Mettre un (tout petit) bruit dans les modèles de dynamique des populations

Claude Lobry
Inria Modémic

Dans un chémostat

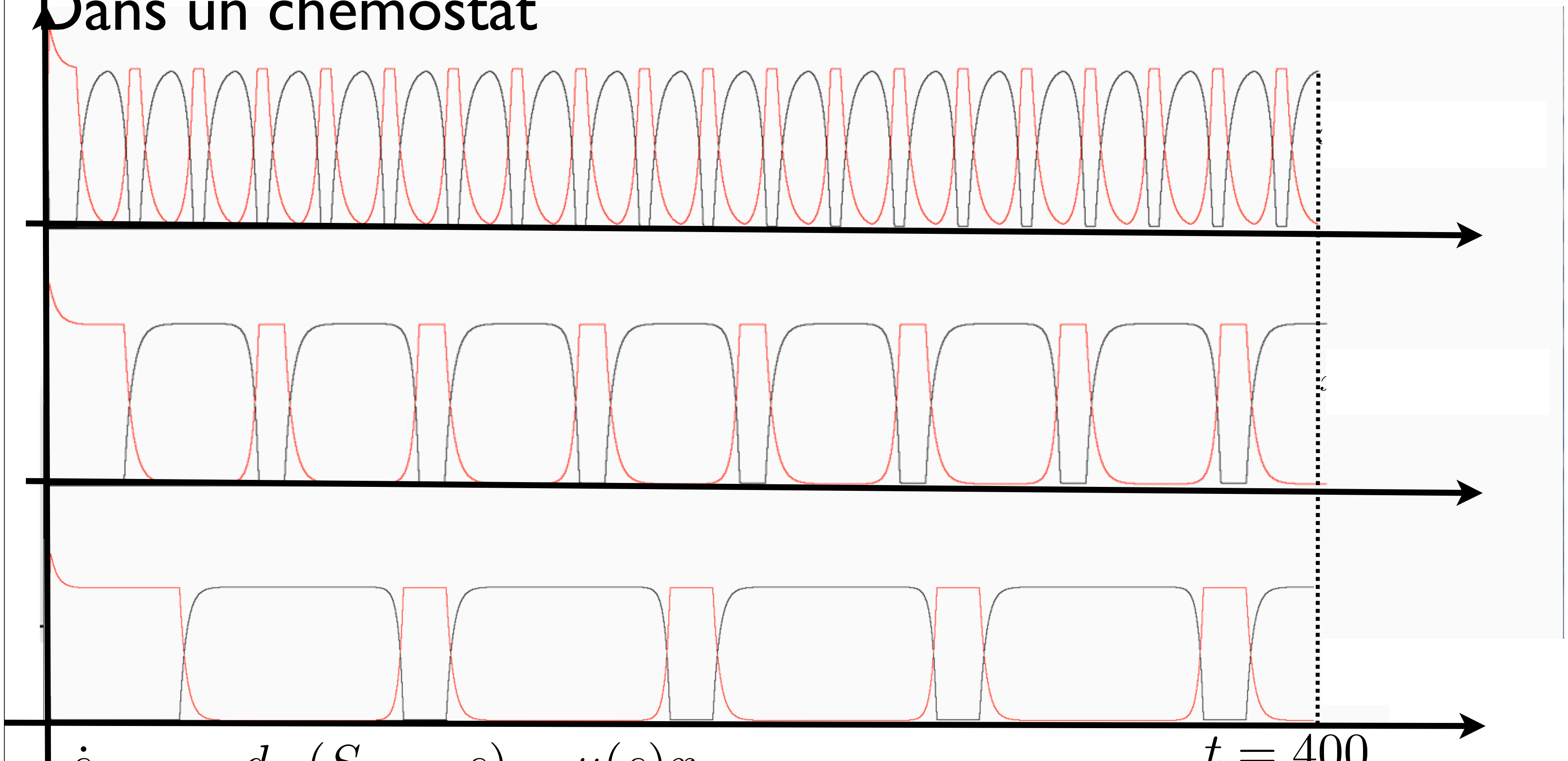
$$\begin{aligned}\dot{s} &= d \cdot (S_{in} - s) - \mu(s)x \\ \dot{x} &= (\mu(s) - d) \cdot x\end{aligned}$$

Jour

$$\begin{aligned}\dot{s} &= d \cdot (S_{in} - s) - \cancel{\mu(s)}x \\ \dot{x} &= (\cancel{\mu}(s) - d) \cdot x\end{aligned}$$

Nuit

Dans un chémostat

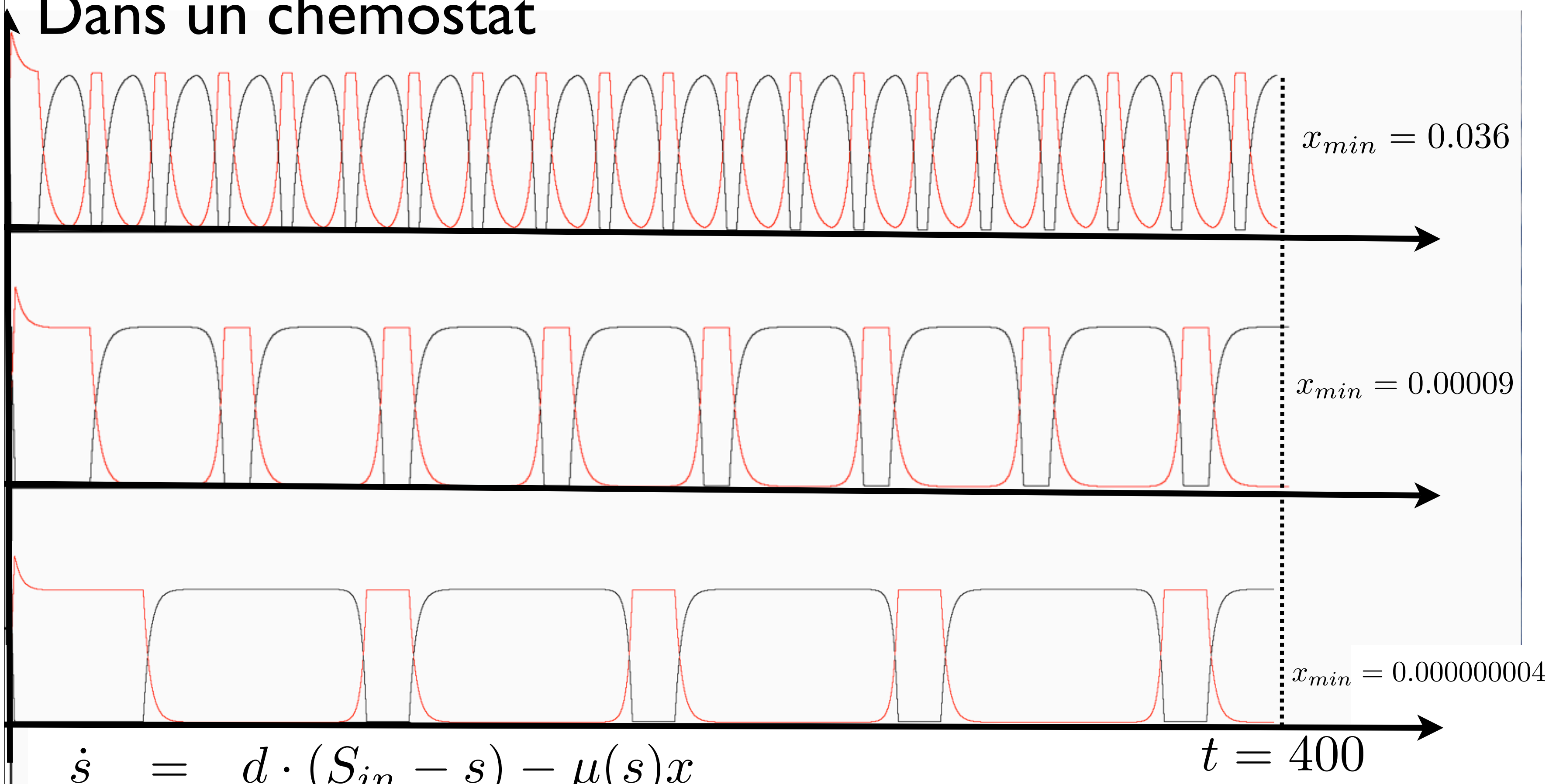


$$\dot{s} = d \cdot (S_{in} - s) - \mu(s)x$$
$$\dot{x} = (\mu(s) - d) \cdot x$$

$$\dot{s} = d \cdot (S_{in} - s) - \cancel{\mu(s)}x$$
$$\dot{x} = (\cancel{\mu(s)} - d) \cdot x$$

$$\dot{s} = 0.4 \cdot (2 - s) - \frac{s}{0.02 + s} \cdot x$$
$$\dot{x} = (\frac{s}{0.02 + s} - 0.4) \cdot x$$

Dans un chémostat



$$\dot{s} = d \cdot (S_{in} - s) - \mu(s)x$$

$$\dot{x} = (\mu(s) - d) \cdot x$$

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$$\dot{s} = 0.4 \cdot (2 - s) - \frac{s}{0.02 + s} \cdot x$$

$$\dot{x} = (\frac{s}{0.02 + s} - 0.4) \cdot x$$

$t \mapsto x(t)$ ne n'est jamais nul *stricto sensu*.

Zéro machine $= 10^{-240}$

Mettre dans les programmes :

If $x <$ seuil then x := 0.

Pour les petites populations il faut prendre :

$$x(t) = N(t)$$

où N est un entier

La croissance exponentielle

Proba de naissance

$$P\left(N(t+dt) = \left\{\begin{array}{l} N(t)+1 \\ N(t) \\ N(t)-1 \end{array}\right\}\right) \begin{array}{l} = N(t)pdt + o(dt) \\ = 1 - N(t)(p+q))dt + o(dt) \\ = N(t)qdt + o(dt) \end{array}$$

Proba de mort

Pour les petites populations il faut prendre :

$$x(t) = N(t)$$

où N est un entier

La croissance exponentielle

$$P\left(N(t+dt) = \left\{\begin{array}{l} N(t)+1 \\ N(t) \\ N(t)-1 \end{array}\right\}\right) \begin{array}{l} = N(t)pdt + o(dt) \\ = 1 - N(t)(p+q))dt + o(dt) \\ = N(t)qdt + o(dt) \end{array}$$

$$N(t+dt) = N(t) + W$$

$$W = \begin{pmatrix} +1 \\ 0 \\ -1 \end{pmatrix} \quad \begin{array}{rcl} P(W=+1) & = & N(t)pdt + o(dt) \\ P(W=\ \ 0) & = & 1 - N(t)(p+q)dt + o(dt) \\ P(W=-1) & = & N(t)qdt + o(dt) \end{array}$$

La croissance exponentielle

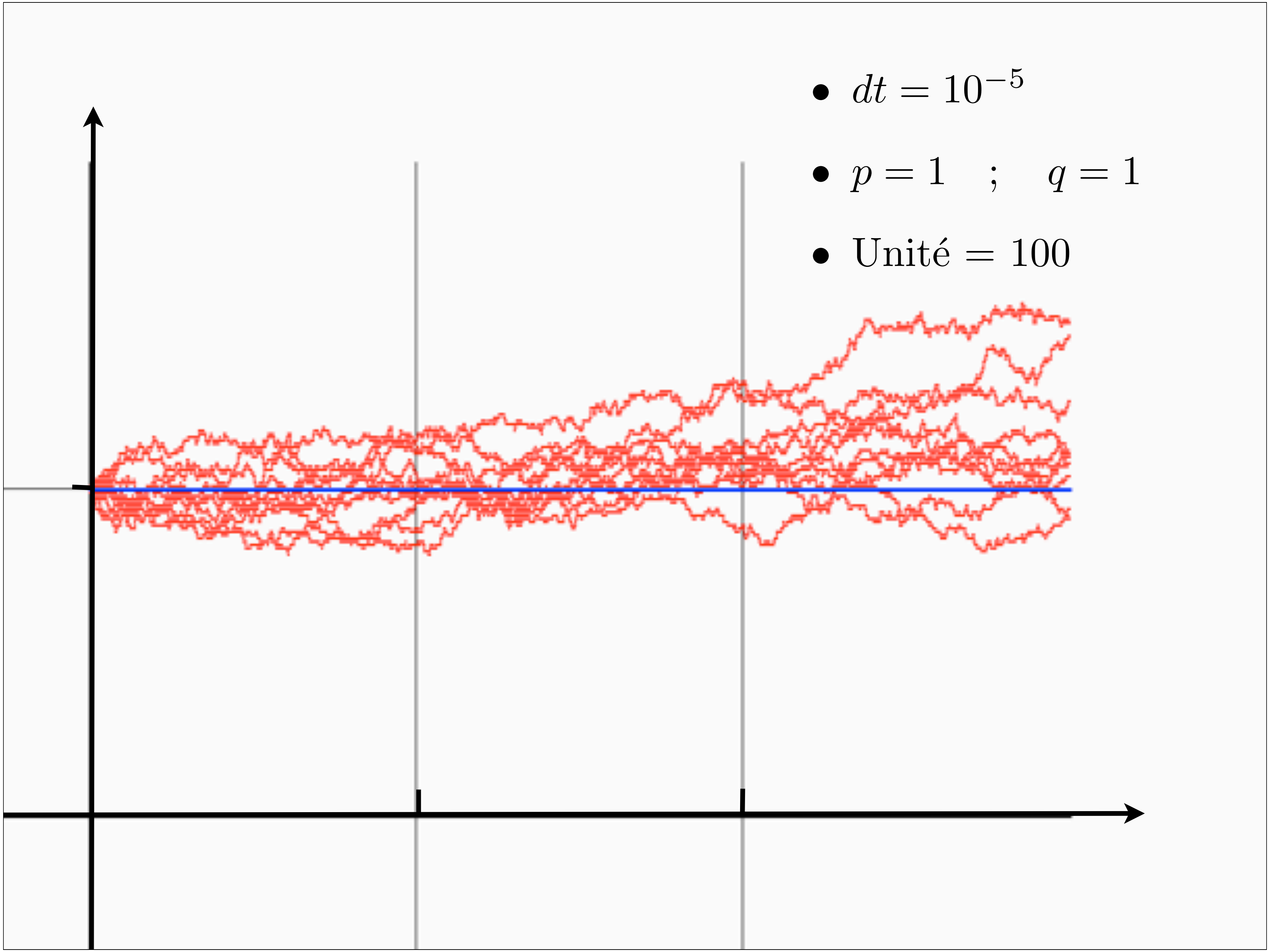
$$N(t+dt) = N(t) + W$$

$$W = \begin{pmatrix} +1 \\ 0 \\ -1 \end{pmatrix} \qquad \begin{aligned} P(W=+1) &= N(t)p\,dt + o(dt) \\ P(W=\;\;0) &= 1 - N(t)(p+q)dt + o(dt) \\ P(W=-1) &= N(t)q\,dt + o(dt) \end{aligned}$$

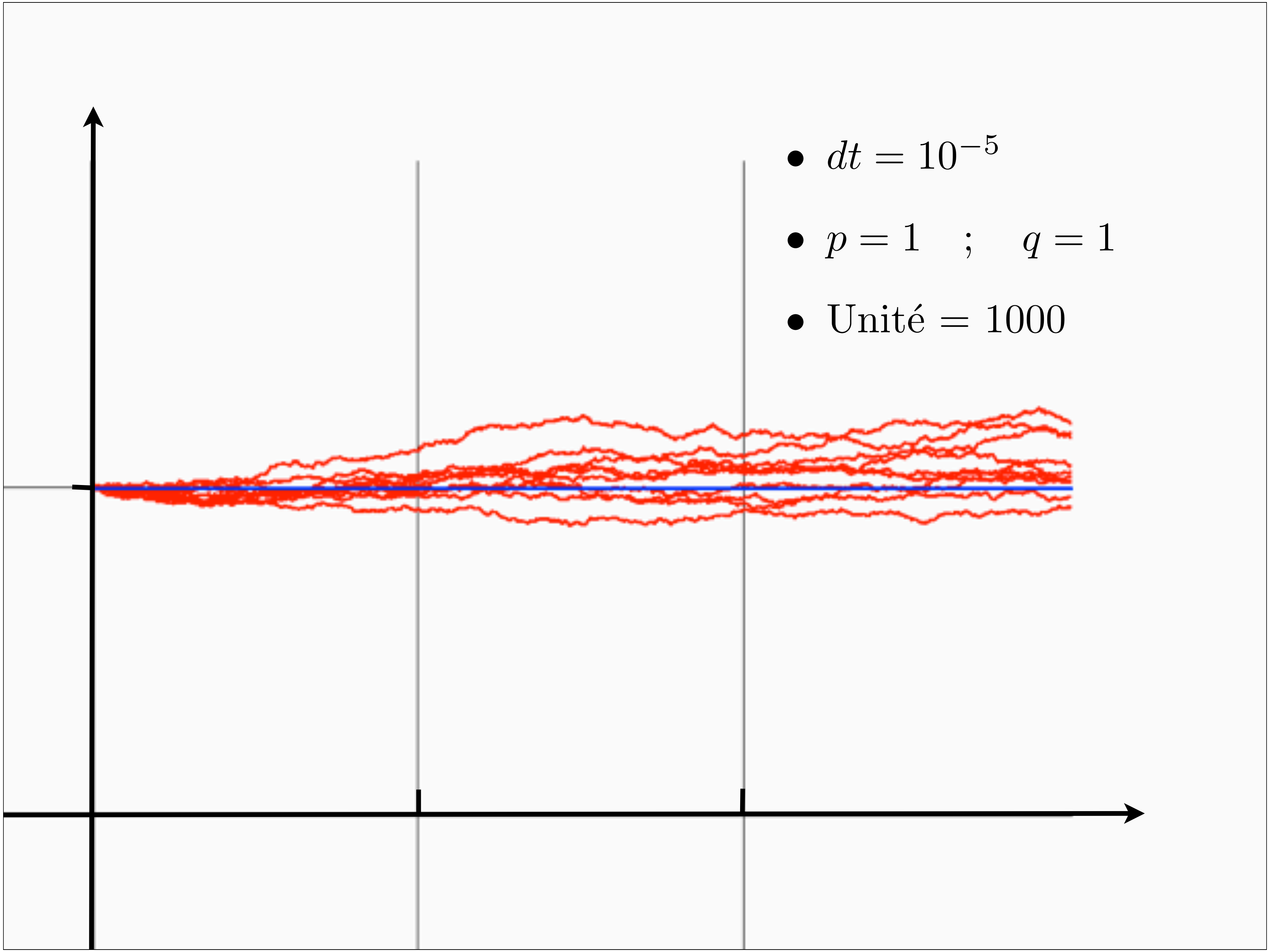
$$E[N(t+dt)] = E[N(t)] + E[N(t)](p-q)dt$$

$$m(t) = E[N(t)] \Longrightarrow m'(t) = (p-q)m(t)$$

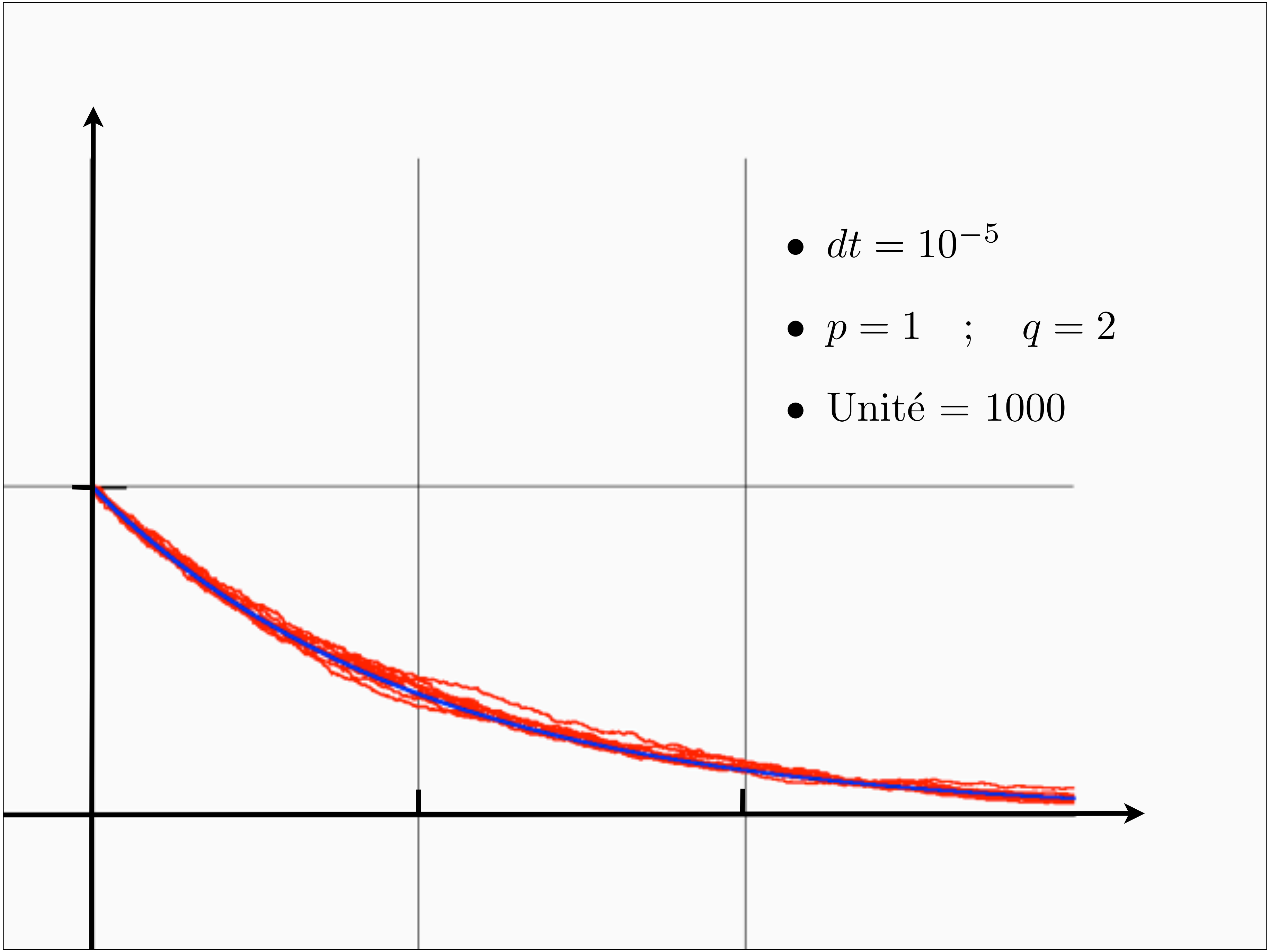
Dépend de la différence des 2 paramètres

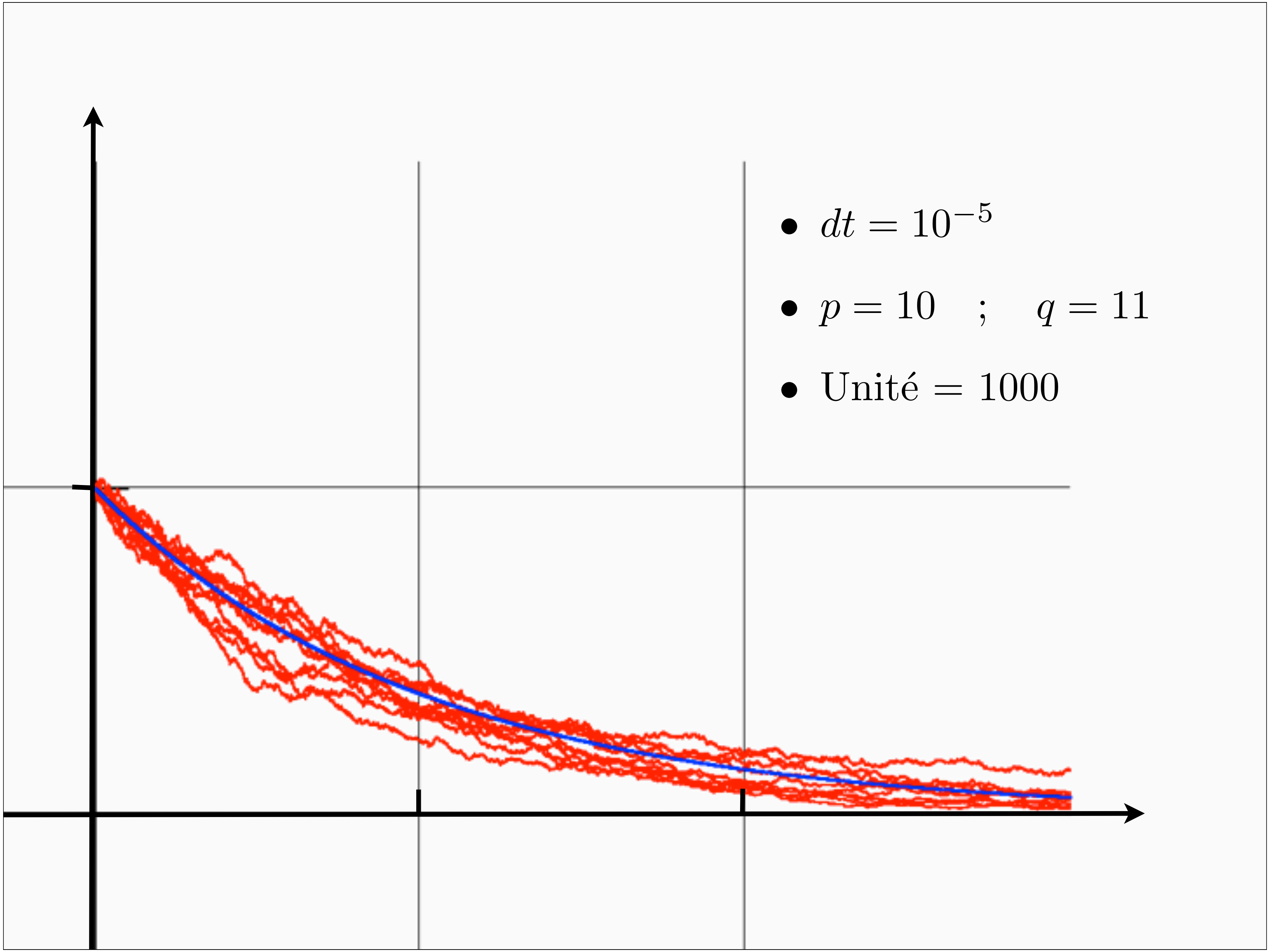


• $dt = 10^{-5}$
• $p = 1 \quad ; \quad q = 1$
• Unité = 100

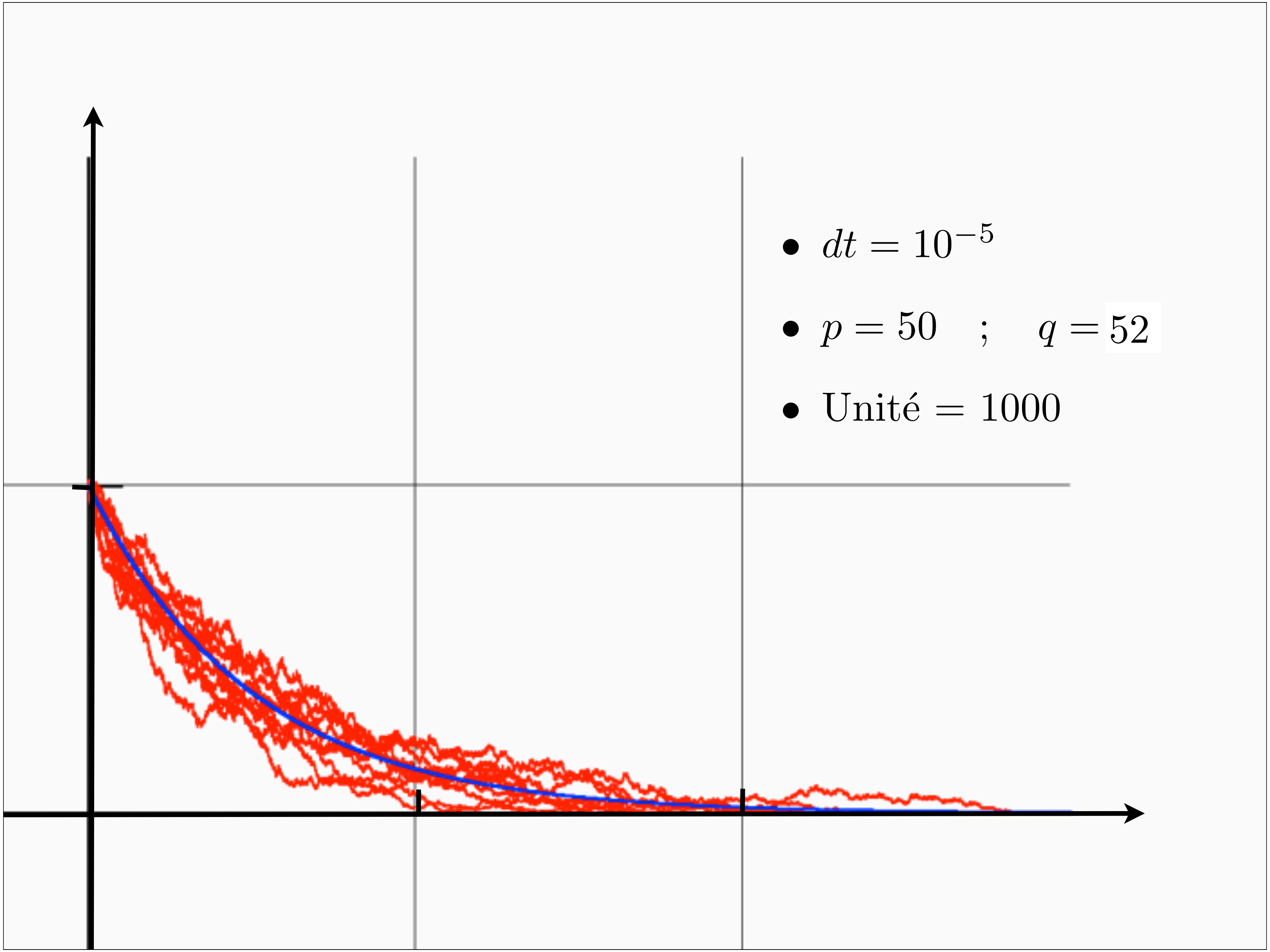


• $dt = 10^{-5}$
• $p = 1$; $q = 1$
• Unité = 1000


• $dt = 10^{-5}$
• $p = 1 \quad ; \quad q = 2$
• Unité $= 1000$



- $dt = 10^{-5}$
- $p = 10 \quad ; \quad q = 11$
- Unité $= 1000$

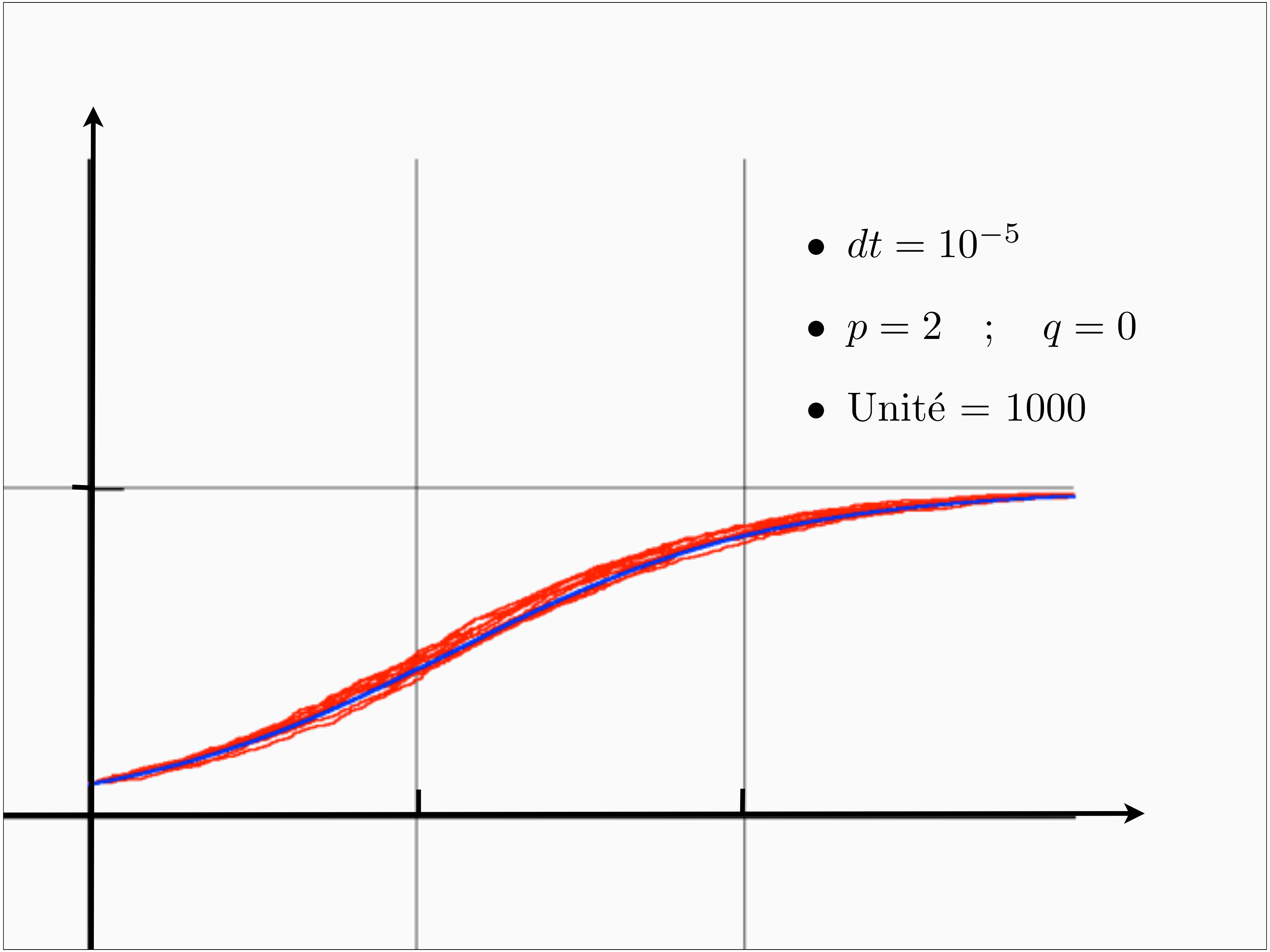

• $dt = 10^{-5}$
• $p = 50$; $q = 52$
• Unité = 1000

Logistique Vie-et-Mort

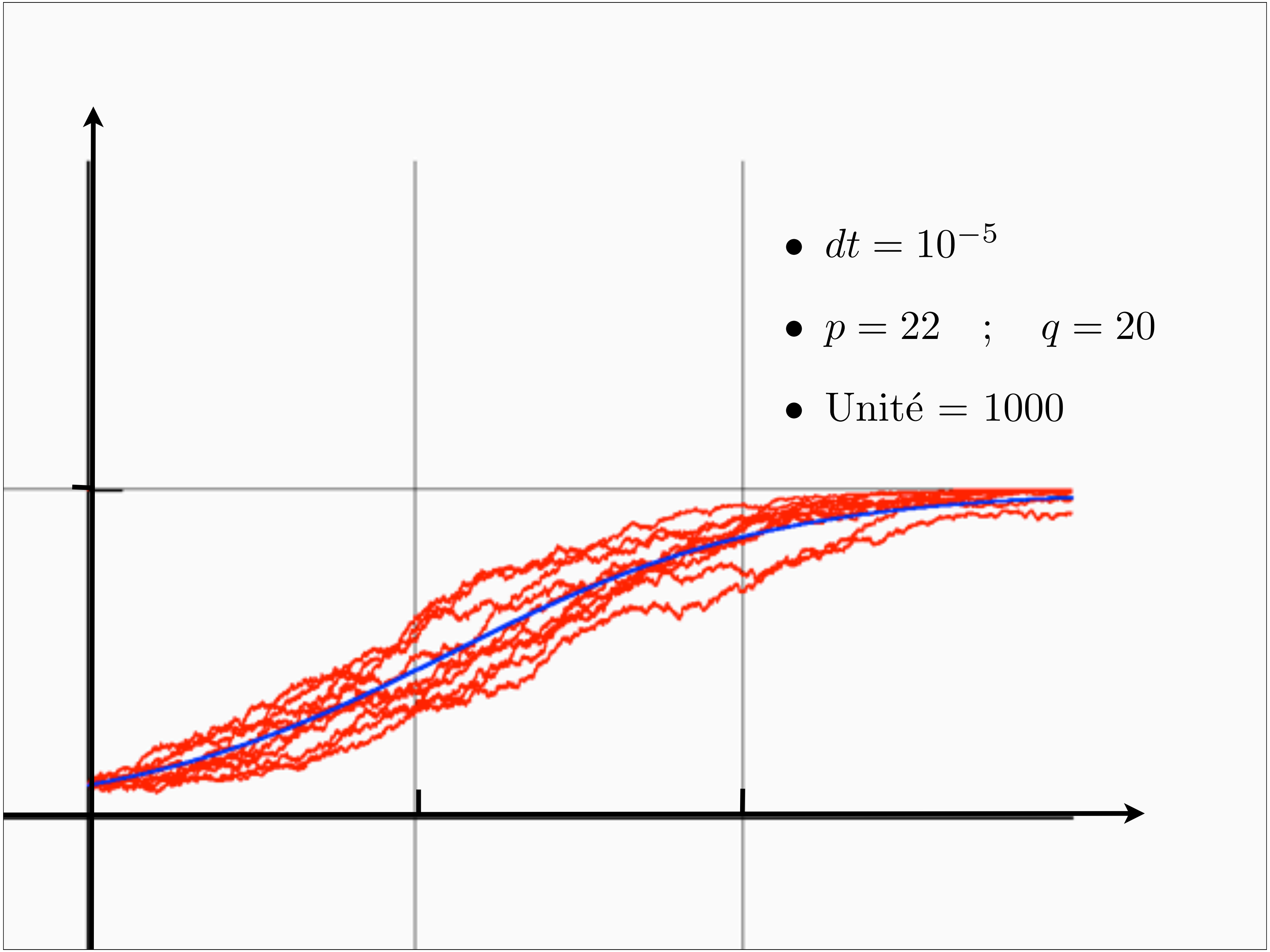
$$\dot{x} \quad = \quad rx(1-x)$$

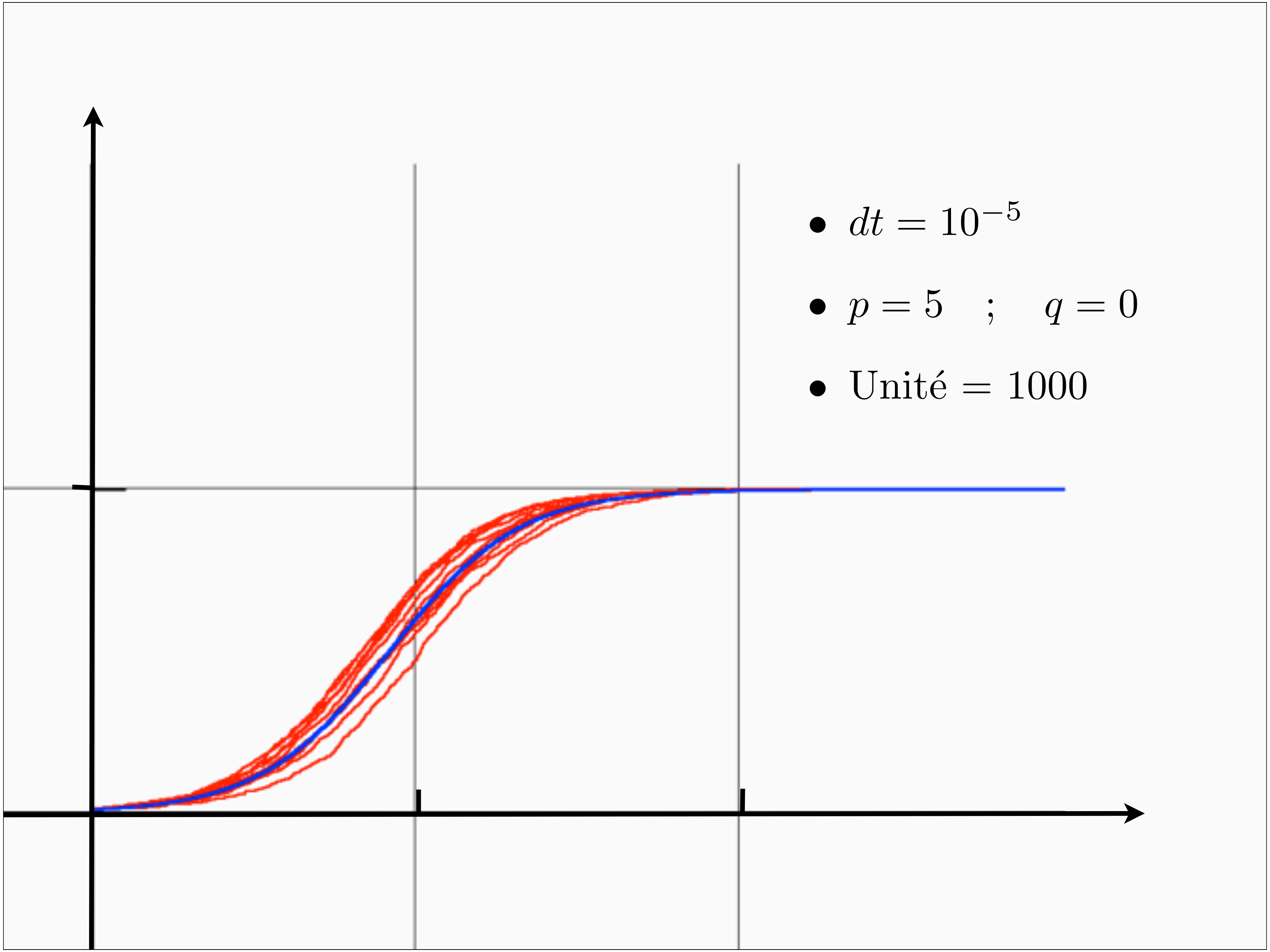
$$P\left(N(t+dt) = \left\{\begin{array}{l} N(t)+1 \\ N(t) \\ N(t)-1 \end{array}\right\}\right) \begin{array}{l} = N(t)pdt + o(dt) \\ = 1 - N(t)(p+q))dt + o(dt) \\ = N(t)qdt + o(dt) \end{array}$$

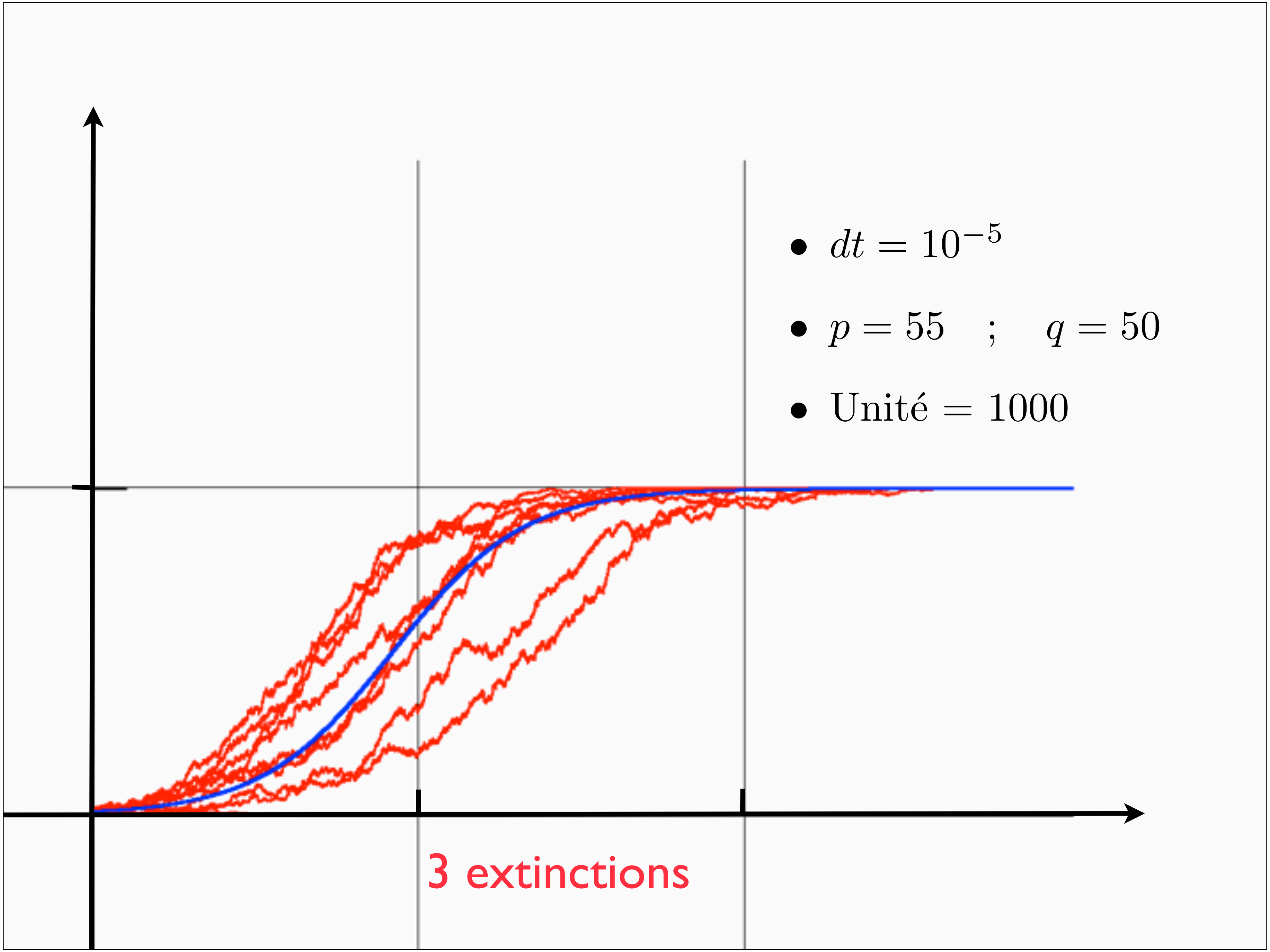
$$P\left(N(t+dt) = \left\{\begin{array}{l} N(t)+1 \\ N(t) \\ N(t)-1 \end{array}\right\}\right) \begin{array}{l} = N(t)(1-N(t)/Nb)pdt + o(dt) \\ = 1 - N(t)(1-N(t)/Nb)(p+q))dt + o(dt) \\ = N(t)(1-N(t)/Nb)qdt + o(dt) \end{array}$$



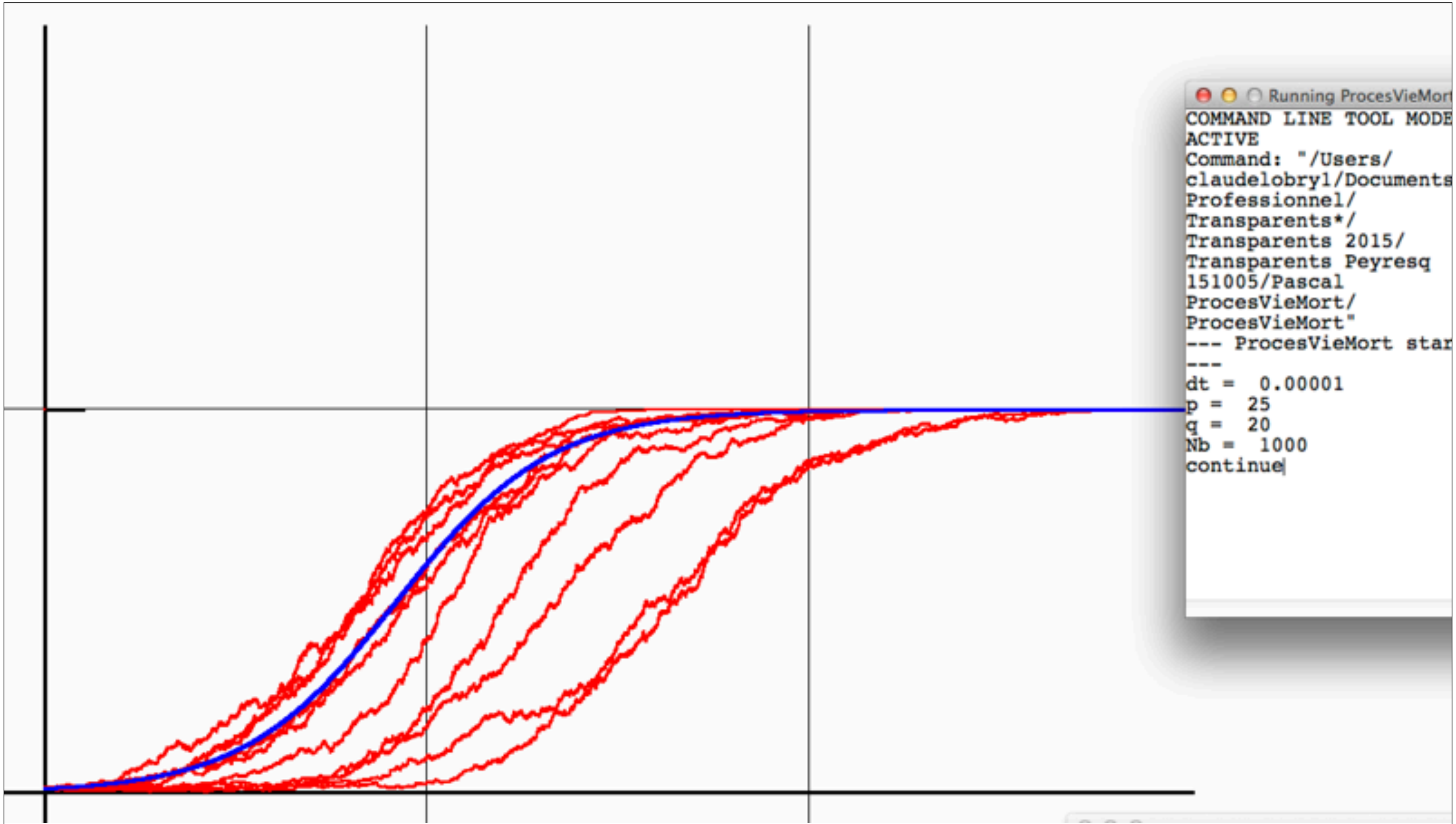
• $dt = 10^{-5}$
• $p = 2 \quad ; \quad q = 0$
• Unité = 1000

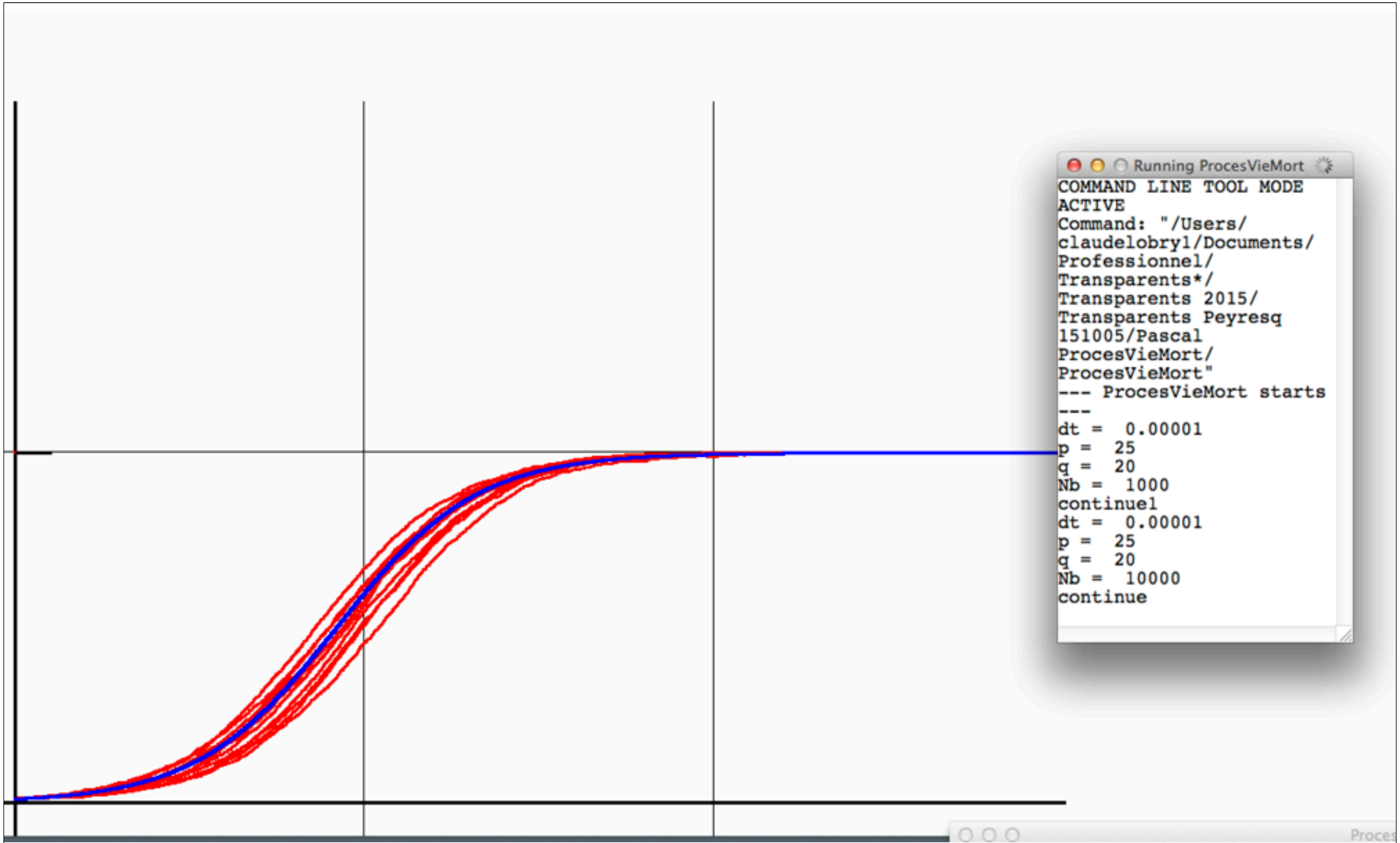

$dt = 10^{-5}$
$p = 22 \quad ; \quad q = 20$
Unité $= 1000$


$dt = 10^{-5}$
$p = 5 \quad ; \quad q = 0$
Unité $= 1000$

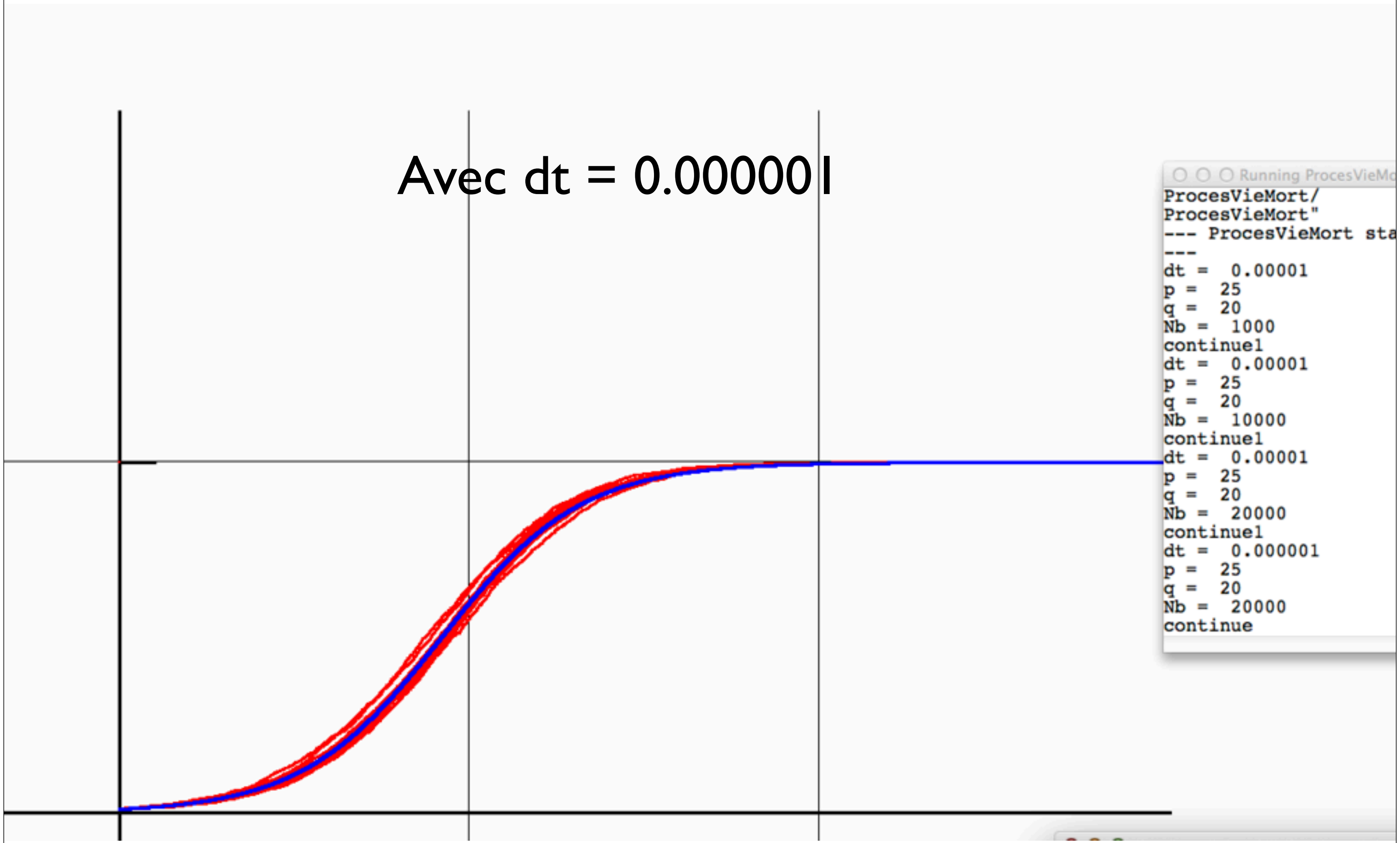


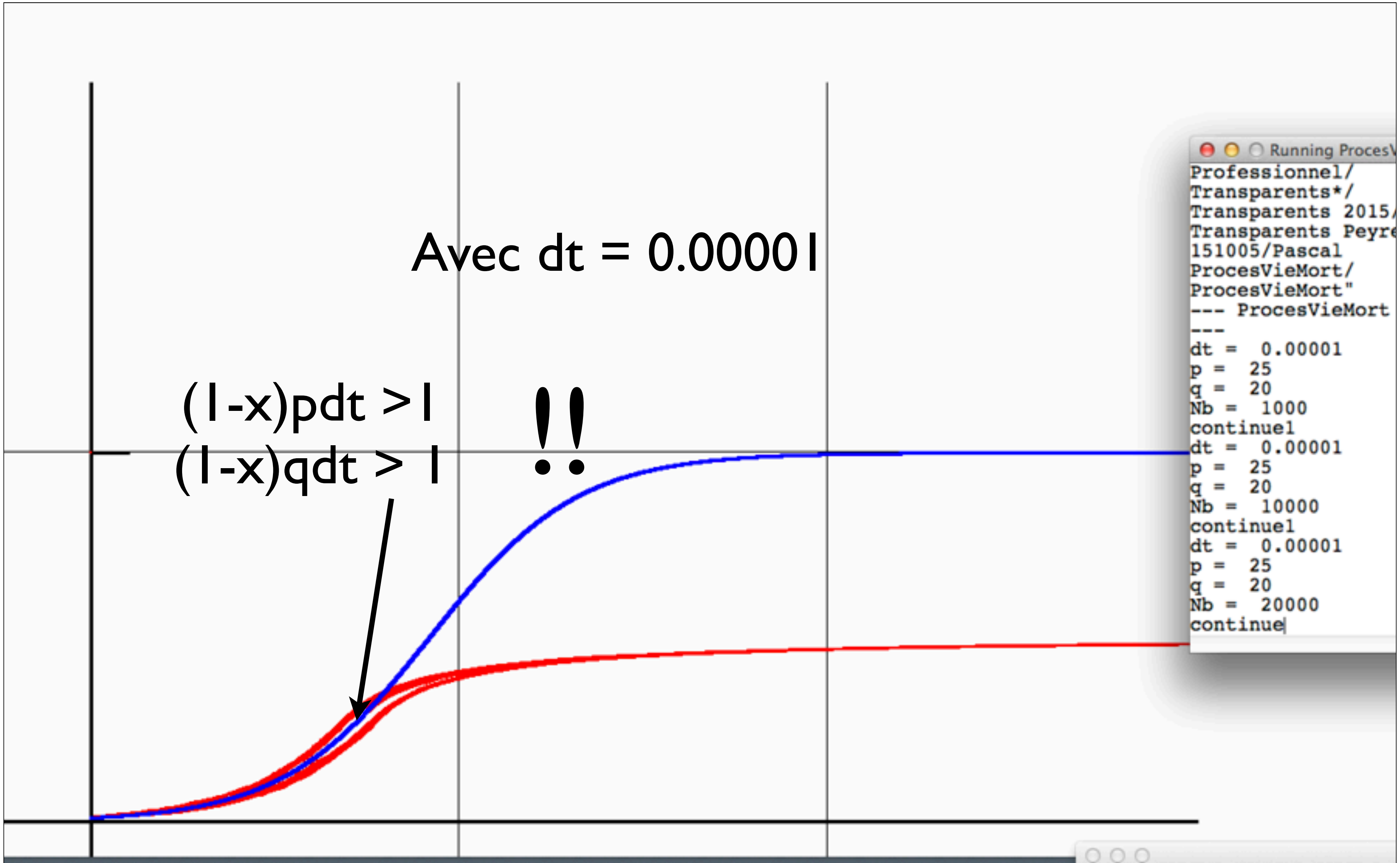
• $dt = 10^{-5}$
• $p = 55$; $q = 50$
• Unité $= 1000$
3 extinctions


Running ProcesVieMort
COMMAND LINE TOOL MODE
ACTIVE
Command: "/Users/
claudelobry1/Documents
Professionnel/
Transparents*/
Transparents 2015/
Transparents Peyresq
151005/Pascal
ProcesVieMort/
ProcesVieMort"
--- ProcesVieMort star
---
dt = 0.00001
p = 25
q = 20
Nb = 1000
continue


Running ProcesVieMort
COMMAND LINE TOOL MODE
ACTIVE
Command: "/Users/
claudelobry1/Documents/
Professionnel/
Transparents*/
Transparents 2015/
Transparents Peyresq
151005/Pascal
ProcesVieMort/
ProcesVieMort"
--- ProcesVieMort starts
---
dt = 0.00001
p = 25
q = 20
Nb = 1000
continue1
dt = 0.00001
p = 25
q = 20
Nb = 10000
continue

Avec dt = 0.000001



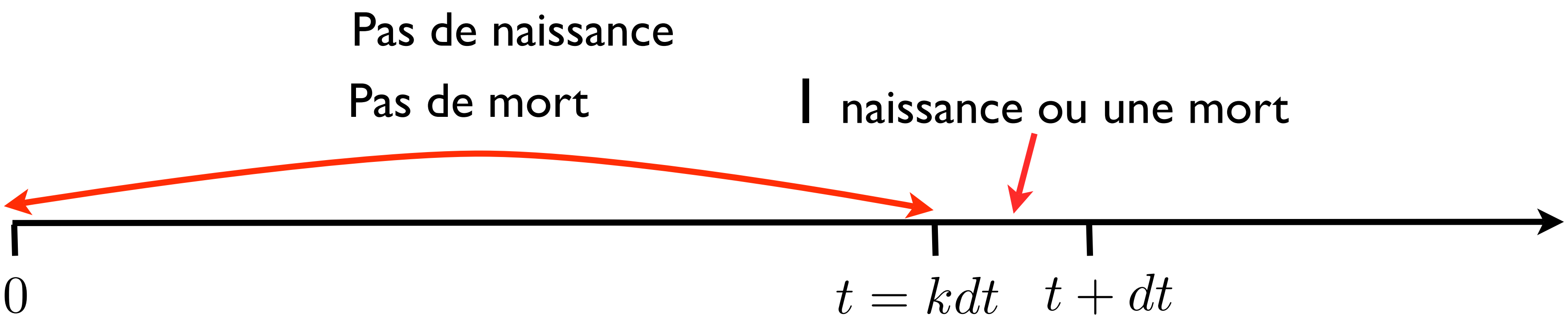

Avec dt = 0.00001
(1-x)pdt >1
(1-x)qdt > 1
!!
Running ProcesV
Professionnel/
Transparents*/
Transparents 2015/
Transparents Peyre
151005/Pascal
ProcesVieMort/
ProcesVieMort"
--- ProcesVieMort
---
dt = 0.00001
p = 25
q = 20
Nb = 1000
continue1
dt = 0.00001
p = 25
q = 20
Nb = 10000
continue1
dt = 0.00001
p = 25
q = 20
Nb = 20000
continue

On transforme “vie et mort” en un processus équivalent dit de Feller-Gillespie

$$P\left(N(t+dt) = \left\{\begin{array}{l} N(t)+1 \\ N(t) \\ N(t)-1 \end{array}\right\}\right) \begin{array}{l} = N(t)pdt + o(dt) \\ = 1 - N(t)(p+q))dt + o(dt) \\ = N(t)qdt + o(dt) \end{array}$$

T v.a. “ attente du prochain évènement “

Prochain évènement à l’instant t $= P(T \in [t; t+dt[)$



$$P(t \in [t, t+dt]) \approx \left(1 - \frac{N(t)(p+q)dt}{-}\right)^k \cdot N(t)(p+q)dt$$

$$P(T \in [t, t+dt]) = \int_t^{t+dt} N(t)(p+q)\,\mathrm{e}^{-N(t)(p+q)s} ds$$

$$P\left(N(t+dt)=\left\{\begin{array}{l} N(t)+1 \\ N(t) \\ N(t)-1 \end{array}\right\}\right) \begin{array}{l} = N(t)pdt+o(dt) \\ = 1-N(t)(p+q))dt+o(dt) \\ = N(t)qdt+o(dt) \end{array}$$

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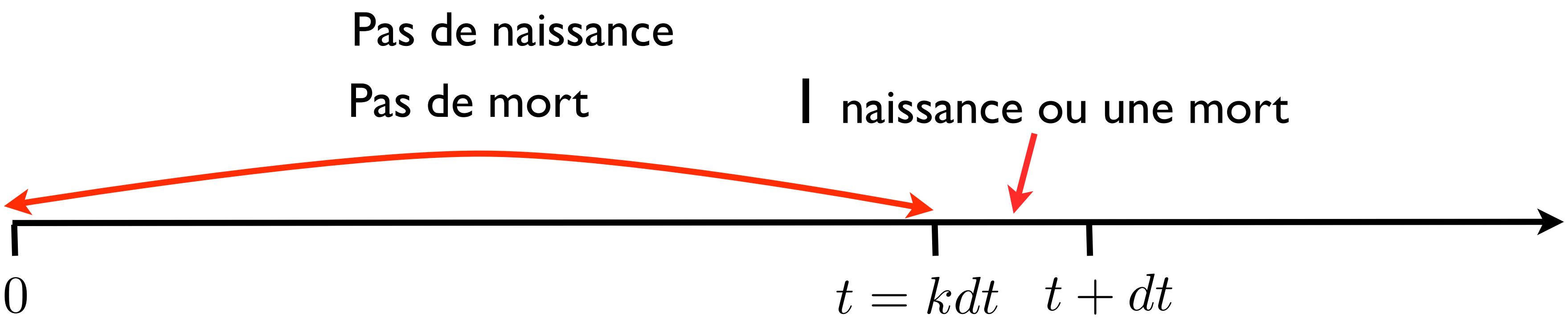


$$P(t \in [t, t+dt]) \approx \left(1 - \frac{N(t)(p+q)dt}{k}\right)^k \cdot N(t)(p+q)dt$$

$$P(T \in [t, t+dt]) \approx \int_t^{t+dt} N(t)(p+q)\,\mathrm{e}^{-N(t)(p+q)s}ds$$

$$P(T \in [t, t+dt]) = \int_t^{t+dt} N(t)(p+q)\, \mathrm{e}^{-N(t)(p+q)s} ds$$

à la limite (ou pour dt i.p.) *T* suit une loi exponentielle de paramètre

$$\lambda = N(t)(p+q)$$

$$E[T] = \frac{1}{\lambda} = \frac{1}{N(t)(p+q)}$$

$$\sigma^2(t) = \frac{1}{\lambda^2} = \frac{1}{(N(t)(p+q))^2}$$

Processus de Feller-Gillespie

C'est un algorithme :

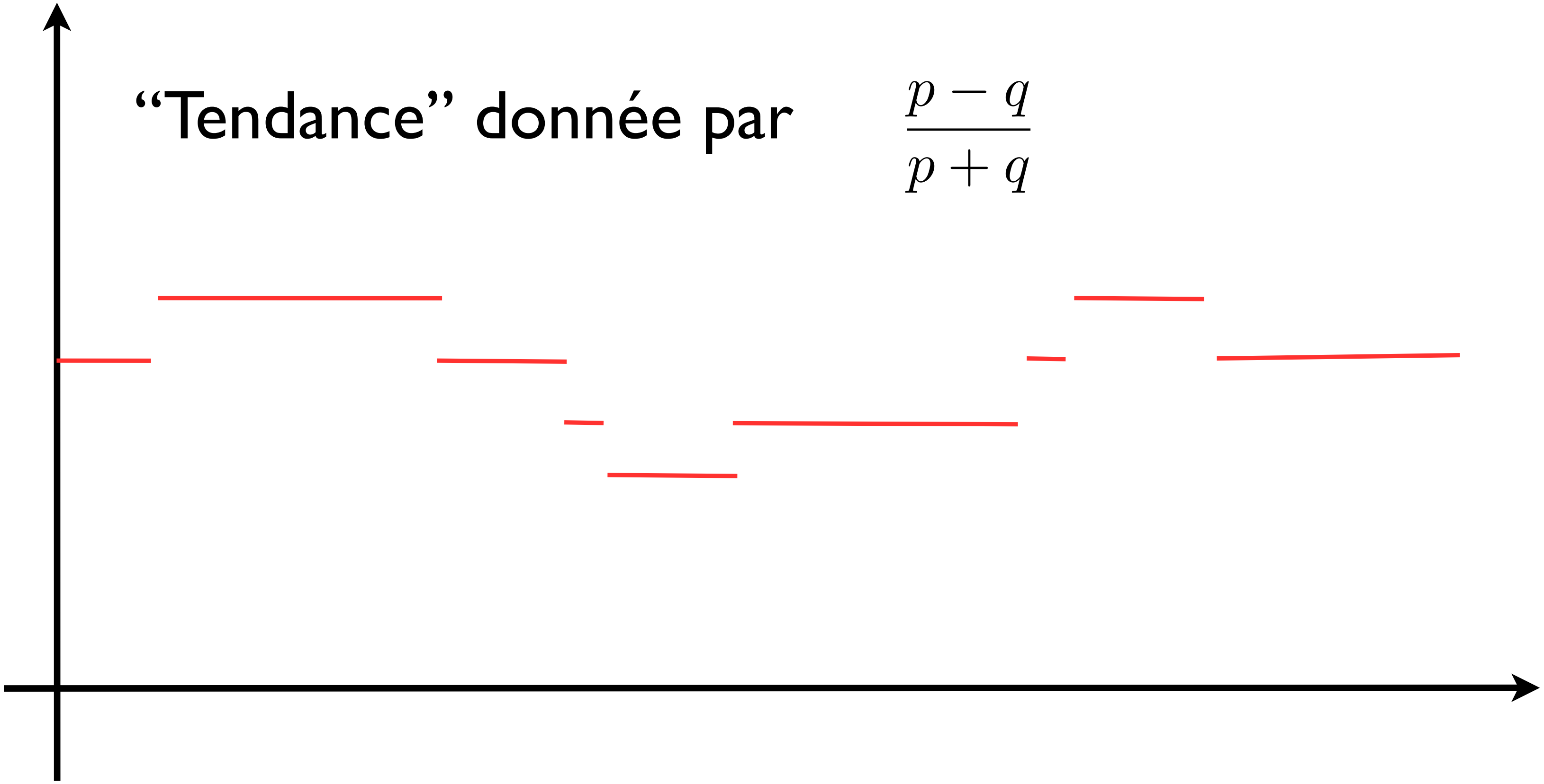
1. $t \leftarrow 0$;
2. $N \leftarrow N(0)$;
3. 0n tire T selon une loi exponentielle de paramètre $N(0)(p+q)$;
4. $W \leftarrow +1$ ou -1 selon $p/(p+q)$ et $q/(p+q)$;
5. $t \leftarrow T$;
6. $\leftarrow T + W$
7. On recommence...

Processus de Feller-Gillespie

Temps moyen entre deux évènements de l'ordre de $\dfrac{1}{N(t)(p+q)}$

"Fréquence" $\approx N(t)(p+q)$

"Tendance" donnée par $\dfrac{p-q}{p+q}$

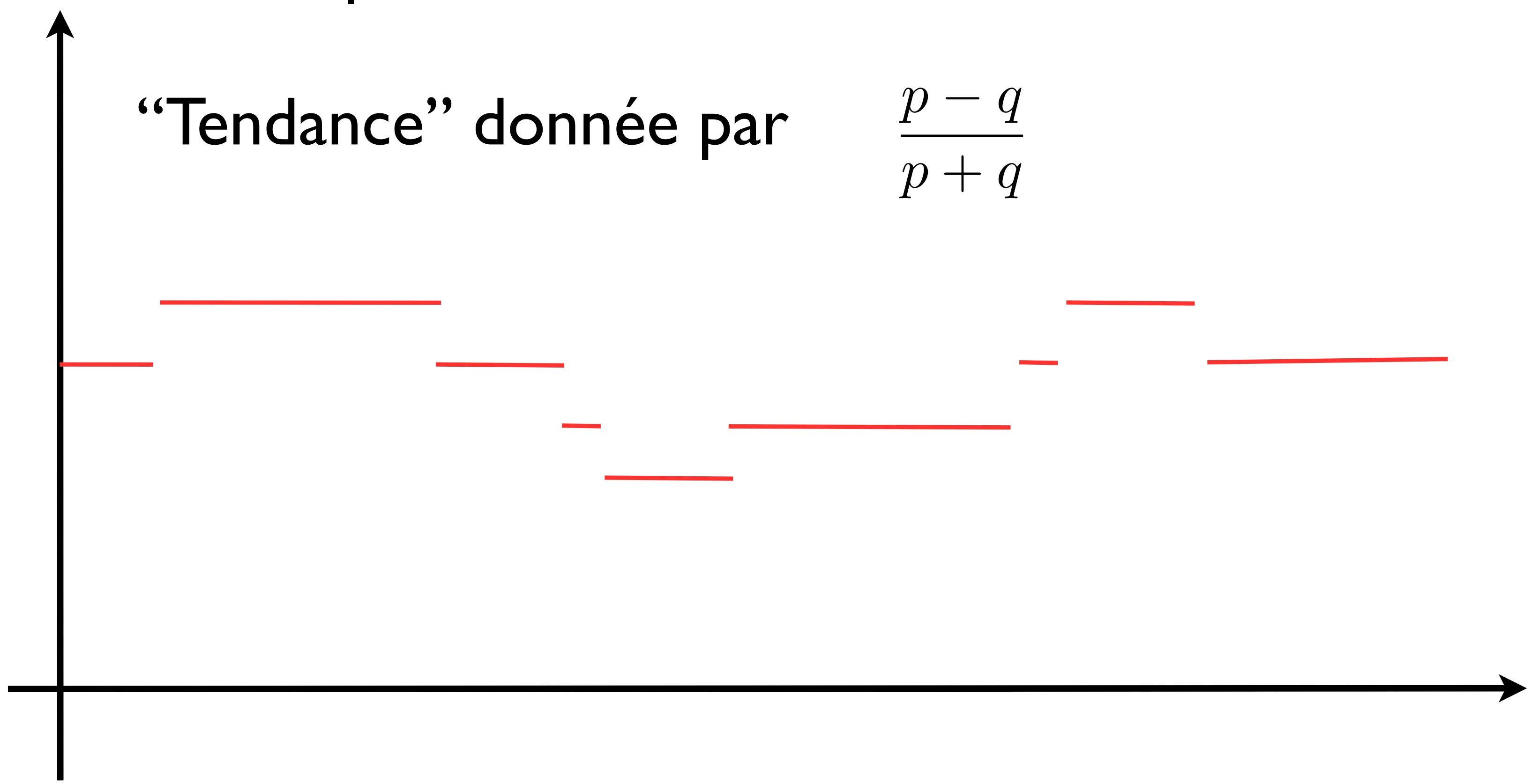


Processus de Feller-Gillespie

Temps moyen entre deux évènements de l'ordre de $\frac{1}{N(t)(p+q)}$

"Fréquence" $\approx N(t)(p+q)$

"Tendance" donnée par $\frac{p-q}{p+q}$



Processus de Feller-Gillespie

Rien n'empêche *p* et *q* de dépendre de *N(t)* et *t*

C'est un algorithme : On se donne $p(N)$ et $q(N)$

$$(N(0)(p(N(0),0)q(N(0),0))$$

1. $t \leftarrow 0$;
2. $N \leftarrow N(0)$;
3. On tire T selon une loi exponentielle de paramètre $N(0)(p+q)$;
4. $W \leftarrow +1$ ou -1 selon $p/(p+q)$ et $q/(p+q)$;
5. $t \leftarrow T$;
6. $\leftarrow T + W$
7. On recommence...

$$\frac{p(N(0),0)}{p(N(0),0)+q(N(0),0)}$$

Approximation diffusion

Pour *N(t)* grand le temps de simulation est trop grand

1. $x(t) = \dfrac{N(t)}{\omega}$: Une unité de x représente ω individus.
2. On se donne $p = p(x,t)$; $q = q(x,t)$
3. On évalue l'accroissement de x sur une durée Δt telle que

$$N(t)(p+q)\delta t = \omega x(t)\Delta t$$

   soit grand (de l'ordre de 10^3).

1. $\# =$ "nombre (aléatoire) d'évènements sur" $[t, t + \Delta t[$
2. $x(t)$ est approximativement constant sur $[t, t + \Delta t[$ si Δt n'est pas trop grand.
3. $N(t + Deltat) - N(t) = \sum_{i=1}^{\#} W_i \quad ; \quad P(W_i = 1) = \frac{p}{p+q}$; $p(W_i = -1) = \frac{q}{p+q}$
4. comme # est grand $W = \sum_{i=1}^{\#} W_i$ est approximativement une gaussienne.
5. $E[W] = \# \frac{p-q}{p+q}$; $\sigma^2(W) = \# \frac{4pq}{(p+q)^2} \Longrightarrow W \approx \# \frac{p-q}{p+q} + \sqrt{\#}\sqrt{\frac{4pq}{(p+q)^2}}\mathcal{N}(0,1)$
6. "en moyenne" $\# = N(t)(p + q)\Delta t = \omega x(t)(p + q)\Delta t$
7. $x(t + \Delta t) - x(t) = x(t)(p - q)\Delta t + \frac{\sqrt{\Delta t}}{\sqrt{\omega}}\sqrt{\frac{4x(t)p(x(t),t)q(x(t),t)}{p(x(t),t)+q(x(t),t)}}\mathcal{N}(0,1)$

- $x(t+\Delta t) - x(t) = x(t)(p-q)\Delta t + \frac{\sqrt{\Delta t}}{\sqrt{\omega}}\sqrt{\frac{4x(t)p(x(t),t)q(x(t),t)}{p(x(t),t)+q(x(t),t)}}\mathcal{N}(0,1)$
- $x(t+\Delta t) = x(t) + f(x(t))\Delta t + \frac{\sqrt{\Delta t}}{\sqrt{\omega}} g(x(t))\mathcal{N}(0,1)$

Processus de diffusion

Schéma d'Euler + (petit) bruit en $\sqrt{\Delta t}$

Processus de diffusion

Exemple de processus de diffusion

$$x(t+dt) = x(t) + dt\, f(x(t)) + \sigma(x(t))\sqrt{dt}\,\mathcal{N}(0,1)$$

Pour respecter la positivité :

$$x(t+dt) = \max\left\{x(t) + dt\, f(x(t)) + \sigma(x(t)\sqrt{dt}\,\mathcal{N}(0,1)\ ;\ 0\right\}$$

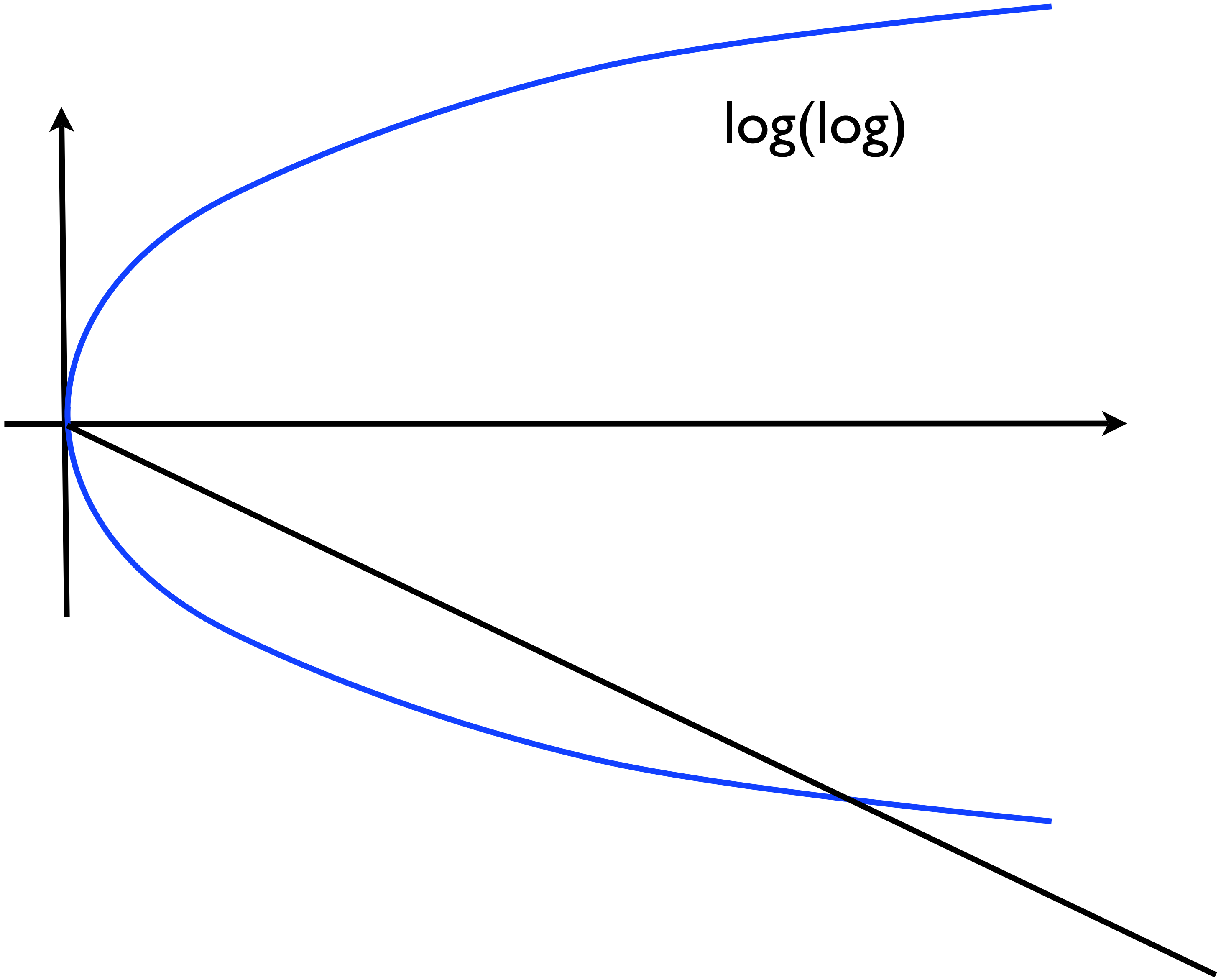
La croissance exponentielle “diffusion”

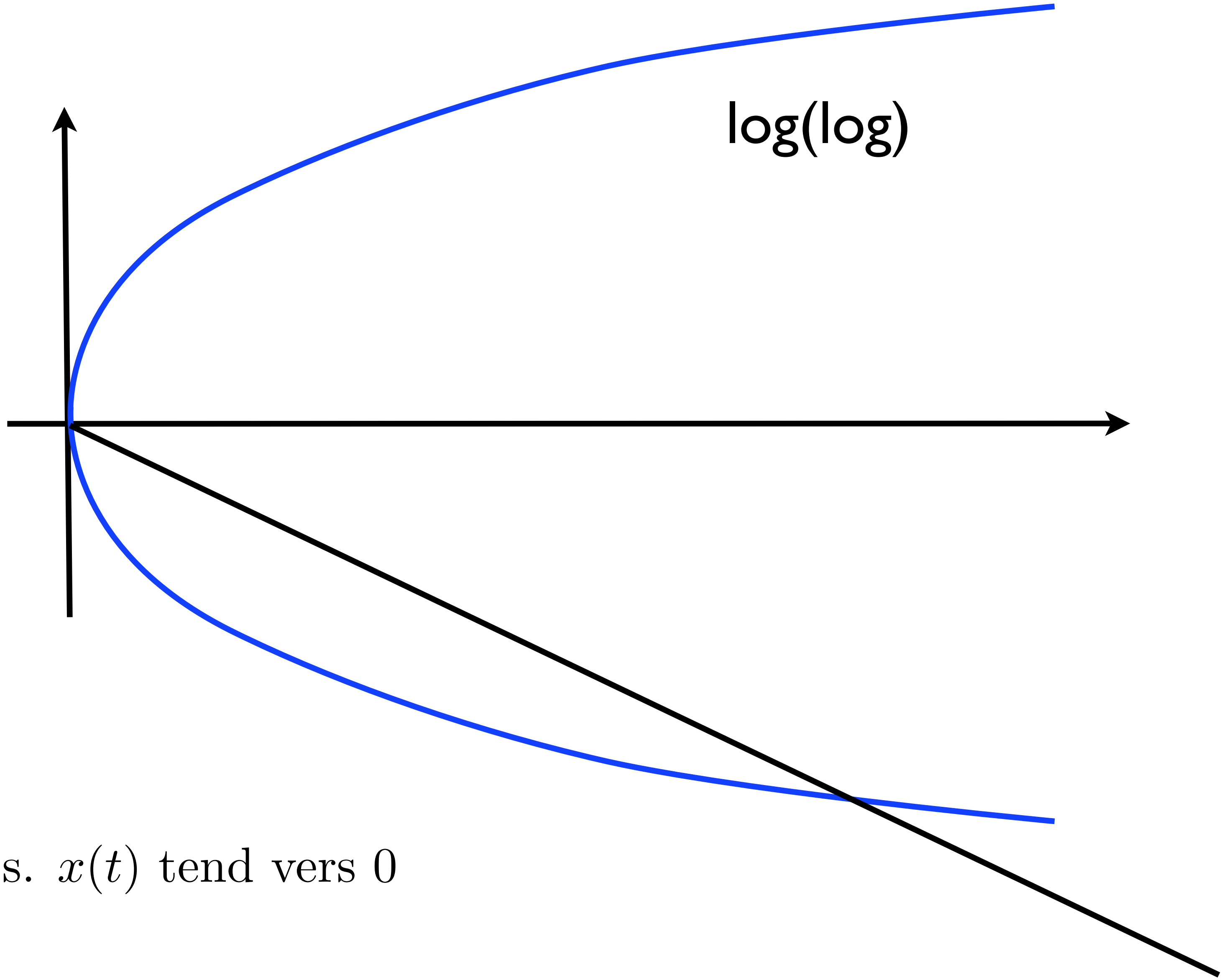
$$x(t+dt) = x(t) + dt\, r\, x(t) + (\pm)\sigma\, x(t)\sqrt{dt}$$

$$y = \ln(x)$$

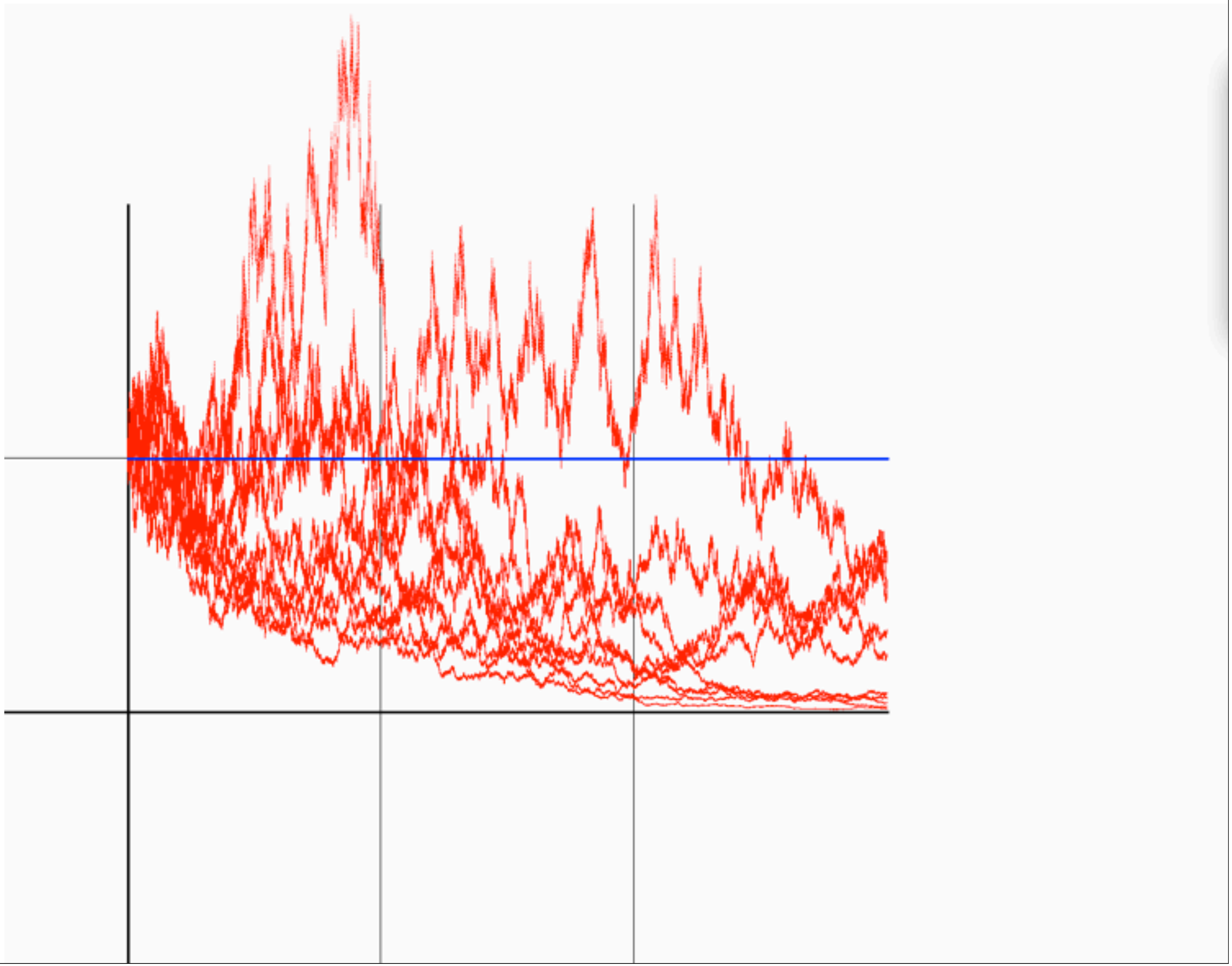
$$y(t+dt) = y(t) + (r\,dt + (\pm)\sigma\sqrt{dt}) - \frac{1}{2}(r\,dt + (\pm)\sigma\sqrt{dt})^2$$

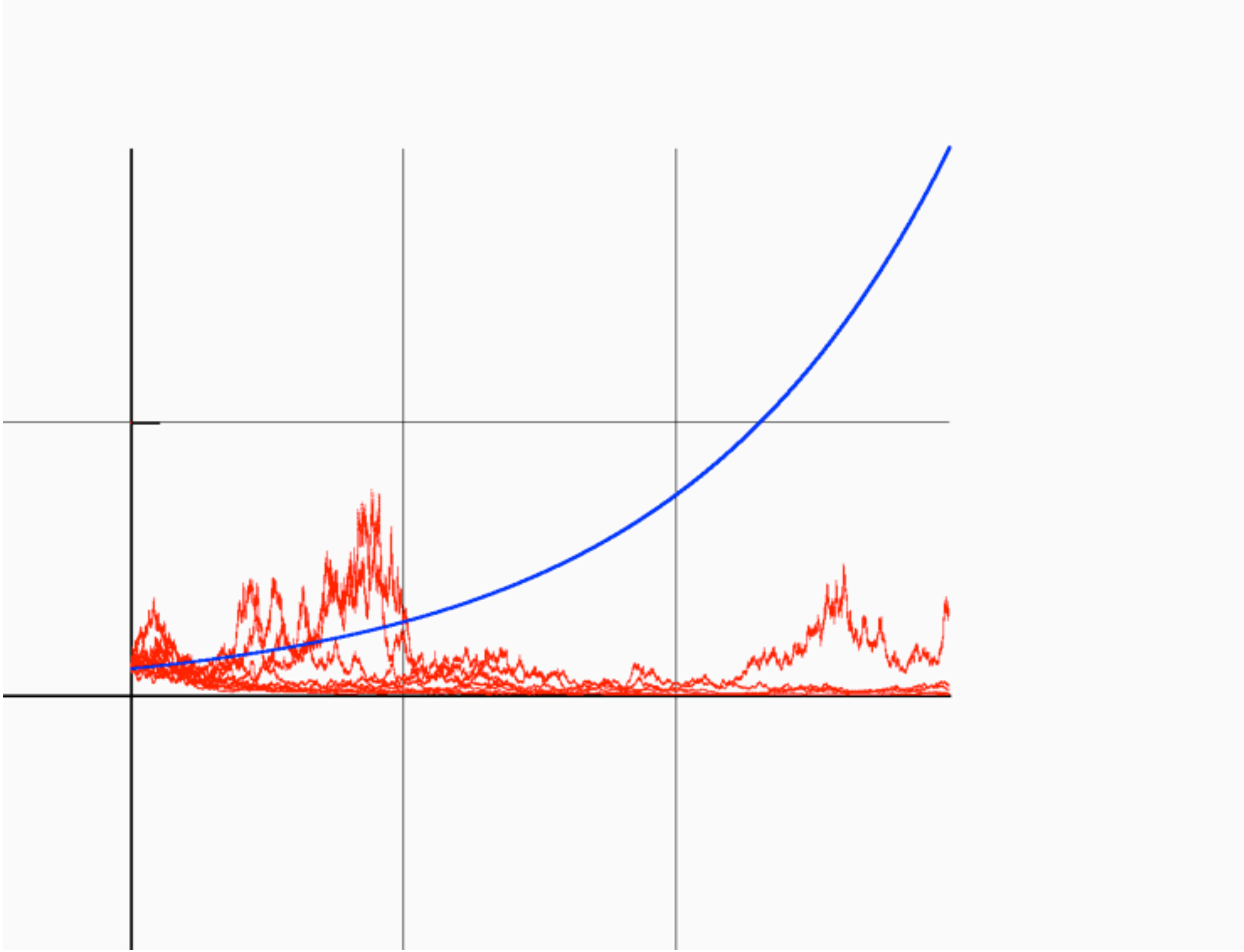
$$y(t+dt) = y(t) + (r - \sigma^2/2)dt + (\pm)\sigma\sqrt{dt}) + ...$$


log(log)



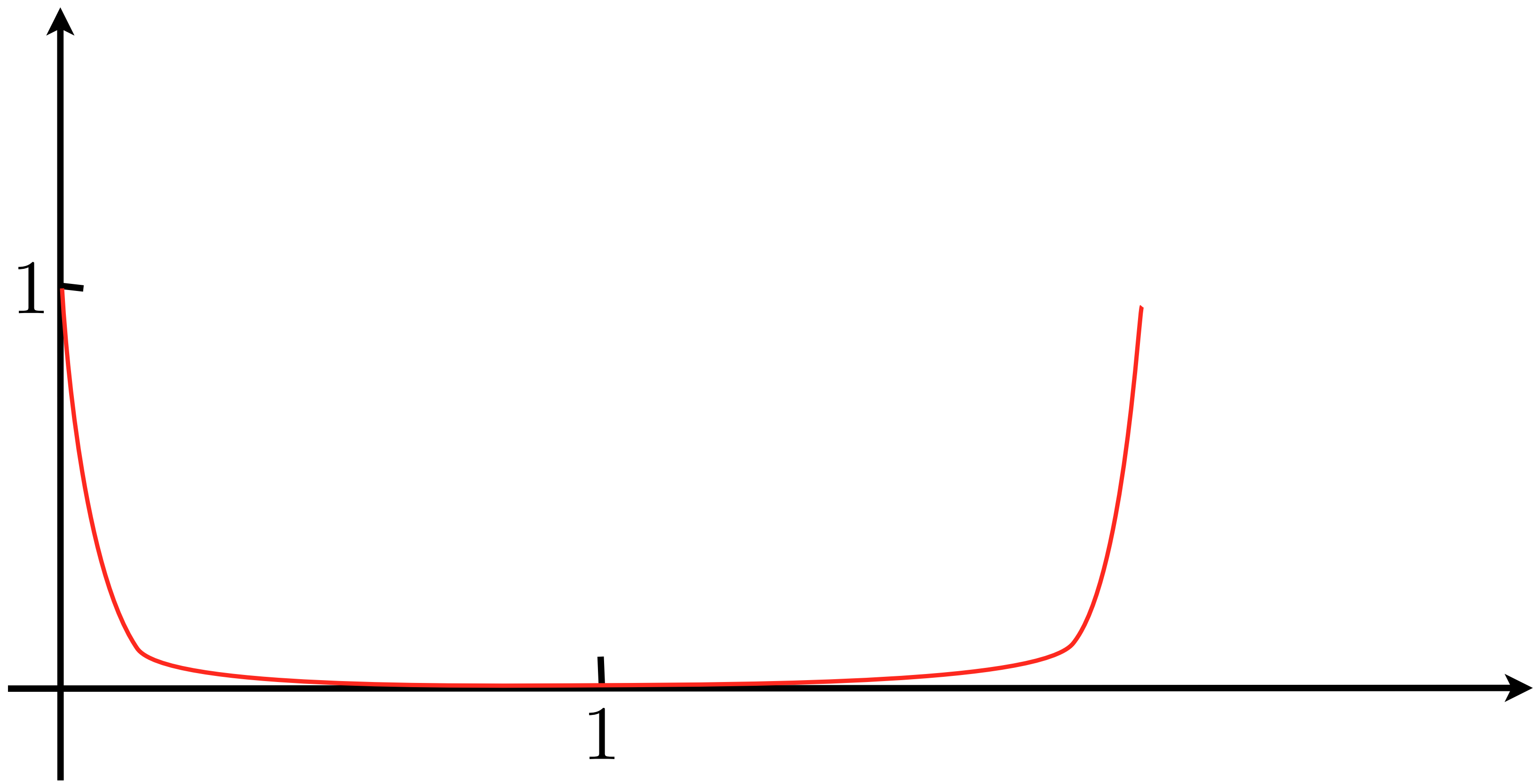
p.s. $x(t)$ tend vers 0

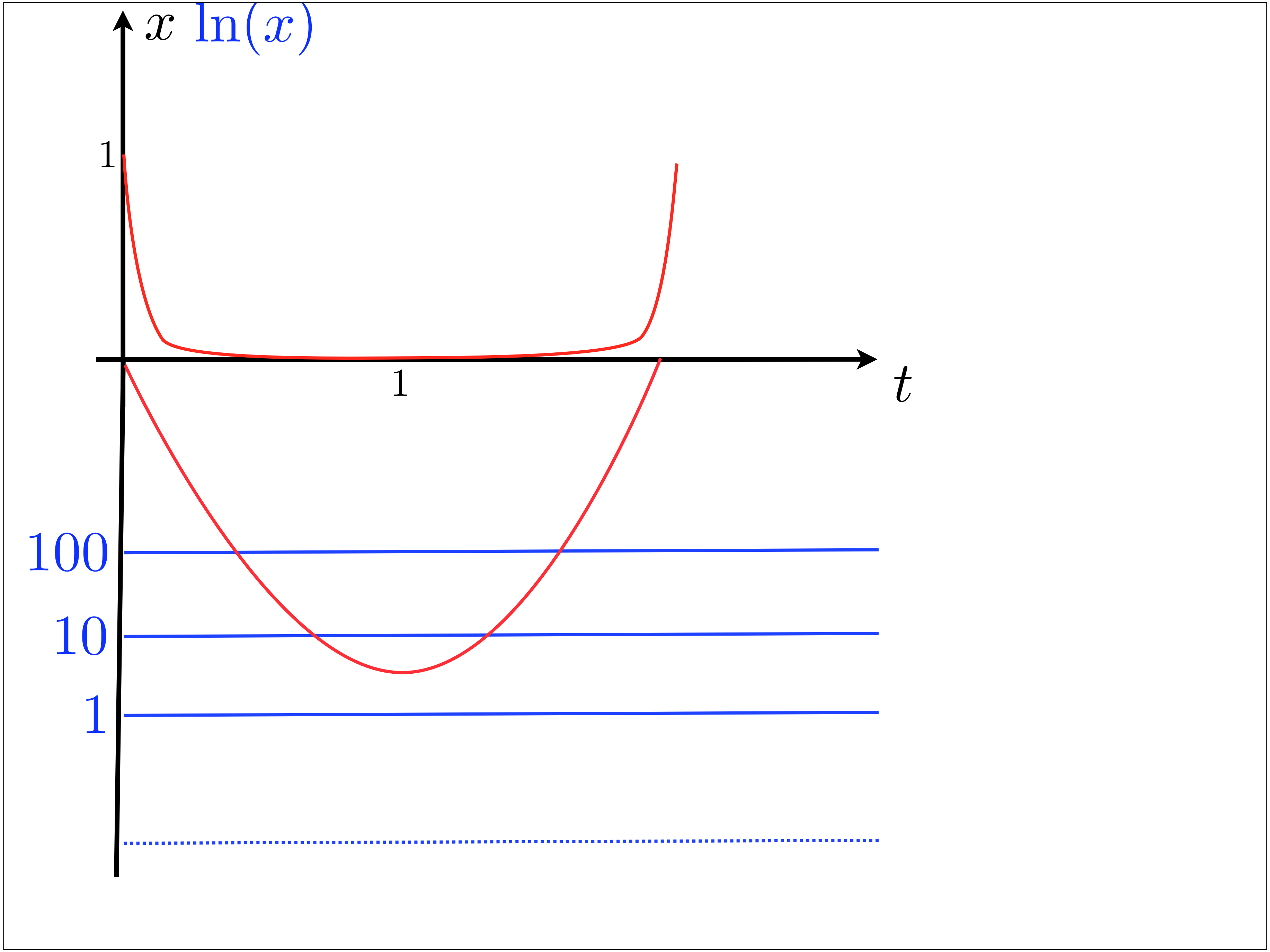




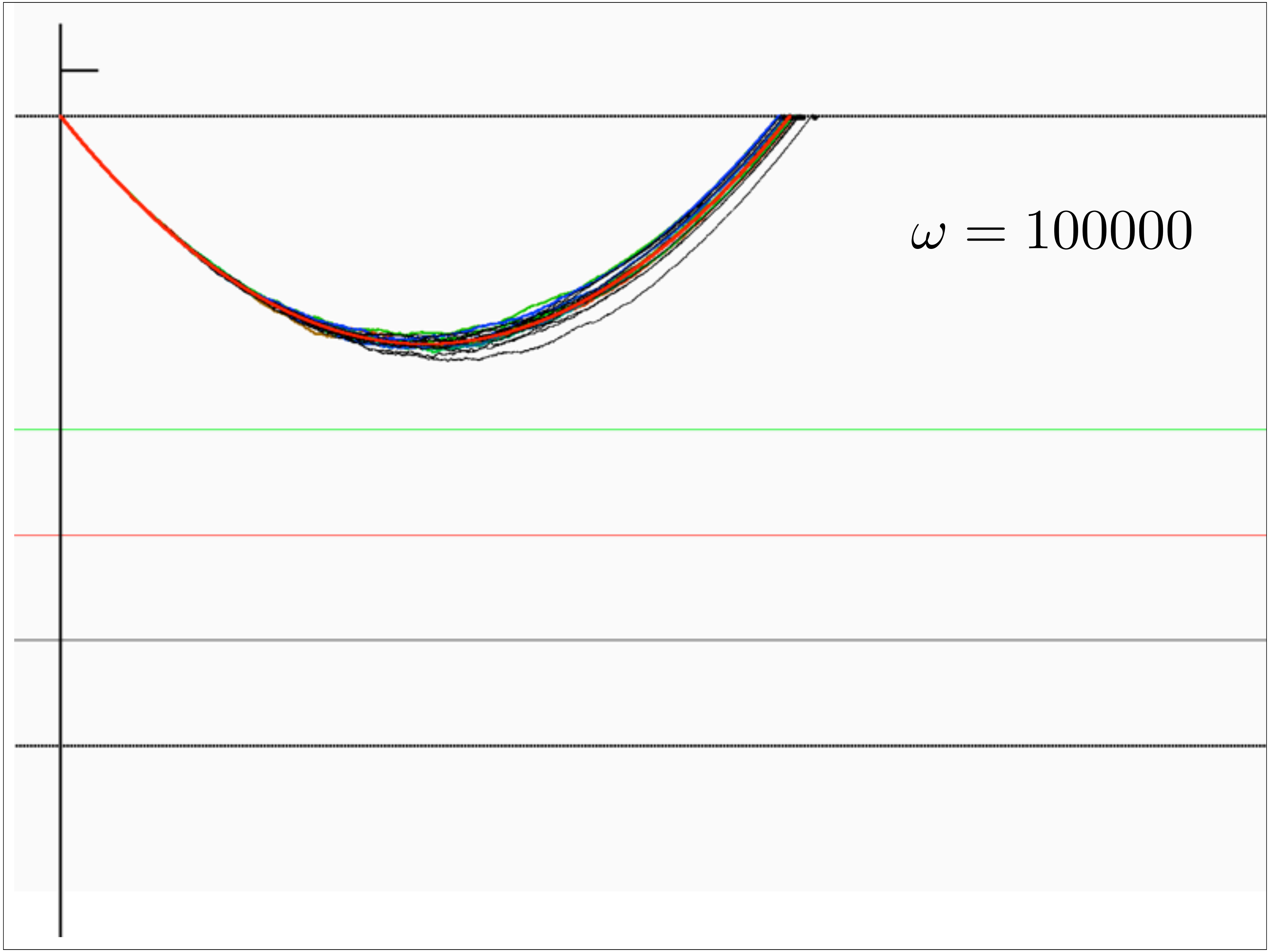
Expériences numériques

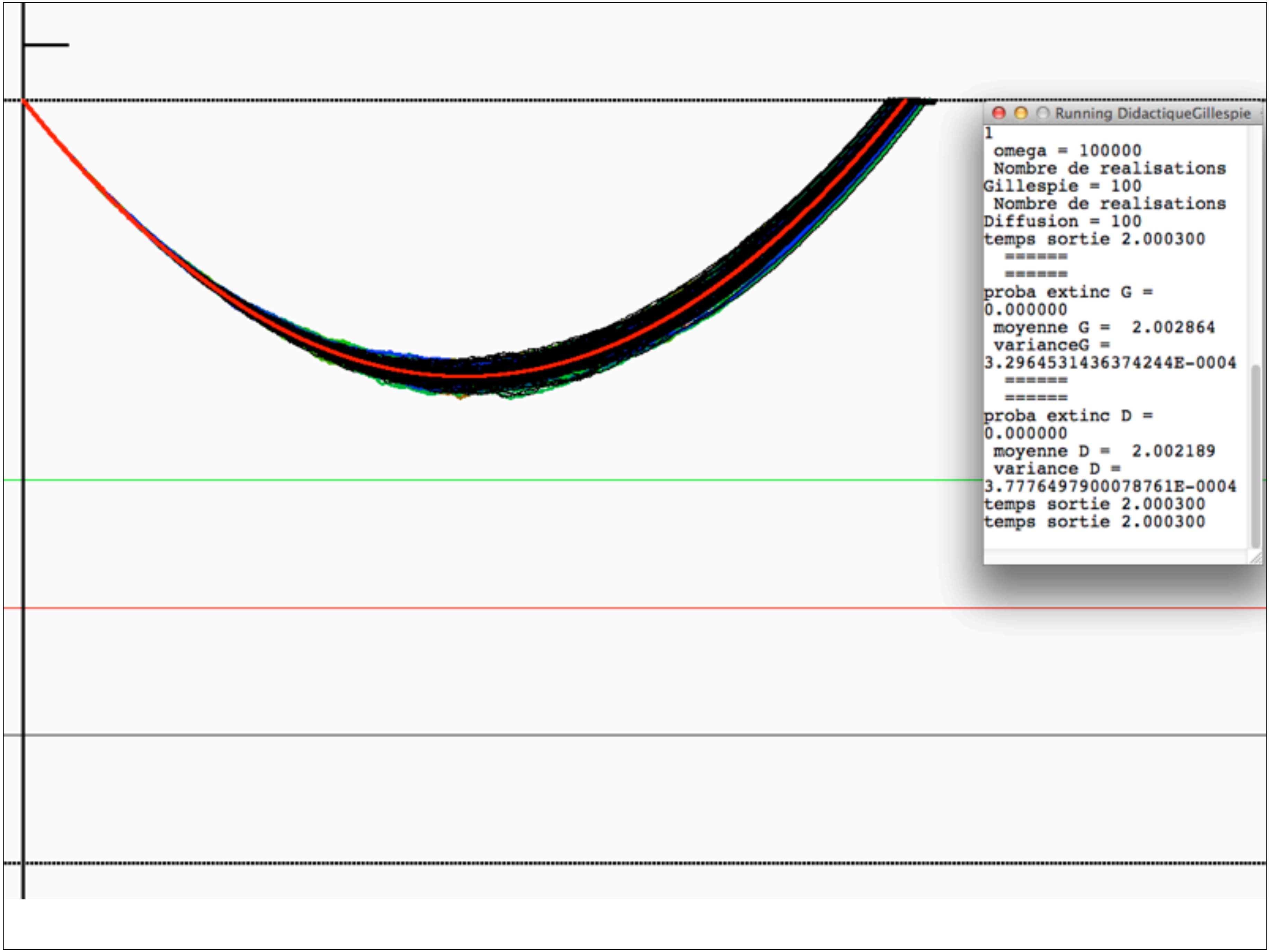
- $p(t) = 10(1 + t/2)$; $q(t) = 10(2 - t/2)$
- Approximation déterministe : $\dot{x}(t) = 10(-1 + t)x(t)$

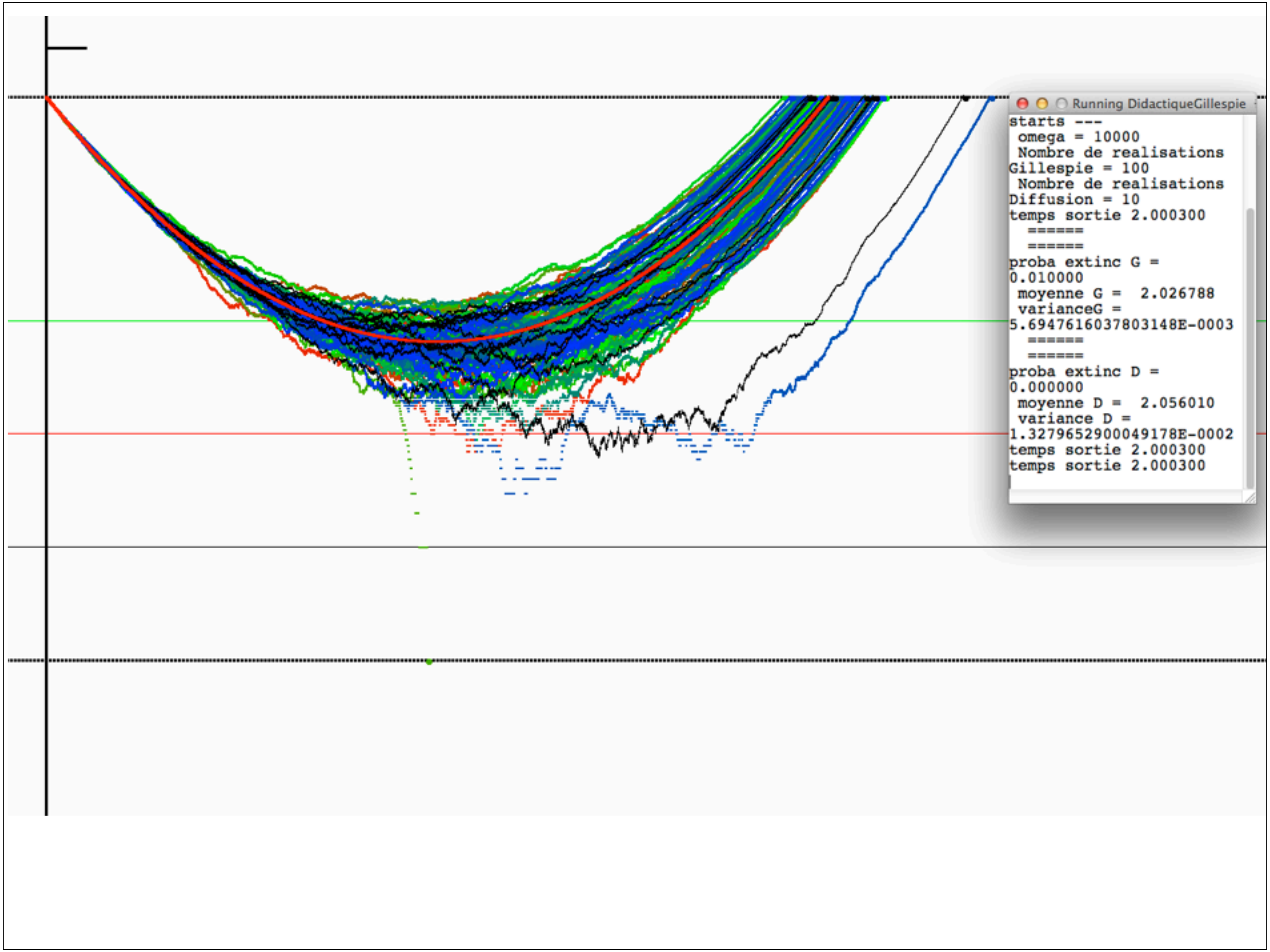


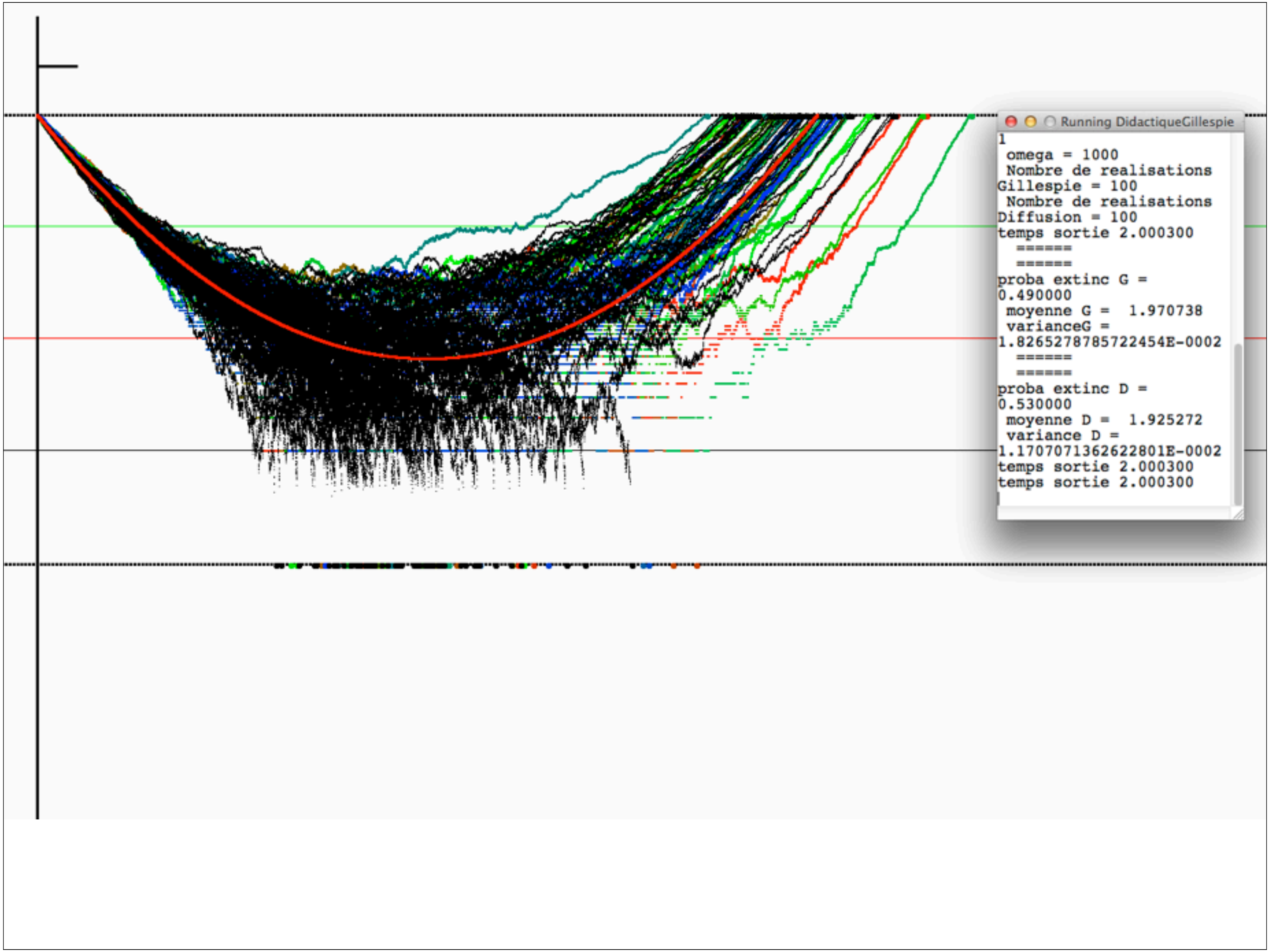


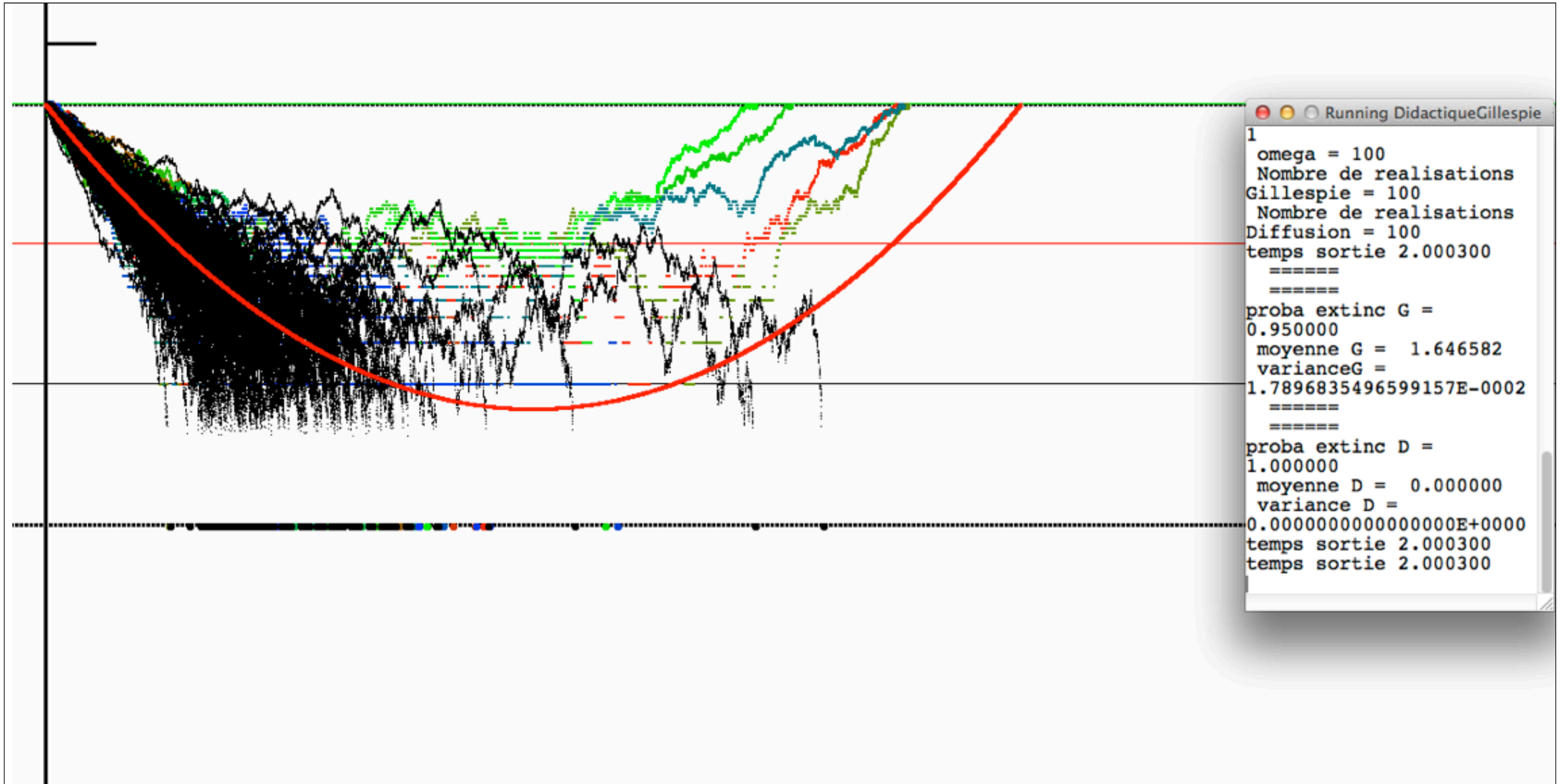
x $\ln(x)$
1
1
t
100
10
1


$\omega = 100000$

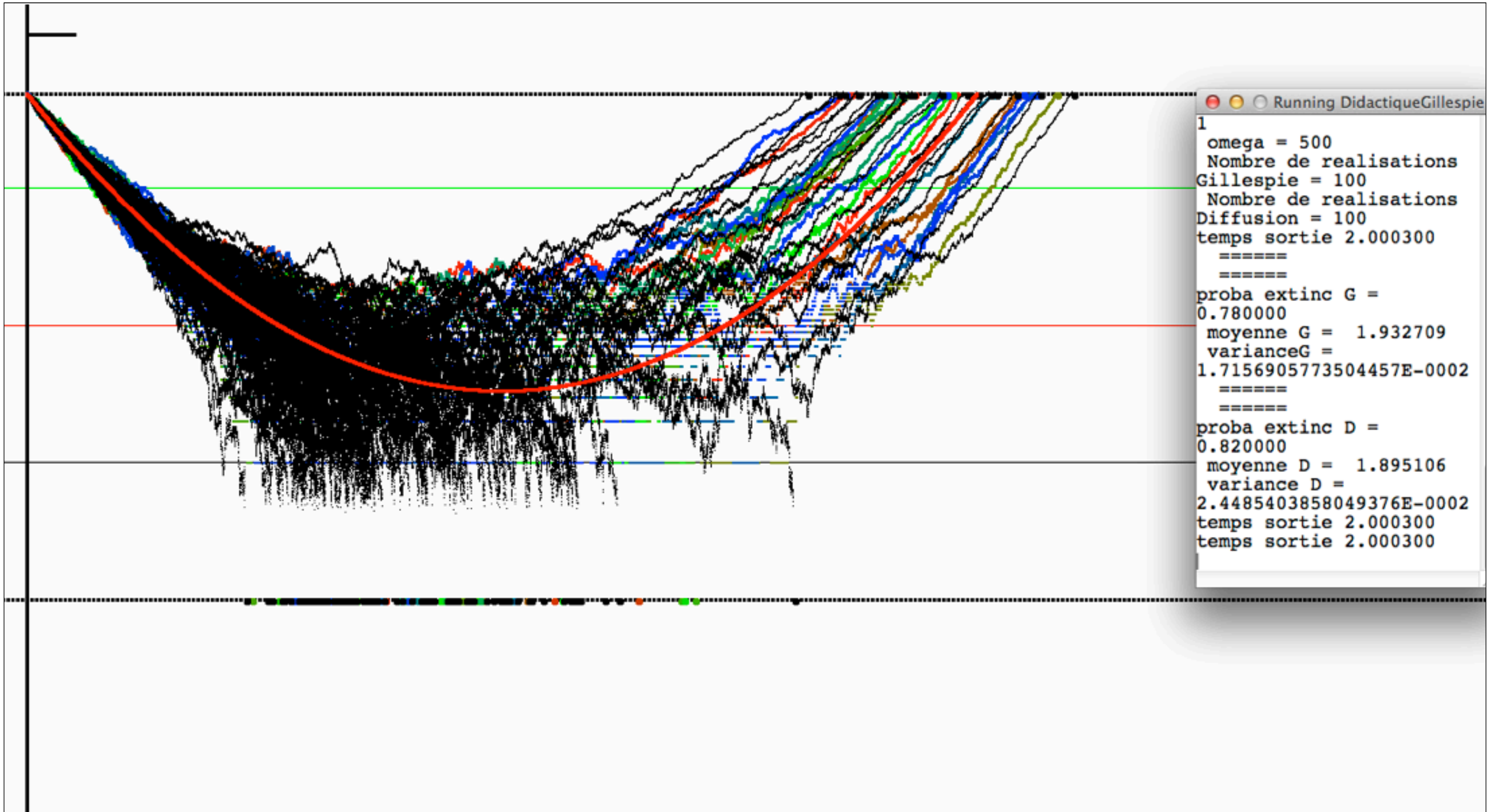

Running DidactiqueGillespie
1
omega = 100000
Nombre de realisations
Gillespie = 100
Nombre de realisations
Diffusion = 100
temps sortie 2.000300
======
======
proba extinc G =
0.000000
moyenne G = 2.002864
varianceG =
3.2964531436374244E-0004
======
======
proba extinc D =
0.000000
moyenne D = 2.002189
variance D =
3.7776497900078761E-0004
temps sortie 2.000300
temps sortie 2.000300

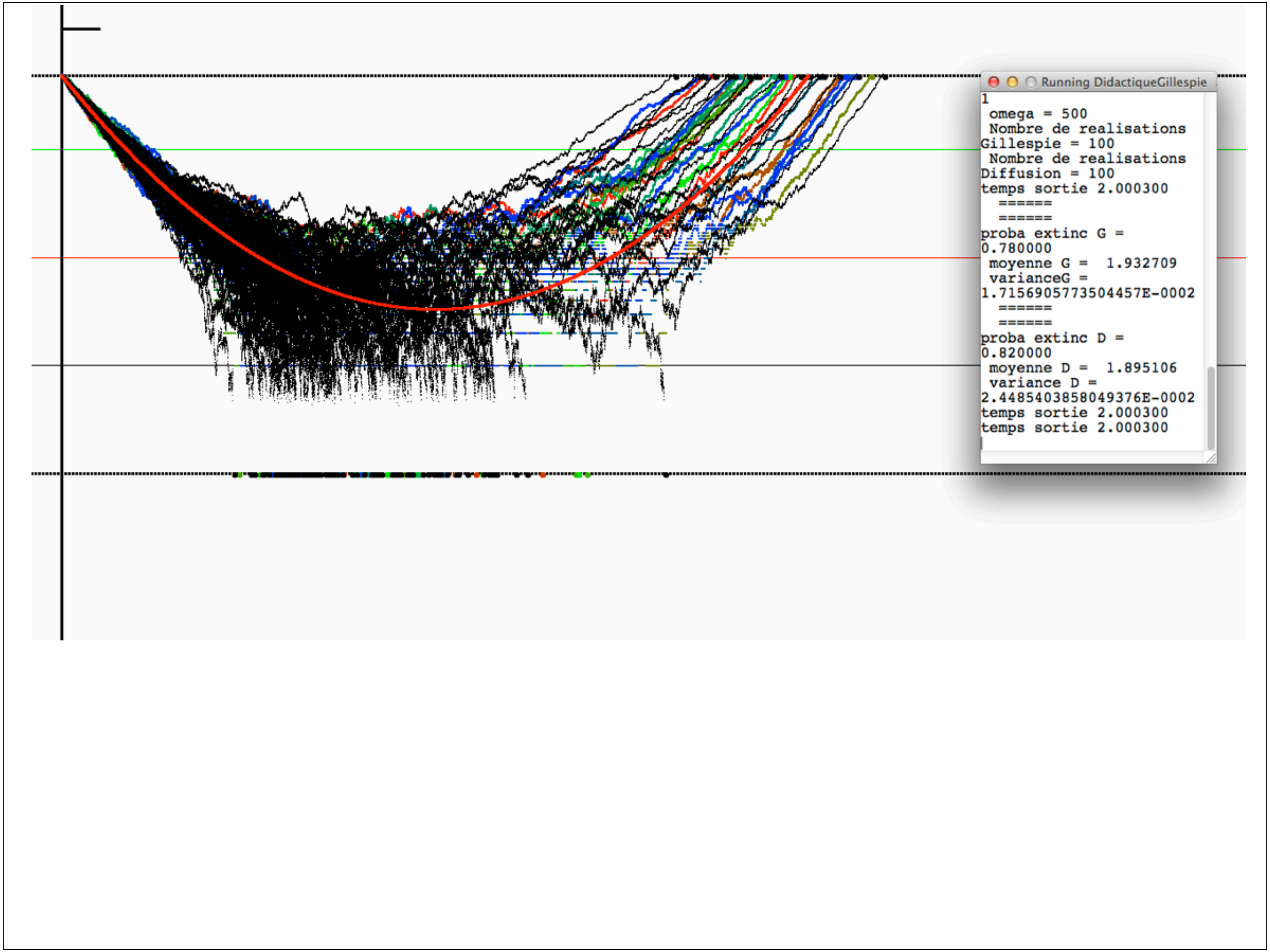

Running DidactiqueGillespie
starts ---
omega = 10000
Nombre de realisations
Gillespie = 100
Nombre de realisations
Diffusion = 10
temps sortie 2.000300
======
======
proba extinc G =
0.010000
moyenne G = 2.026788
varianceG =
5.6947616037803148E-0003
======
======
proba extinc D =
0.000000
moyenne D = 2.056010
variance D =
1.3279652900049178E-0002
temps sortie 2.000300
temps sortie 2.000300

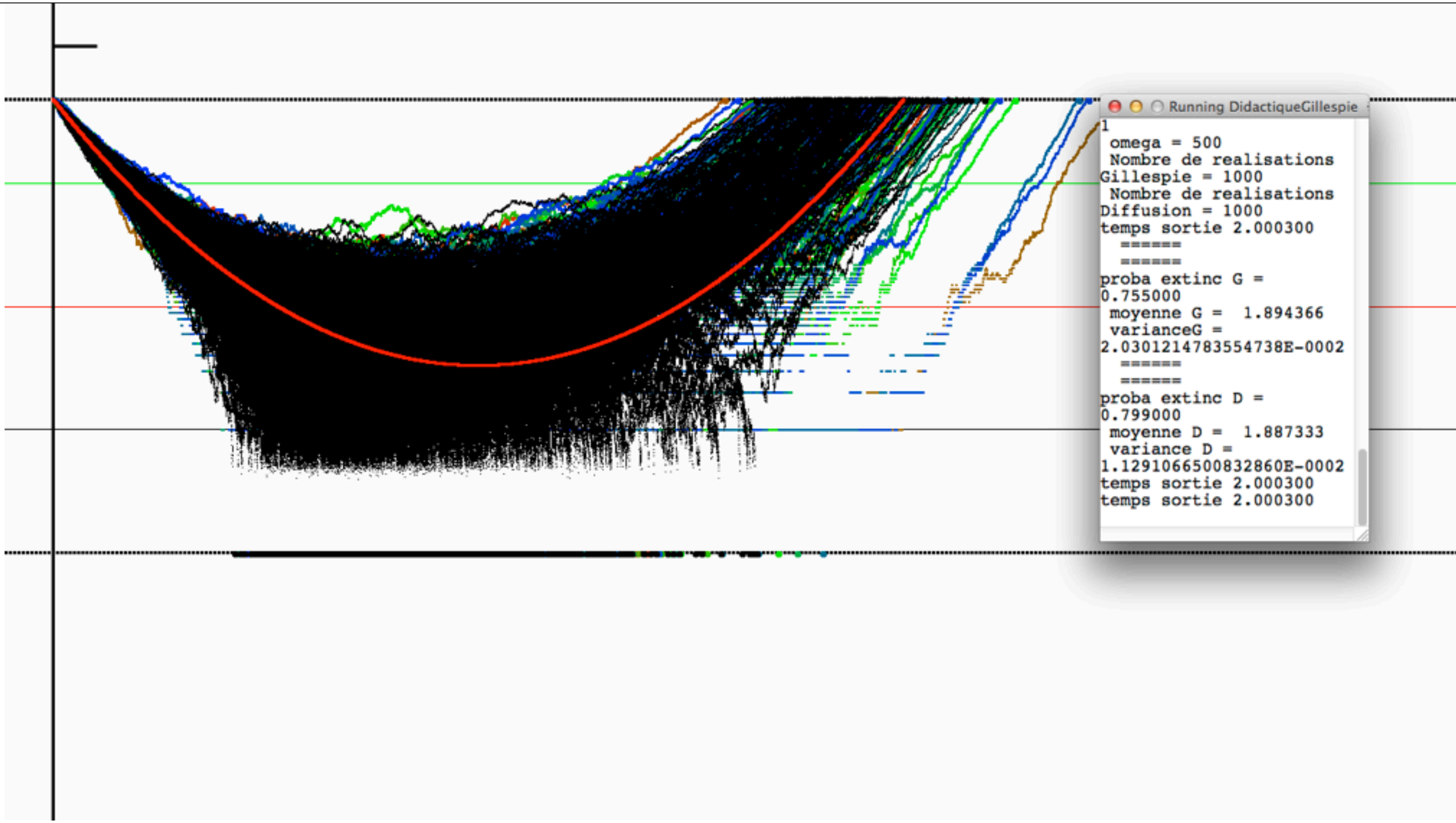

Running DidactiqueGillespie
1
omega = 1000
Nombre de realisations
Gillespie = 100
Nombre de realisations
Diffusion = 100
temps sortie 2.000300
======
======
proba extinc G =
0.490000
moyenne G = 1.970738
varianceG =
1.8265278785722454E-0002
======
======
proba extinc D =
0.530000
moyenne D = 1.925272
variance D =
1.170707136262280lE-0002
temps sortie 2.000300
temps sortie 2.000300

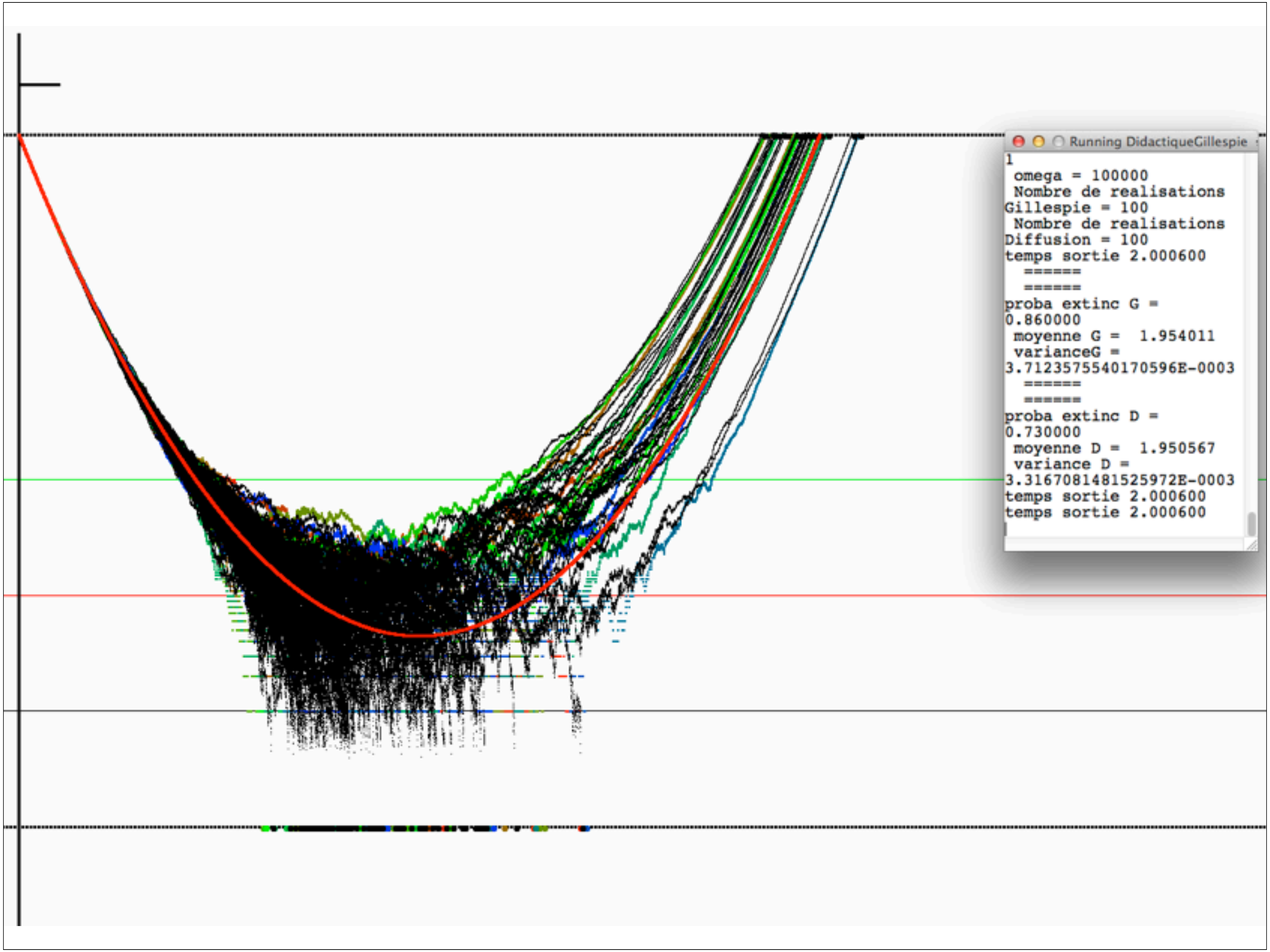


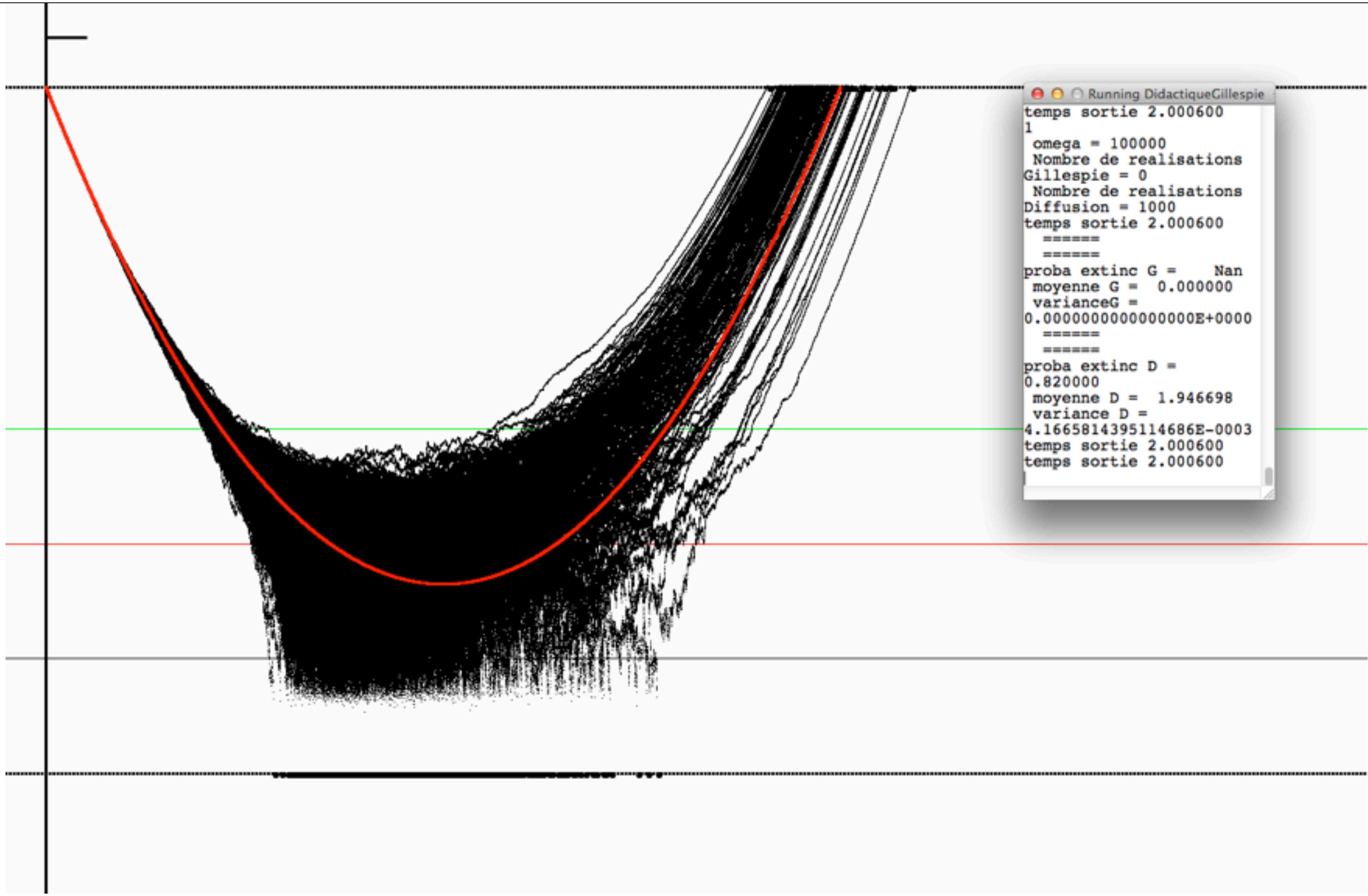
Running DidactiqueGillespie
1
omega = 100
Nombre de realisations
Gillespie = 100
Nombre de realisations
Diffusion = 100
temps sortie 2.000300
======
======
proba extinc G =
0.950000
moyenne G = 1.646582
varianceG =
1.7896835496599157E-0002
======
======
proba extinc D =
1.000000
moyenne D = 0.000000
variance D =
0.0000000000000000E+0000
temps sortie 2.000300
temps sortie 2.000300

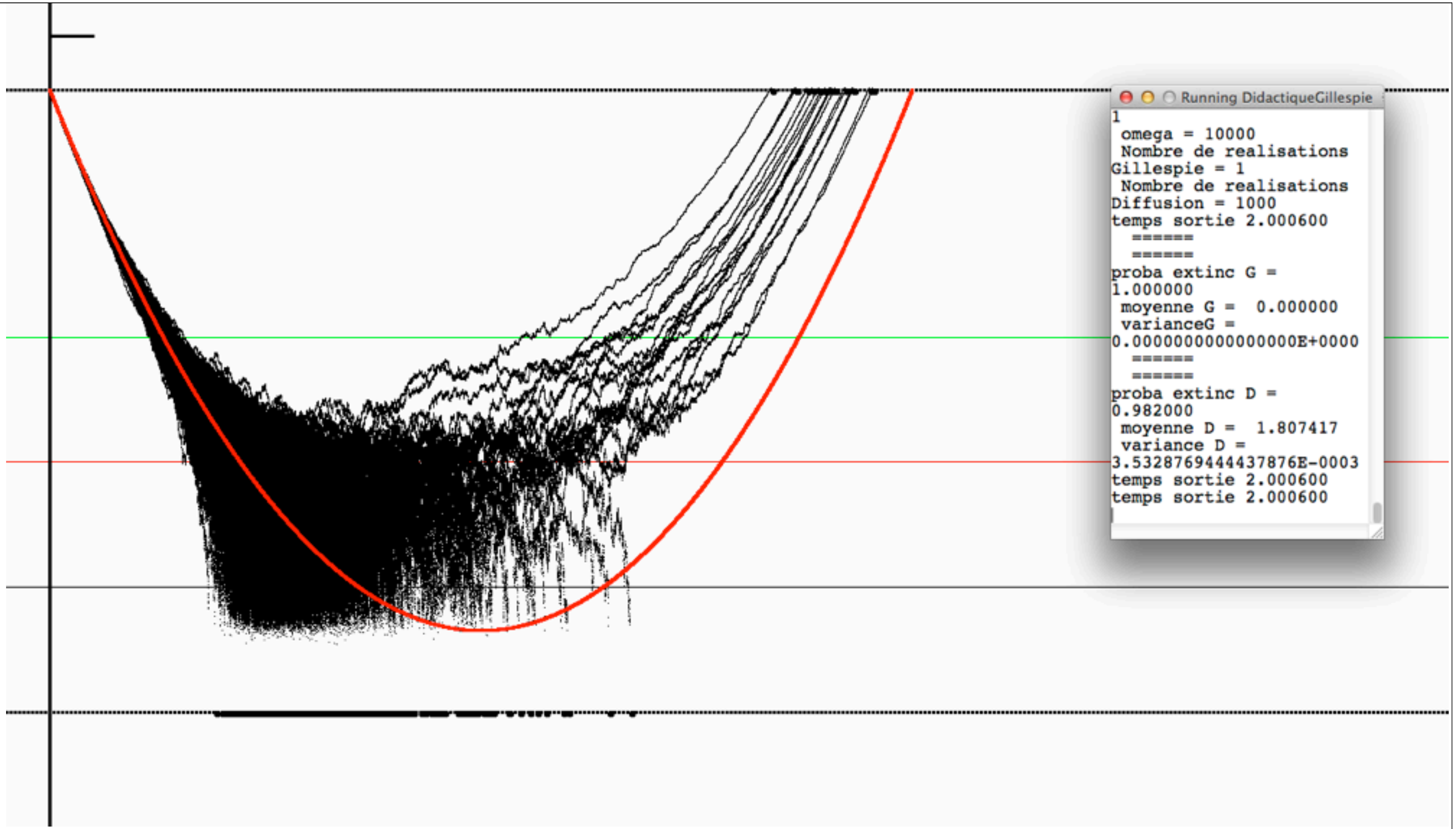

Running DidactiqueGillespie
1
omega = 500
Nombre de realisations
Gillespie = 100
Nombre de realisations
Diffusion = 100
temps sortie 2.000300
======
======
proba extinc G =
0.780000
moyenne G = 1.932709
varianceG =
1.7156905773504457E-0002
======
======
proba extinc D =
0.820000
moyenne D = 1.895106
variance D =
2.4485403858049376E-0002
temps sortie 2.000300
temps sortie 2.000300

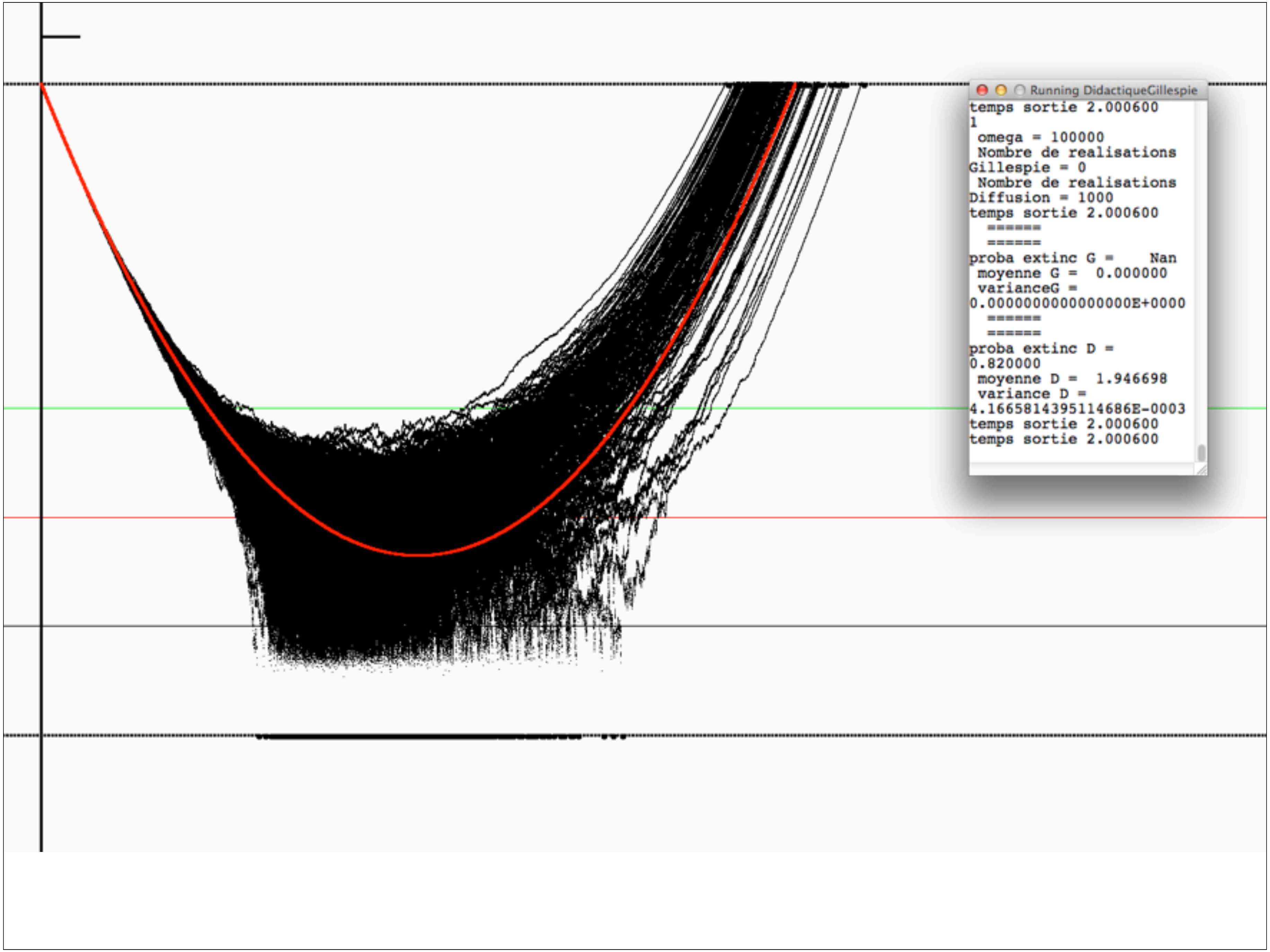

Running DidactiqueGillespie
1
omega = 500
Nombre de realisations
Gillespie = 100
Nombre de realisations
Diffusion = 100
temps sortie 2.000300
======
======
proba extinc G =
0.780000
moyenne G = 1.932709
varianceG =
1.7156905773504457E-0002
======
======
proba extinc D =
0.820000
moyenne D = 1.895106
variance D =
2.4485403858049376E-0002
temps sortie 2.000300
temps sortie 2.000300


Running DidactiqueGillespie
1
omega = 500
Nombre de realisations
Gillespie = 1000
Nombre de realisations
Diffusion = 1000
temps sortie 2.000300
======
======
proba extinc G =
0.755000
moyenne G = 1.894366
varianceG =
2.0301214783554738E-0002
======
======
proba extinc D =
0.799000
moyenne D = 1.887333
variance D =
1.1291066500832860E-0002
temps sortie 2.000300
temps sortie 2.000300


Running DidactiqueGillespie
1
omega = 100000
Nombre de realisations
Gillespie = 100
Nombre de realisations
Diffusion = 100
temps sortie 2.000600
======
======
proba extinc G =
0.860000
moyenne G = 1.954011
varianceG =
3.7123575540170596E-0003
======
======
proba extinc D =
0.730000
moyenne D = 1.950567
variance D =
3.3167081481525972E-0003
temps sortie 2.000600
temps sortie 2.000600


Running DidactiqueGillespie
temps sortie 2.000600
1
omega = 100000
Nombre de realisations
Gillespie = 0
Nombre de realisations
Diffusion = 1000
temps sortie 2.000600
======
======
proba extinc G = Nan
moyenne G = 0.000000
varianceG =
0.0000000000000000E+0000
======
======
proba extinc D =
0.820000
moyenne D = 1.946698
variance D =
4.1665814395114686E-0003
temps sortie 2.000600
temps sortie 2.000600


Running DidactiqueGillespie
1
omega = 10000
Nombre de realisations
Gillespie = 1
Nombre de realisations
Diffusion = 1000
temps sortie 2.000600
======
======
proba extinc G =
1.000000
moyenne G = 0.000000
varianceG =
0.0000000000000000E+0000
======
======
proba extinc D =
0.982000
moyenne D = 1.807417
variance D =
3.5328769444437876E-0003
temps sortie 2.000600
temps sortie 2.000600


Running DidactiqueGillespie
temps sortie 2.000600
1
omega = 100000
Nombre de realisations
Gillespie = 0
Nombre de realisations
Diffusion = 1000
temps sortie 2.000600
======
======
proba extinc G = Nan
moyenne G = 0.000000
varianceG =
0.0000000000000000E+0000
======
======
proba extinc D =
0.820000
moyenne D = 1.946698
variance D =
4.1665814395114686E-0003
temps sortie 2.000600
temps sortie 2.000600

Un modèle proie-prédateur

$$\begin{cases} \dfrac{dx}{dt} & = & \frac{1}{\varepsilon}[f(x) - \mu(x)y] \\ \dfrac{dy}{dt} & = & (\mu(x) - m)y \end{cases}$$

- $f(x) = \dfrac{1}{2}x\,(2-x)$
- $\mu(x) = \dfrac{x}{0.4+x}$
- $\varepsilon = 0.02$

$$\begin{cases} \dfrac{dx}{dt} & = & \frac{1}{\varepsilon}[f(x) - \mu(x)y] \\ \dfrac{dy}{dt} & = & (\mu(x) - m)y \end{cases}$$

- $f(x) = \dfrac{1}{2}x\,(2-x)$
- $\mu(x) = \dfrac{x}{0.4+x}$
- $\varepsilon = 0.02$

$$x(t+dt) - x(t) \approx dt\frac{1}{\varepsilon}[f(x(t)) - \mu(x(t))\,y(t)] + \sqrt{dt\frac{4}{\omega\varepsilon}\;\frac{f(x(t))\mu(x(t))\,y(t)}{(f(x(t)) + \mu(x(t))\,y(t))}}W_t$$

$$y(t+dt) - y(t) \approx dt[\mu(x(t) - m]y(t) - \sqrt{dt\frac{\varepsilon}{\omega}\;\frac{f(x(t))\mu(x(t))\,y(t)}{(f(x(t)) + \mu(x(t))\,y(t))}}W_t$$

$$\begin{cases} \dfrac{dx}{dt} = \frac{1}{\varepsilon}[f(x) - \mu(x)y] \\ \dfrac{dy}{dt} = (\mu(x) - m)y \end{cases}$$

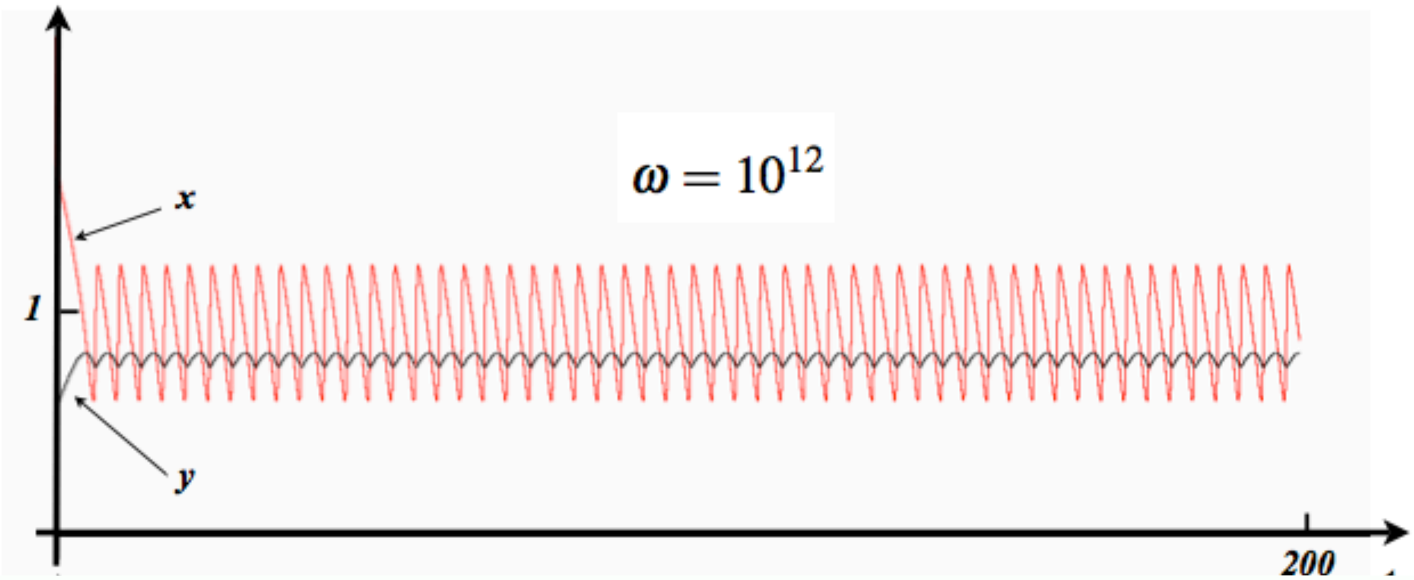
- $f(x) = \dfrac{1}{2}x\,(2-x)$
- $\mu(x) = \dfrac{x}{0.4+x}$
- $\varepsilon = 0.02$

$$x(t+dt) - x(t) \approx dt\frac{1}{\varepsilon}[f(x(t)) - \mu(x(t))\,y(t)] + \sqrt{dt\frac{4}{\omega\varepsilon}\;\frac{f(x(t))\mu(x(t))\,y(t)}{(f(x(t)) + \mu(x(t))\,y(t))}}W_t$$

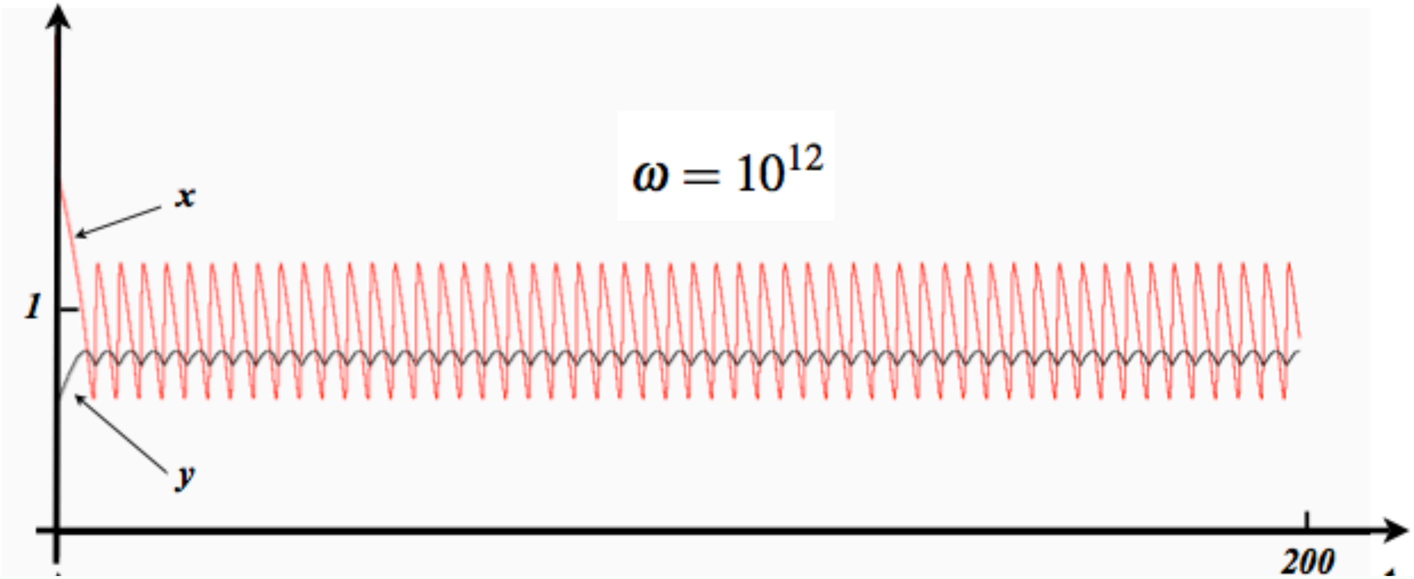
$$y(t+dt) - y(t) \approx dt[\mu(x(t) - m]y(t) - \sqrt{dt\frac{\varepsilon}{\omega}\;\frac{f(x(t))\mu(x(t))\,y(t)}{(f(x(t)) + \mu(x(t))\,y(t))}}W_t$$

1 unité de x = ω individus

Simulations



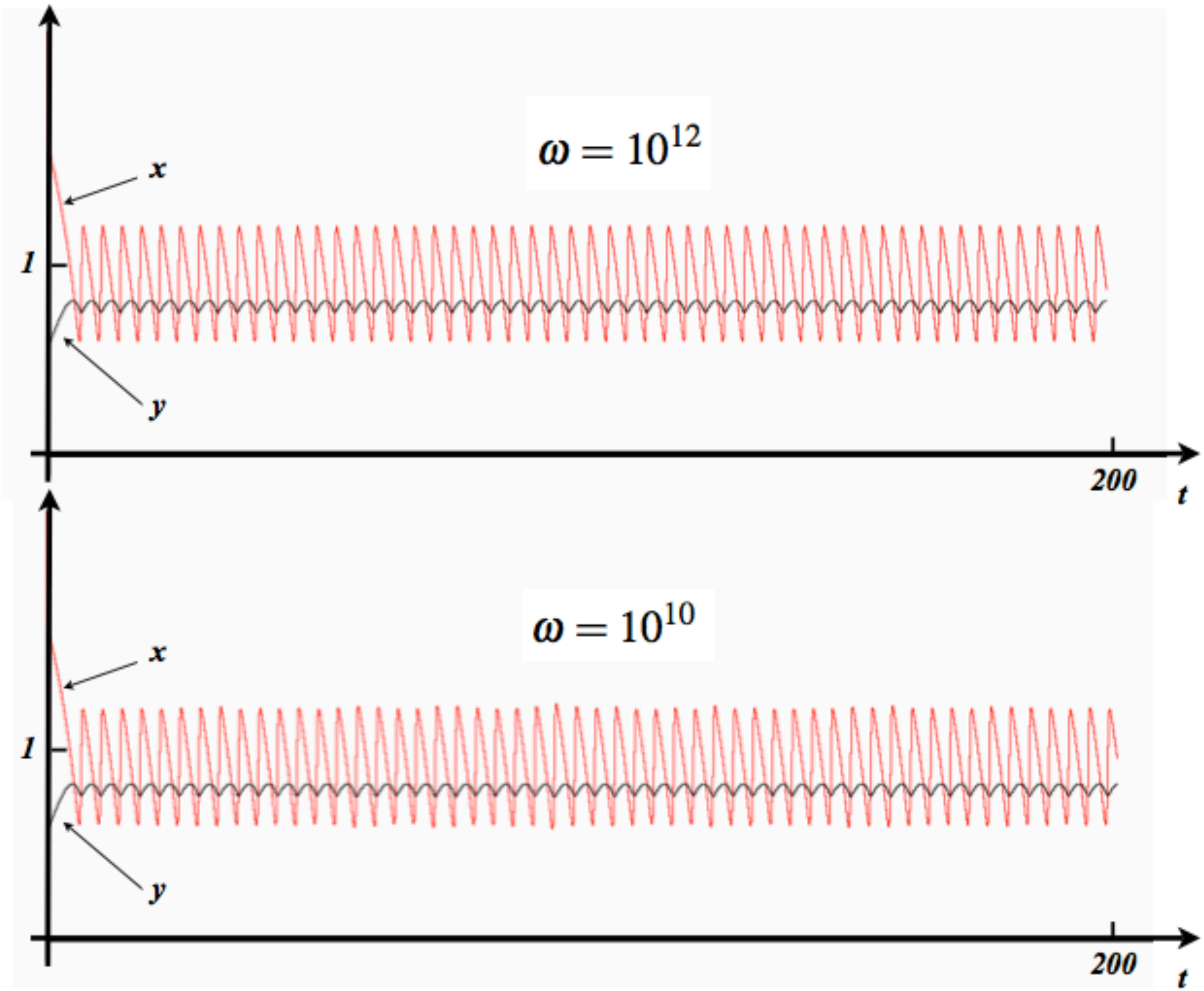
$\omega = 10^{12}$
x
1
y
200



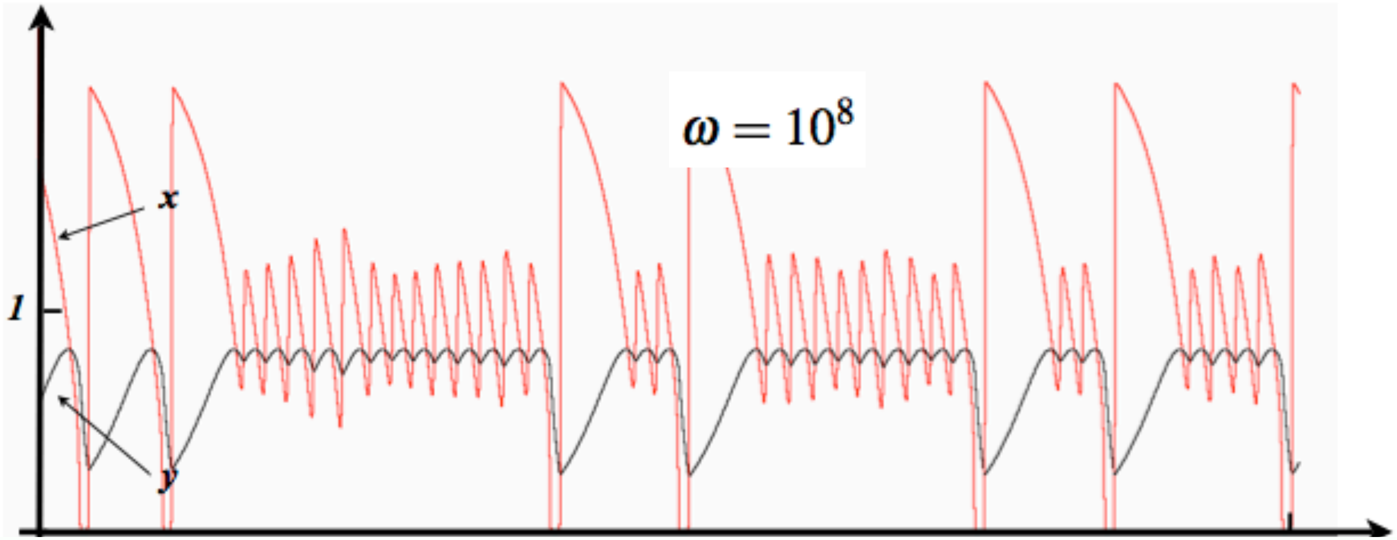
Ce n'est pas une surprise

$$\begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon}[f(x) - \mu(x)y] \\ \frac{dy}{dt} = (\mu(x) - m)y \end{cases}$$

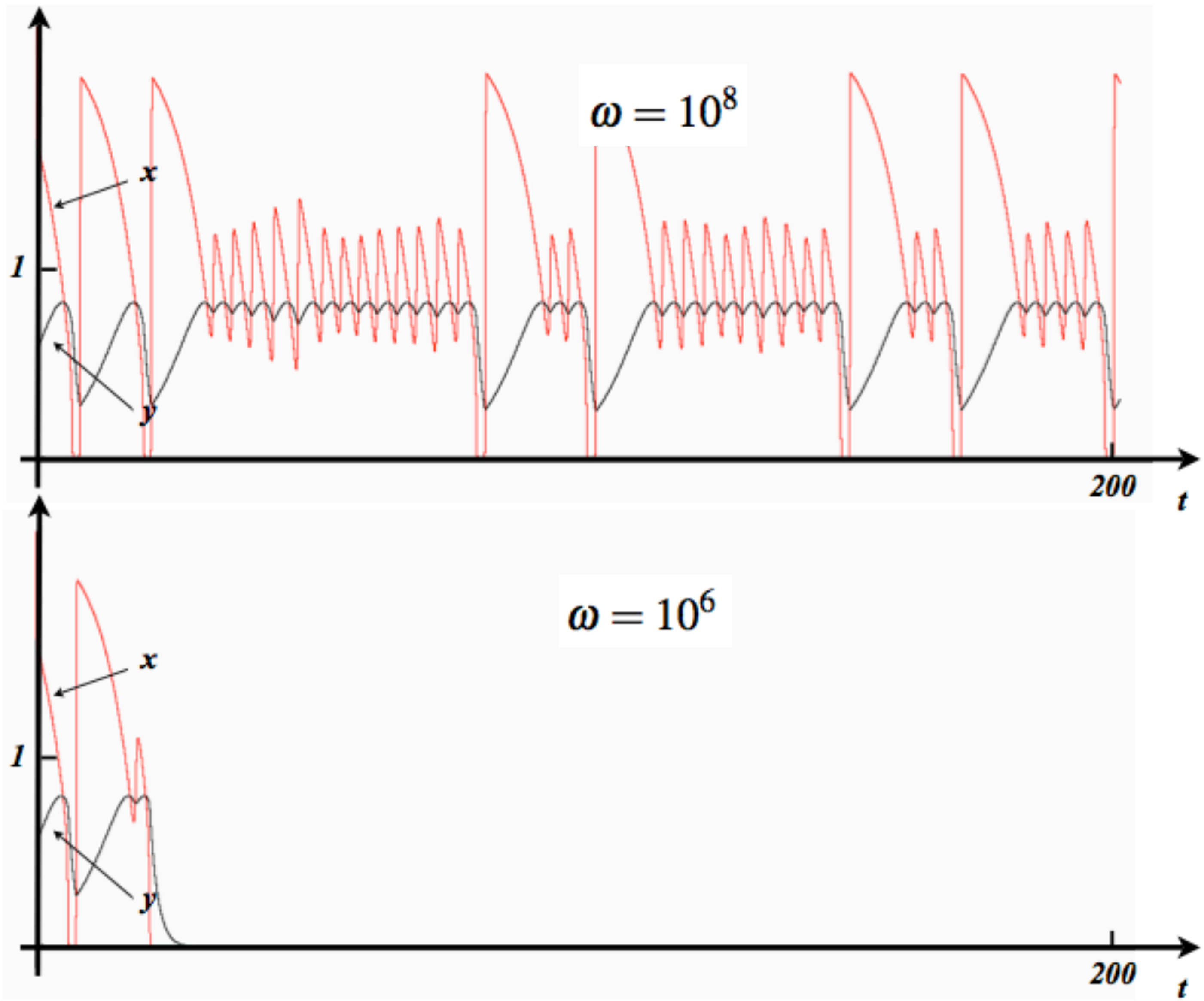
+bruit très (très) petit



$\omega = 10^{12}$
x
1
y
200
t
$\omega = 10^{10}$
x
1
y
200
t



C'est une (grosse surprise) car le nombre d'individus est toujours très grand.



$\omega = 10^8$
x
y
1
200
t
$\omega = 10^6$
x
y
1
200
t

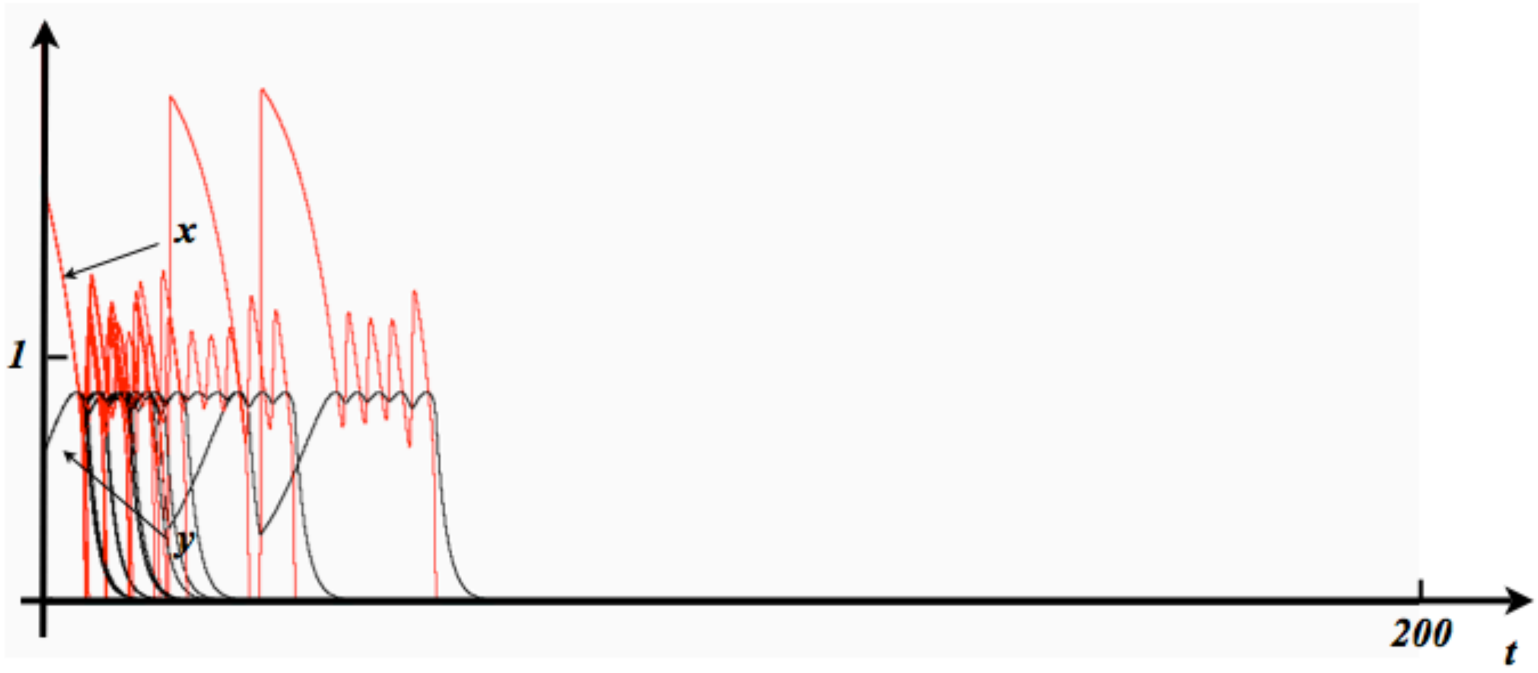


FIG. 3 – Twenty runs with $\omega = 10^6$

ω	$E[T]$	$\sigma(T)$	$P(T \leq 1000)$
10^5	25.8	3.3	1
10^6	32	9.5	1
10^7	143	118	1
$1.5\ 10^7$	410	346	0.95
$1.6\ 10^7$			0.82
$1.7\ 10^7$			0.70
$2\ 10^7$			0.55
$3\ 10^7$			0.06
10^8			0

Probabilité empirique d'extinction en fonction de ω
(1000 réalisations)

Extinction fixée à 1000 individus

$$x(t+dt)-x(t) \approx dt\frac{1}{\varepsilon}[f(x(t))-\mu(x(t))y(t)]+\sqrt{dt\frac{4}{\omega\varepsilon}\frac{f(x(t))\mu(x(t))y(t)}{(f(x(t))+\mu(x(t))y(t))}}W_t$$

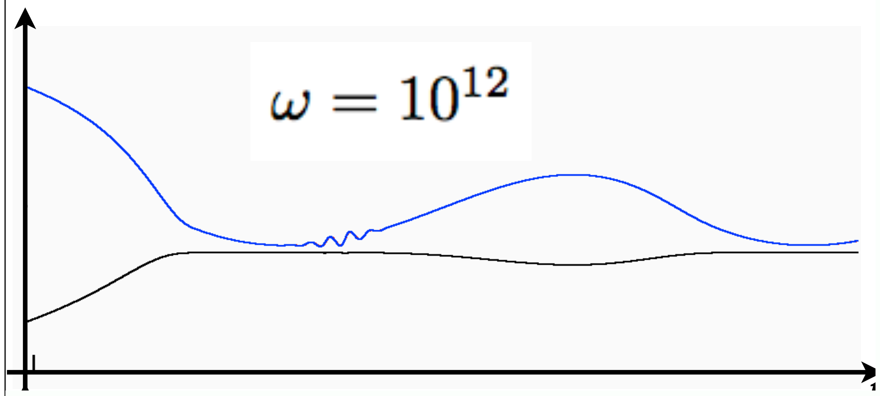
$$y(t+dt)-y(t) \approx dt[\mu(x(t)-m]y(t)+\sqrt{dt\frac{\varepsilon}{\omega}\frac{f(x(t))\mu(x(t))y(t)}{(f(x(t))+\mu(x(t))y(t))}}W_t$$

↓

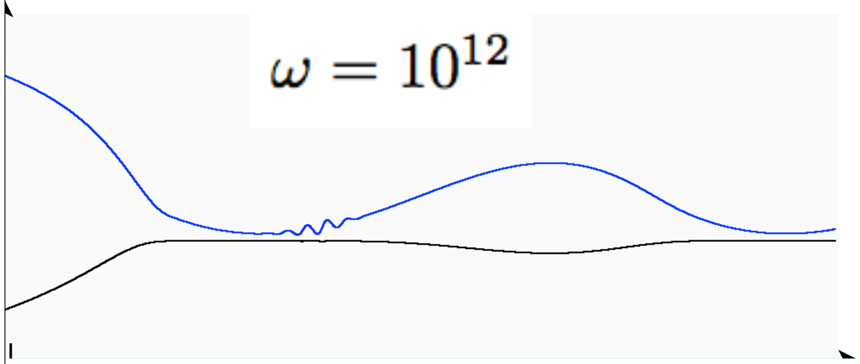
$$m(t) = a + b\cos(r\,t)$$

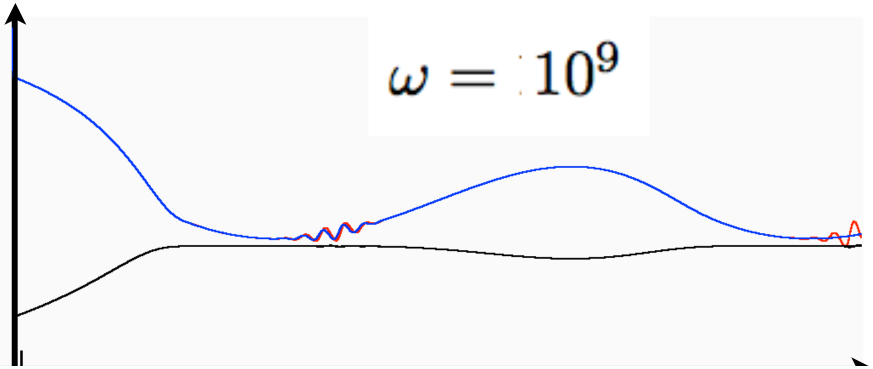
Simulations : **En rouge le modèle complet**

On “repasse”, en bleu, avec la partie déterministe seule

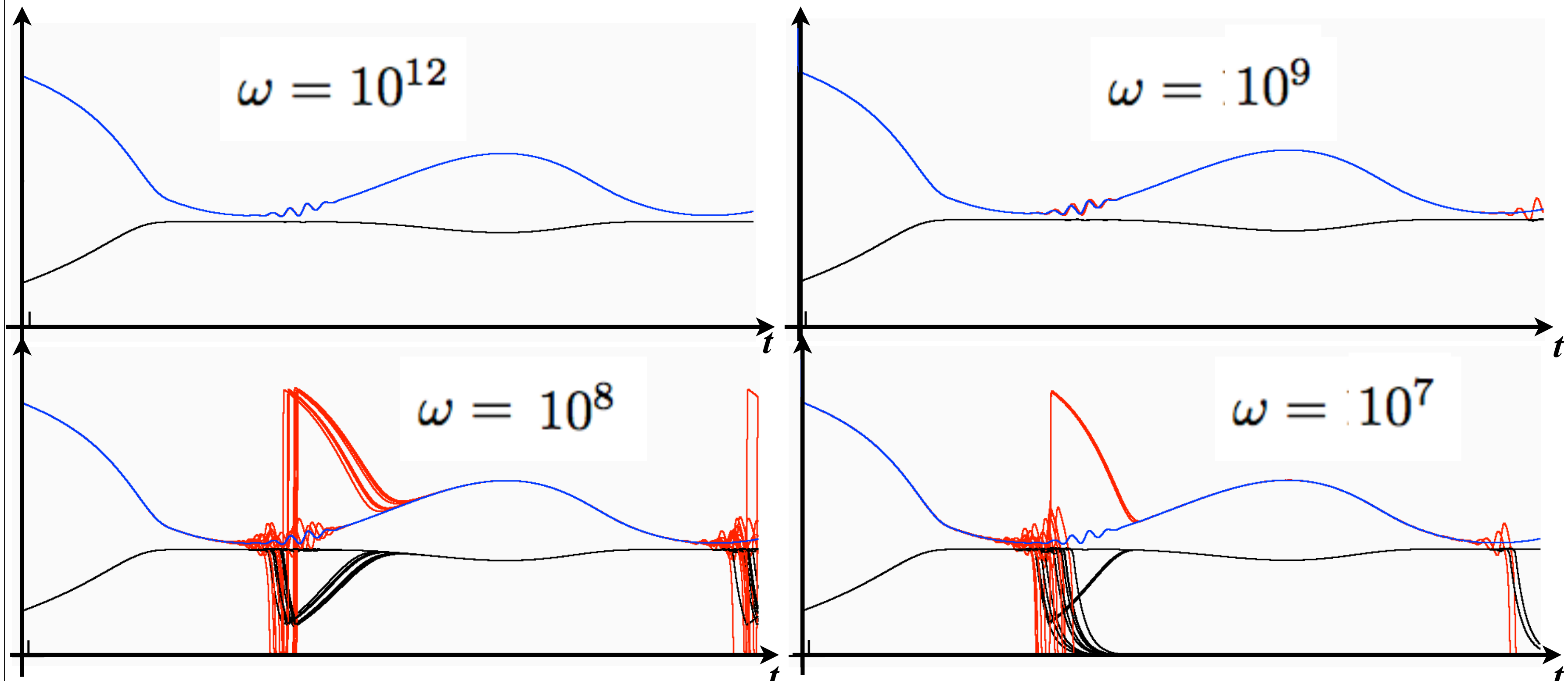


20 réalisations





20 réalisations



20 réalisations

Explication

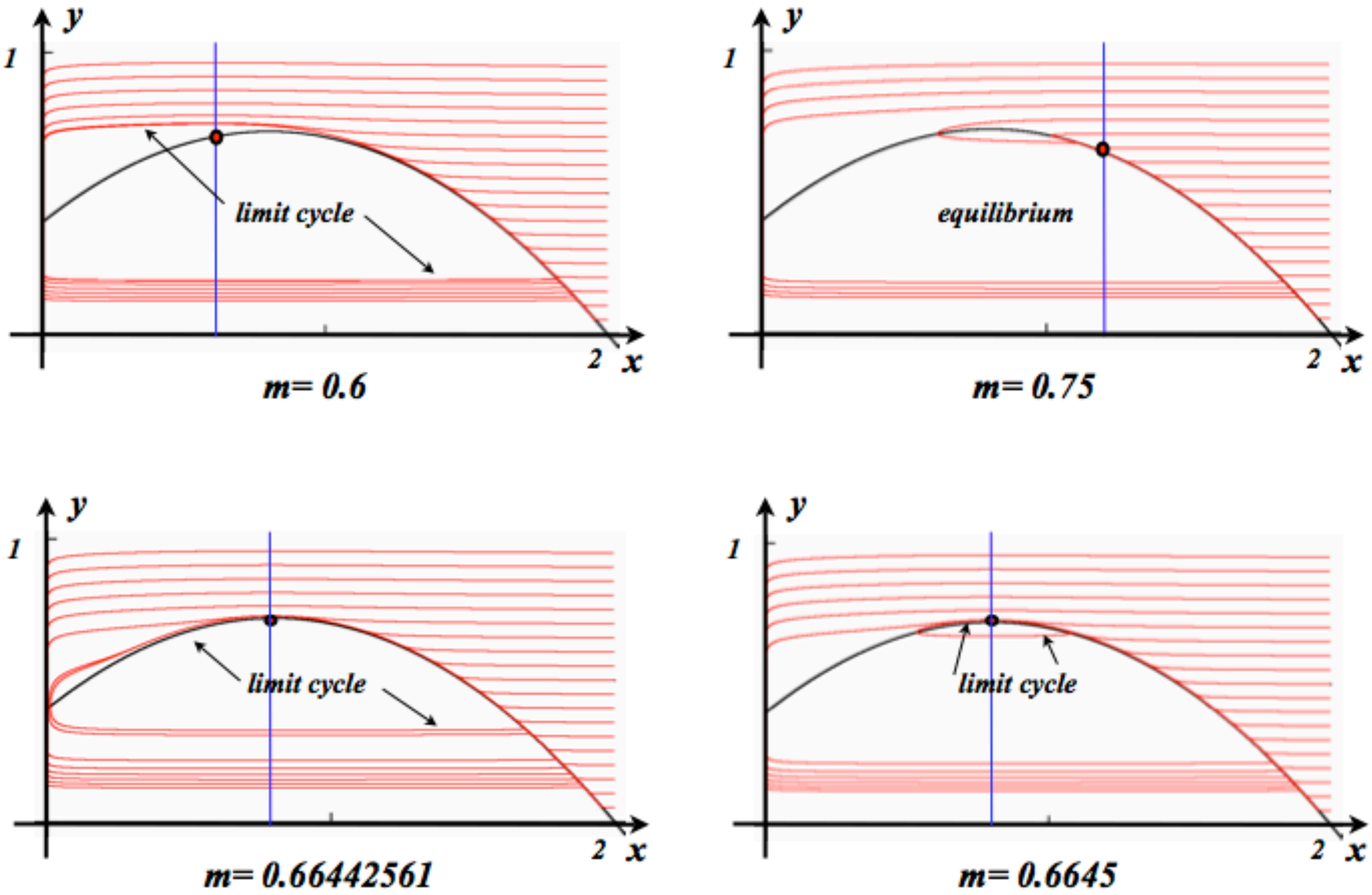


FIG. 6 – Phase portrait of system (7) for different values of m

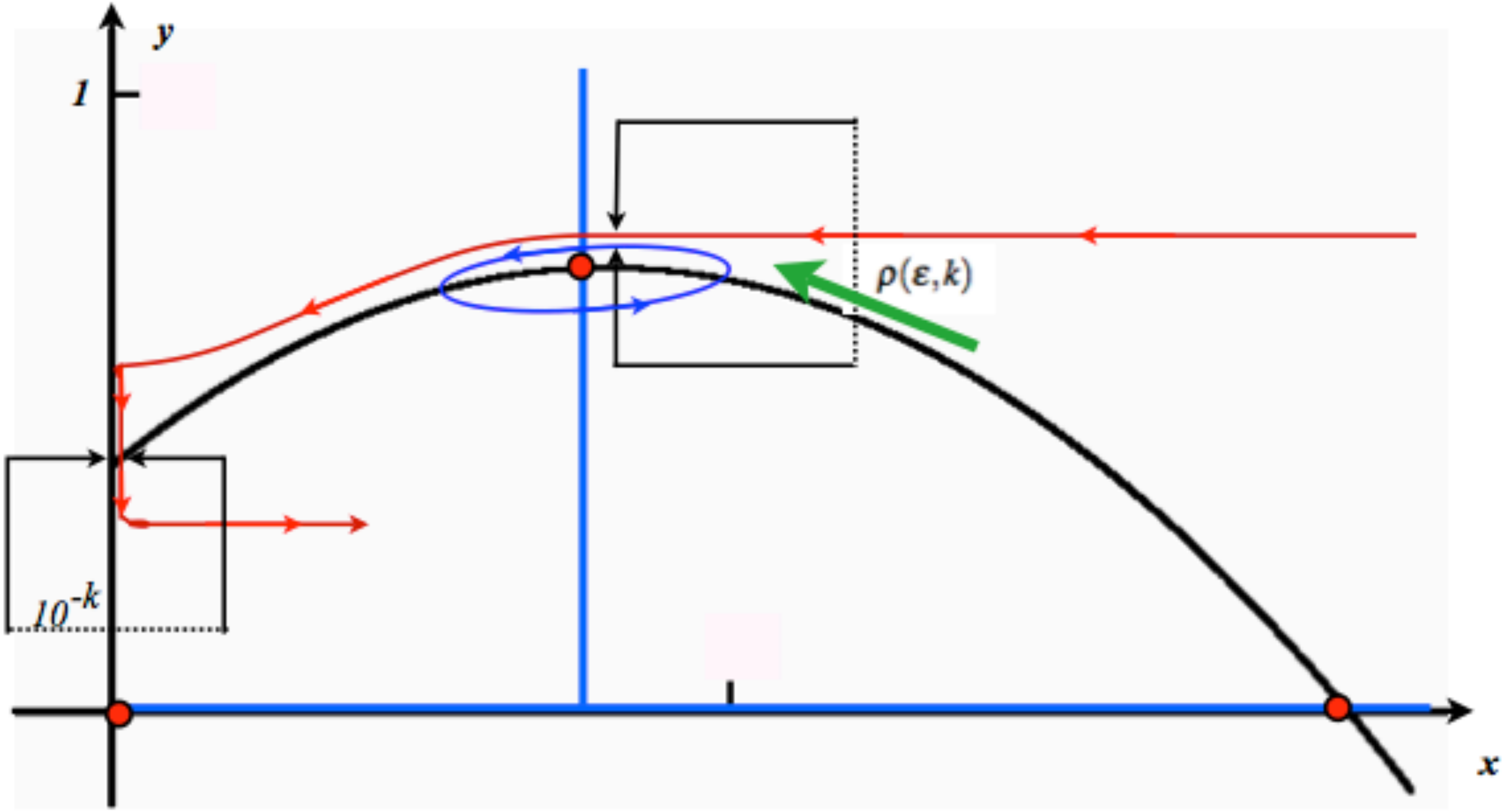


FIG. 9 – The "safety funnel"

ω	10^9	10^8	10^7	10^6
$\rho(\varepsilon,k)$	0.001208	0.000114	0.000054	0.000052

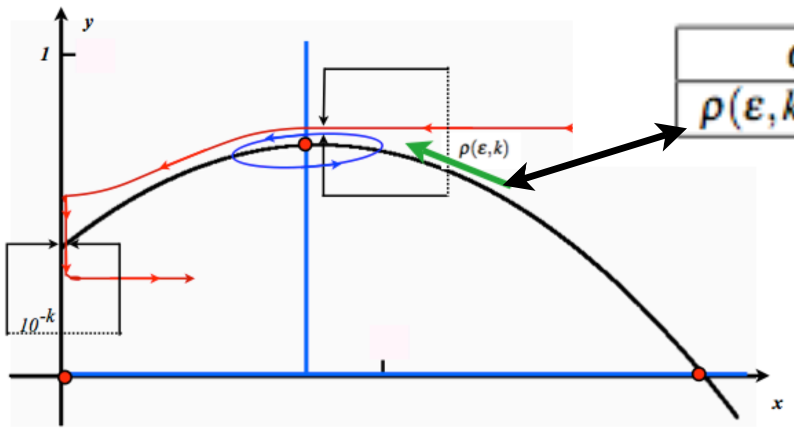
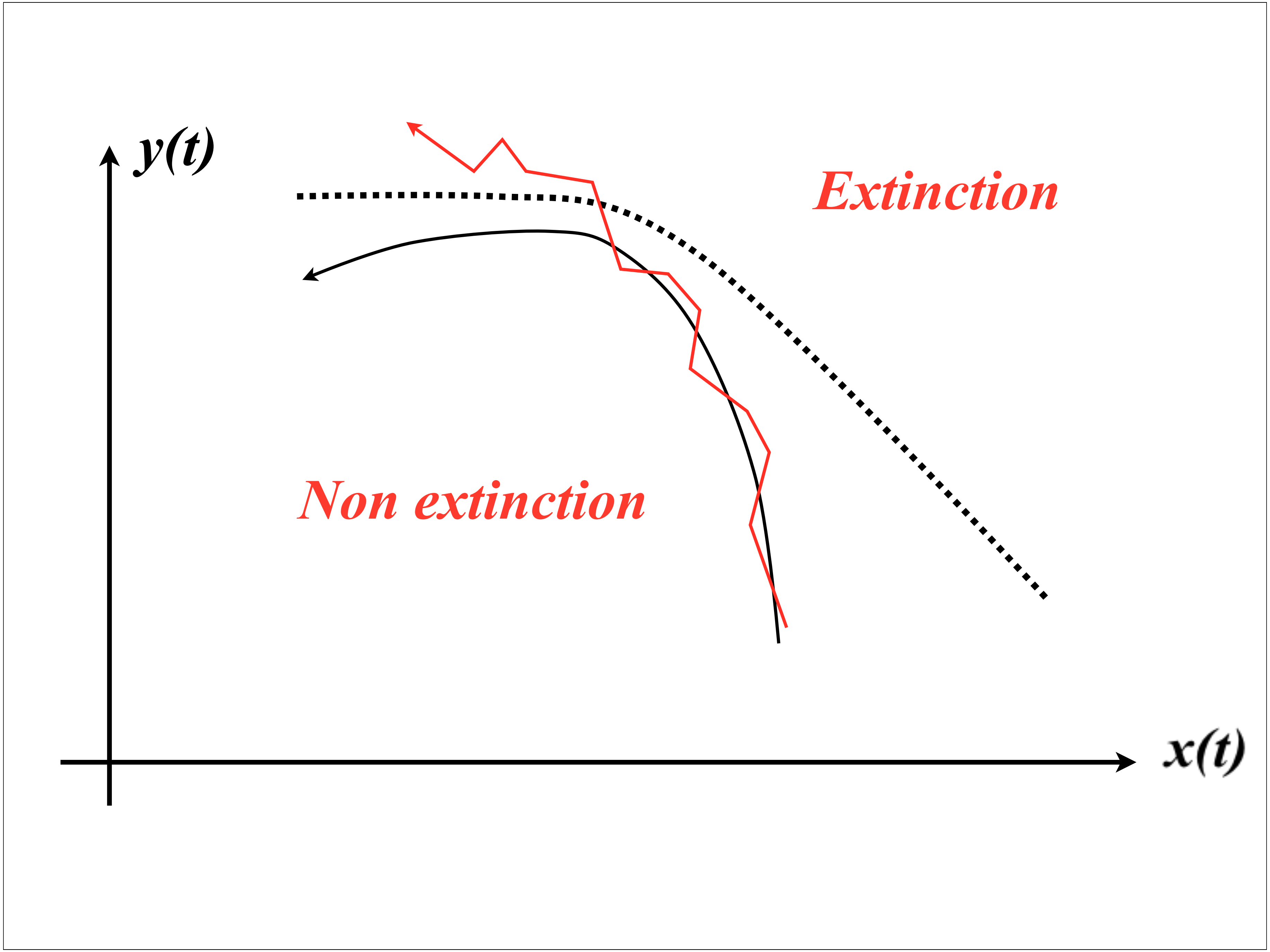


FIG. 9 – The "safety funnel"

ω	10^9	10^8	10^7	10^6
$\rho(\varepsilon,k)$	0.001208	0.000114	0.000054	0.000052

$$x(t+dt)-x(t)\approx dt\frac{1}{\varepsilon}[f(x(t))-mu(x(t))y(t)]+\sqrt{dt\frac{4}{\omega\varepsilon}\frac{f(x(t))\mu(x(t))y(t)}{(f(x(t))+\mu(x(t))y(t))}}W_t$$

$$y(t+dt)-y(t)\approx dt[\mu(x(t)-m]y(t)+\sqrt{dt\frac{\varepsilon}{\omega}\frac{f(x(t))\mu(x(t))y(t)}{(f(x(t))+\mu(x(t))y(t))}}W_t$$


y(t)
Extinction
Non extinction
x(t)