decreasing gradient from the inlet to the outlet
- the concentration of un-degradable species does not change ( $S_i$ ,  $X_i$ )
- *Hyp*: the concentration gradient of biomass  $X_B$  is negligible in most operational conditions (*Actual average concentration is two order of magnitude higher than the amount that grows across the reactor for biomass synthesis*)

# Biological systems

This means that :

- the reactor behaviour is PF for [S]
- is like a CFSTR for all other components

Mass balances are therefore:

$$eq.1 : \frac{d[S]}{r_C} = \frac{d[S]}{\left[ -\frac{\hat{\mu}}{Y} \cdot \left( \frac{[S]}{[S] + k_S} \right) \cdot [X_B] \right]} = d\theta$$

$$eq.2 : 0 = \frac{(0 - [X_B])}{\theta_C} + \left[ \hat{\mu} \cdot \left( \frac{[S]}{[S] + k_S} \right) \cdot [X_B] - b \cdot [X_B] \right]$$

$$eq.3 : 0 = \frac{(0 - [X_P])}{\theta_C} + \left[ \mu \cdot b \cdot [X_B] \right] = 0$$

# Biological systems

Biomass concentration is again:

$$[X_B] = \frac{\theta_C}{\theta_H} \cdot \frac{Y \cdot ([S]_0 - [S]_{outPF})}{1 + b \cdot \theta_C}$$

When integrating eq. 1 :

- $[X]_B$  is assumed as constant along the PF reactor;
- Inlet substrate concentration ( $[S]_{in}$ ) results from the mixing of the inflow and the recycle flow :

$$[S]_{in} = [S]_0 \cdot \frac{1}{1+r} + [S] \cdot \frac{r}{1+r}$$

- the actual residence time for the PF reactor is:  $\theta_H' = \theta_H / (1+r)$ , where  $\theta_H = V/Q$  as usual.

Therefore:

$$\frac{d[S]}{r_C} = \frac{d[S]}{\left[ -\frac{\hat{\mu}}{Y} \cdot \left( \frac{[S]}{[S] + k_S} \right) \cdot [X_B] \right]} = d\theta$$

$$-dS \cdot \left( \frac{[S] + k_S}{[S]} \right) = \frac{\hat{\mu}}{Y} \cdot [X_B] \cdot d\theta$$

# Biological systems

$$-dS \cdot \left( \frac{K_S + [S]}{[S]} \right) = \frac{\hat{\mu}}{Y} \cdot [X_B] \cdot d\theta$$

By integrating from the inlet section  $[S] = [S]_{in}$ , and  $\theta = 0$

and the outlet section:  $[S] = [S]_{outPF}$ , and  $\theta = \theta_H'$  :

$$([S]_{in} - [S]_{outPF}) + K_S \cdot \ln \frac{[S]_{in}}{[S]_{outPF}} = \frac{\hat{\mu}}{Y} \cdot [X_B] \cdot \theta_H'$$

by substituting:  $[X_B] = \frac{\theta_C}{\theta_H} \cdot \frac{Y \cdot ([S]_0 - [S]_{outPF})}{1 + b \cdot \theta_C}$

and:  $\theta_H' = \frac{\theta_H}{1+r}$  and by solving for  $\theta_C$

$$\frac{1}{\theta_C} = \frac{\hat{\mu} \cdot ([S]_0 - [S]_{outPF})}{(1+r) \cdot \left[ ([S]_{in} - [S]_{outPF}) + K_S \cdot \ln \frac{[S]_{in}}{[S]_{outPF}} \right]} - b$$

$[S]_{outPF}$ , can not be explicitly calculated, it depends on  $\theta_C$ , but also from  $[S]_0$  (unlike for CFSTR reactor). Generally, the outlet concentration is lower, the reactor is more efficient, but is also more sensitive to accidental toxicity in the influent.

# Software dynamic simulation

They allow for the setting of:

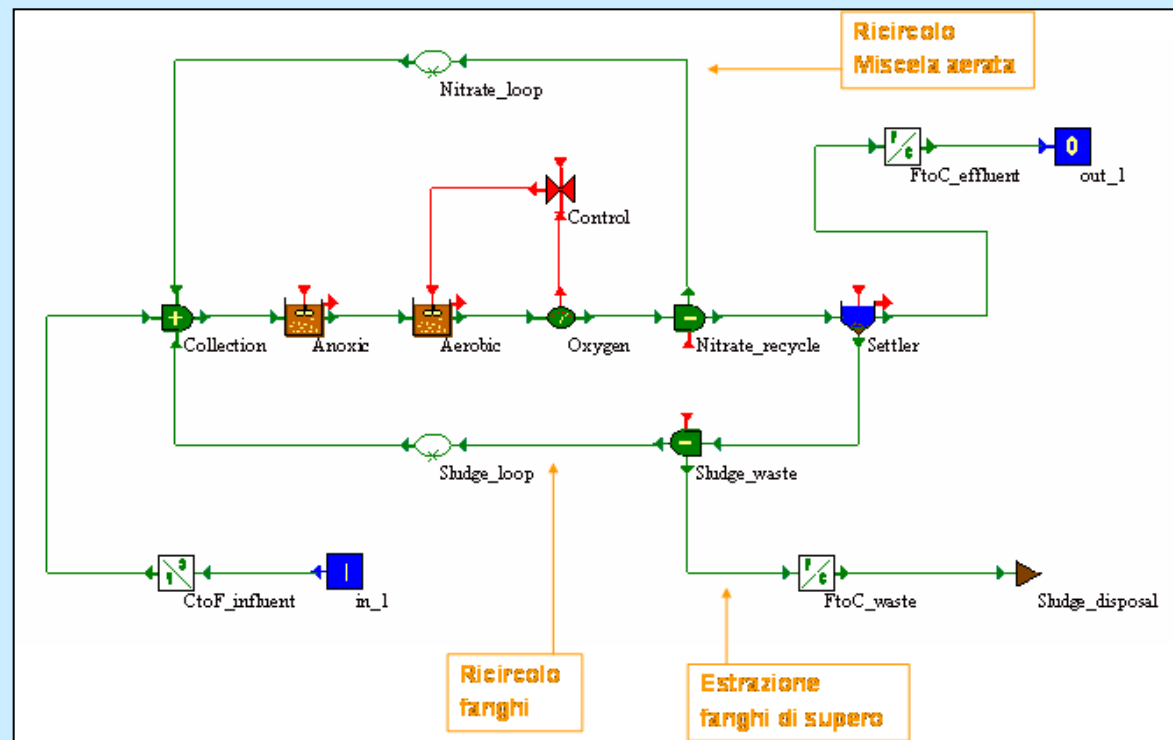
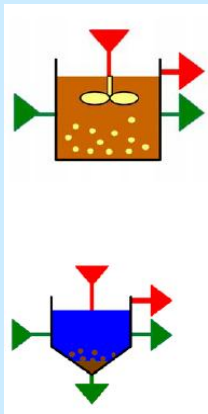
- the hydraulic connections of various reactors
- for each reactor:
  - hydraulic behaviour
  - reactivity (Petersen Matrix):

They allow to:

- Create the set of differential equations that describe the dynamic response of the system
- Integrate it numerically

# Software dynamic simulation

**Example:** WEST (Worldwide Engine For Simulation, Training And Automation)



# Software dynamic simulation

The screenshot displays the WEST Model Editor interface. The main window shows the 'Petersen Matrix' for the 'AerGrowthHetero' process. The matrix is structured as follows:

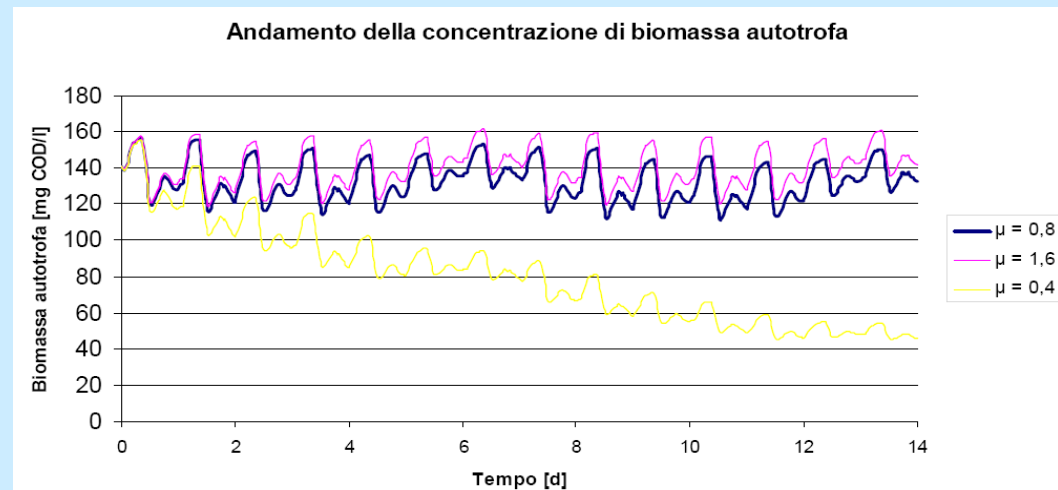
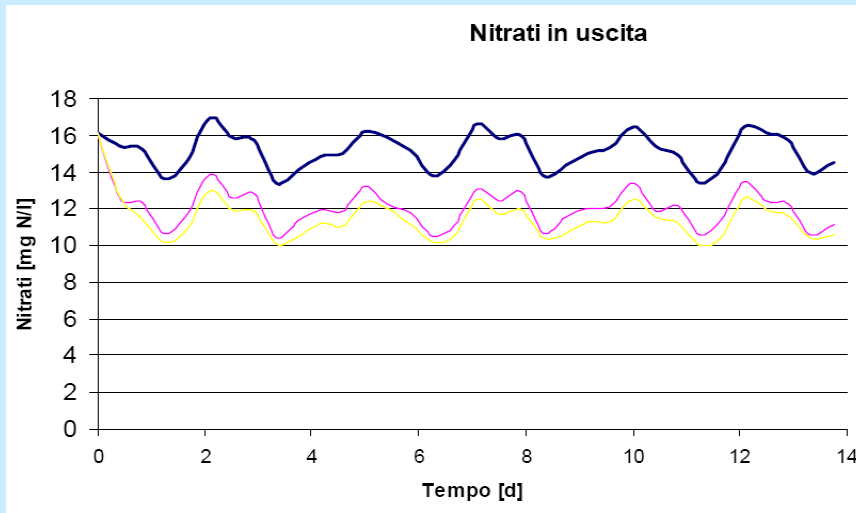
PROCESS	COMPONENTS			PROCESS RATE
	WATER	SOLUBLES	PARTICULATES	
	H2O	S_S	X_BH	
AerGrowthHetero	-1/(Y_H)	-(1-Y_H)/Y_H	1	$\mu_H \cdot C[S_S] / (K_S + C[S_S]) \cdot C[S_O] / (K_{OH} + C[S_O])$
DecayOfHetero			-1	$b_H \cdot C[X_{BH}]$
Aeration		1		$Kla\_Actual \cdot (S_{O\_Sat} - C[S_O])$

Two dialog boxes are overlaid on the main window:

- View Component Group:** A dialog box with checkboxes for component groups: WATER (checked), SOLUBLES (checked), PARTICULATES (checked), GASES (unchecked), OTHER (unchecked), and TSS (checked). Each checked item has a corresponding color swatch and a 'Text' button.
- Define TSS:** A dialog box with two radio buttons: 'Calculate TSS component' (selected) and 'Use TSS component' (unselected). The 'Calculate TSS component' section has 'Begin' and 'End' dropdown menus, both set to 'X\_BH'. A remark at the bottom states: 'Remark : Only components of the PARTICULATES group can be used as TSS component.'

# Software dynamic simulation

Output: dynamic simulation





# Software dynamic simulation

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