Persistence dans les systèmes commutés de Dynamic population

Claude Lobry
Montpellier Octobre 2015

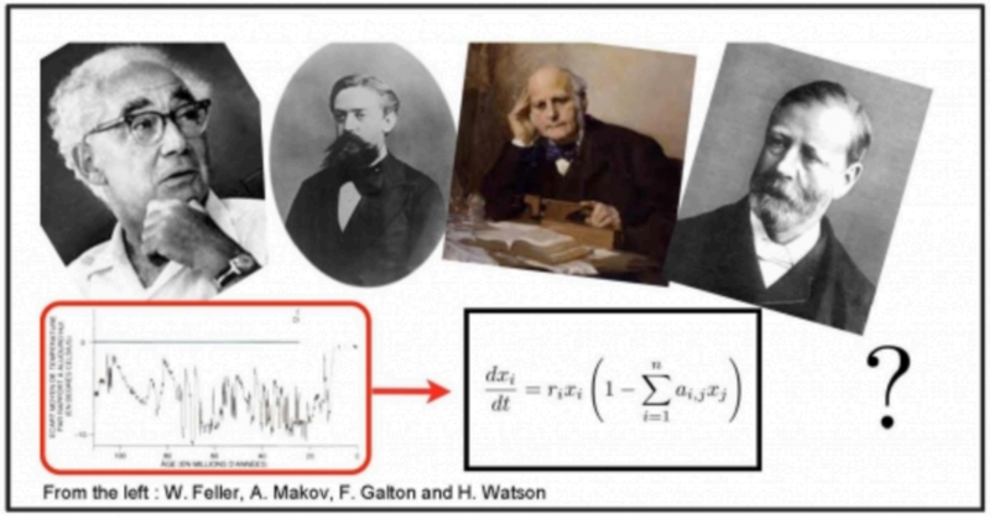
Version française
de....

Version française de....

Persistence in certain Switched Dynamic Population Systems

Fait à l'EPFL le 09-02/15 dans le cadre de

THE ROLE OF MATHEMATICS AND COMPUTER SCIENCE IN ECOLOGICAL THEORY



Persistence of population models in temporally fluctuating environments

A follow-up workshop to the semester
The role of mathematics and computer science in ecological theory
CIB/EPFL

9 - 12 February 2015
Room BI A0 448 - EPFL

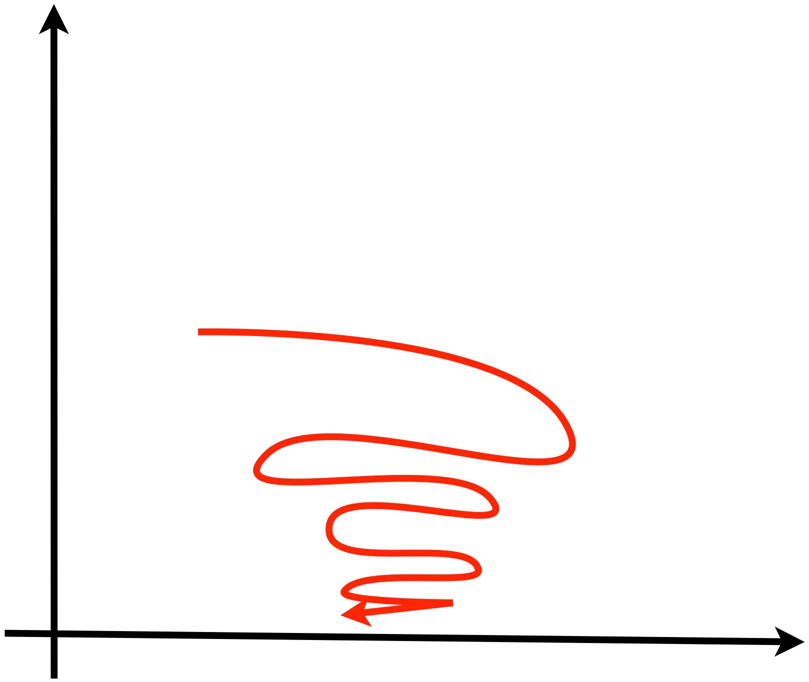
A workshop organised by
Roger ARDITI (Université de Fribourg)
Michel BENAIM (Université de Neuchâtel)
Claude LOBRY (INRIA)

Participants
Vincent Bansaye, Yacine Chitour, Bertrand Cloez, Fritz Colonius, Jérôme Coville, Lorens Imhof, Florent Malrieu, Guilherme Mazanti, Christian Mazza, Alain Rapaport, Gregory Roth, Gauthier Sallet, Nadir Sari, Tewfik Sari, Mario Sigalotti, Pierre-andré Zitt

http://mathcompecol.epfl.ch

$$\frac{dx_i}{dt} = x_i F_i(x_1, x_2, ..., x_n) \qquad x_i \geq 0$$

Pas de migration



$$\frac{dx_i}{dt} = x_i F_i(x_1, x_2, ..., x_n) \qquad x_i \geq 0$$

Persistence faible

$$\forall\ x_i(0) > 0\ \exists\ a > 0; \exists\ M > 0 : a \leq x_i(t) \leq M$$

Persistence forte

$$\exists\ a > 0; \exists\ M > 0\ \forall\ x_i(0) > 0 :$$

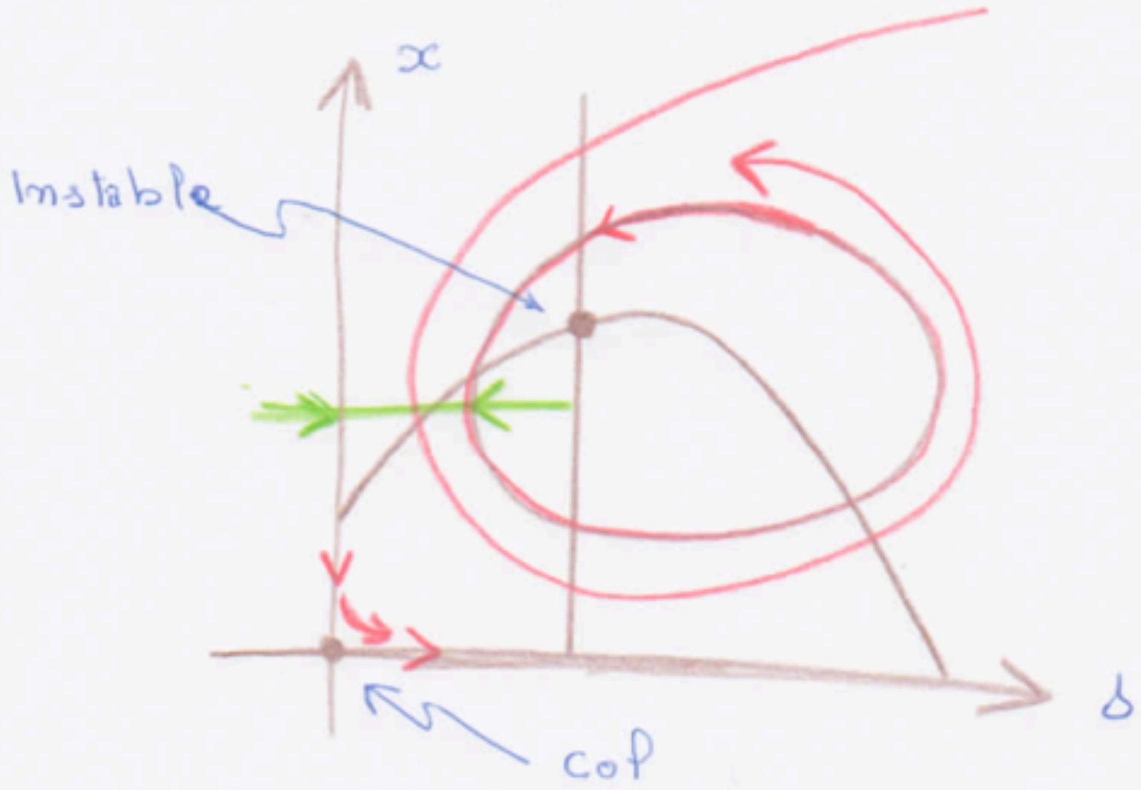
$$a \leq \underline{\lim}\ x_i(t) \leq \overline{\lim}\ x_i(t) \leq M$$

Gause-Rosenzweig-MacArthur
est fortement persistant (équilibre ou cycle limite G.A.S.)

Le modèle de Gause Rosenzweig-Mac Arthur

$$\begin{aligned} \frac{dx}{dt} &= rx\left(1 - \frac{x}{K}\right) - \frac{a\ x}{e+x}y \\ \frac{dy}{dt} &= \varepsilon\frac{a\ x}{e+x}y - \varepsilon m y \end{aligned}$$

$$\begin{aligned} \frac{dx}{dt} &= \frac{1}{\varepsilon}\left(rx\left(1 - \frac{x}{K}\right) - \frac{a\ x}{e+x}y\right) \\ \frac{dy}{dt} &= \frac{a\ x}{e+x}y - m y \end{aligned}$$

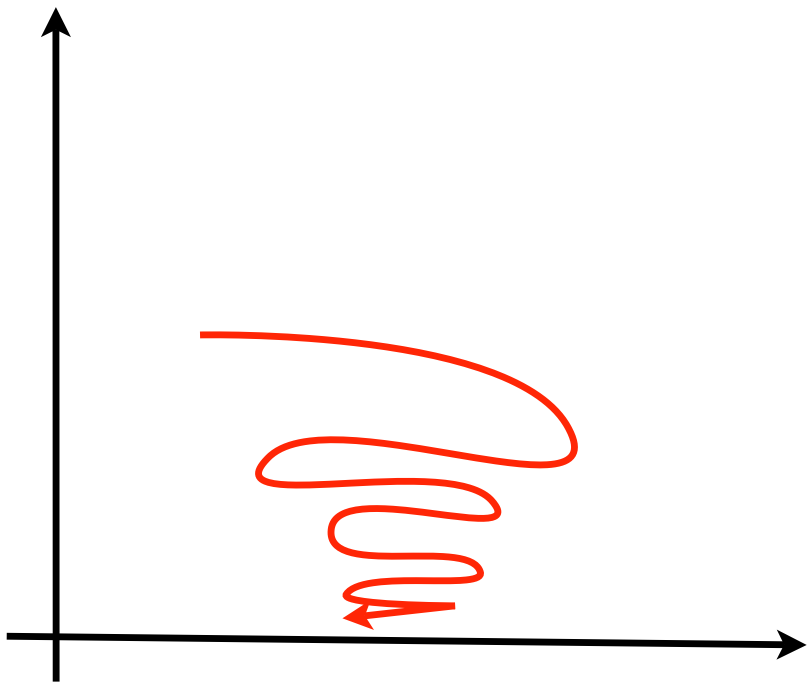


$\liminf x_i(t), \liminf s(t) =$ distance du cycle aux axes

Même question avec 2 environnements

$$\frac{dx_i}{dt} = x_i F_i(x_1, x_2, ..., x_n, u)$$

$$u \in \{1, 2\}$$



Persistence inconditionnelle

$$\frac{dx_i}{dt} = x_i F_i(x_1, x_2, ..., x_n, u) \qquad u \in \{1, 2\}$$

$$\forall t \mapsto u(t) \; \forall x_i(0) \qquad \exists \, a > 0 \; \exists \, M > 0 :$$

$$a \leq \underline{\lim} \, x_i(t) \leq \overline{\lim} \, x_i(t) \leq M$$

$$\exists \, a > 0 \; \exists \, M > 0 \qquad \forall x_i(0) \; ; \; \forall t \mapsto u(t)$$

$$a \leq \underline{\lim} \, x_i(t) \leq \overline{\lim} \, x_i(t) \leq M$$

On suppose que chaque système est persistant :
Est-ce que le système commuté est inconditionnellement persistant ?

Le cas de la persistance forte

C. R. Acad. Sci. Paris, Sciences de la vie/Life sciences, 1994; 317 : 102-7
Écologie/Ecology

Effets paradoxaux des fluctuations de l'environnement sur la croissance des populations et la compétition entre espèces

CLAUDE LOBRY, ANTOINE SCIANDRA, PAUL NIVAL

Observatoire des Sciences de l'Univers, Station Zoologique URA CNRS 716, Université Paris-VI/INSU/CNRS, B.P. 28, 06230, Villefranche-sur-Mer, France.
Reprints : C. Lobry

Le cas de la persistance forte

Modification de l'issue d'une compétition

Nous supposons que deux espèces X et Y d'abondance x et y sont en compétition. En première approximation, nous supposons que la compétition est convenablement modélisée par des équations de type Lotka-Volterra lorsque l'environnement est stable. Nous choisissons les coefficients des deux situations environnementales extrêmes U_1 et U_2 de telle manière que les dynamiques correspondent aux *Figures 4 a* et *4 b*, ce que nous savons faire par l'étude des isoclines des systèmes :

$$U_1 \begin{cases} x'(t) = 5\,x(t)\,(1 - x(t)/2 - y(t)/2) \\ y'(t) = y(t)\,(1 - x(t)/3 - y(t)/3) \end{cases}$$

$$U_2 \begin{cases} x'(t) = x(t)\,(1 - 2\,x(t) - 2\,y(t)) \\ y'(t) = 5\,y(t)\,(1 - x(t) - y(t)) \end{cases}$$

$$\Sigma_1 \begin{cases} \dot{x} = 5\,x(1 - x - y) \\ \dot{y} = 1\,y(1 - 0.8x - 1.1y) \end{cases}$$

$$\Sigma_2 \begin{cases} \dot{x} = 1\,x(1 - 1.2x - 1.2y) \\ \dot{y} = 5\,y(1 - 1.1x - 1.25y) \end{cases}$$

Le cas de la persistance forte

C. Lobry *et al.*

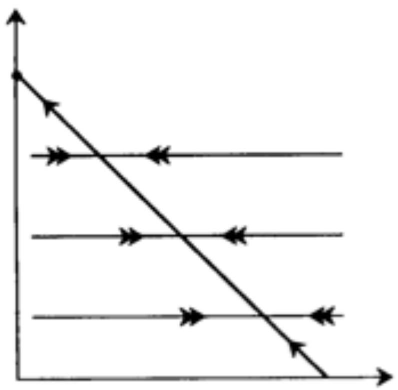


Figure 4 a.

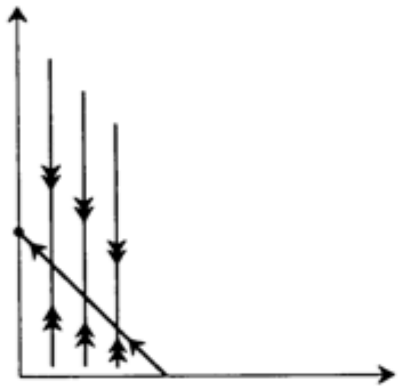


Figure 4 b.

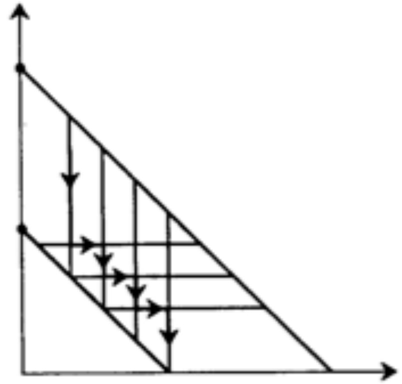


Figure 4 c.

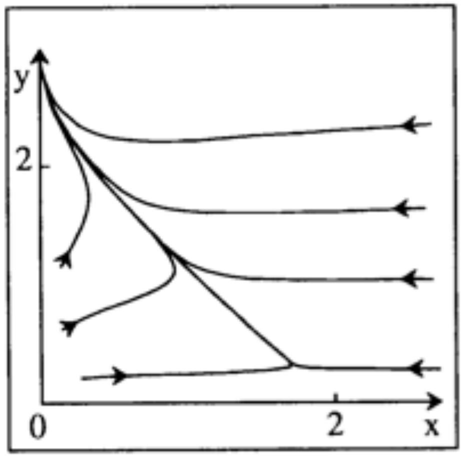


Figure 5 a.

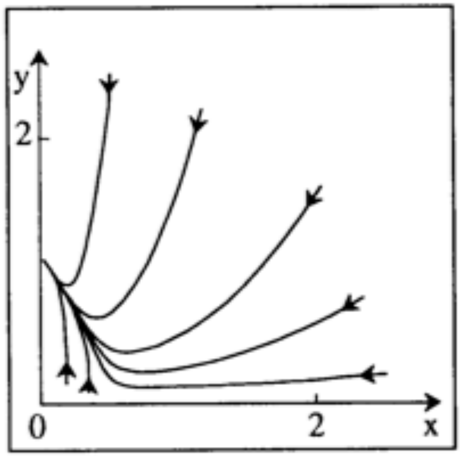


Figure 5 b.

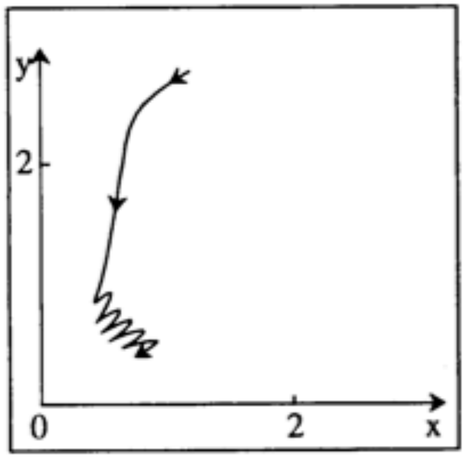
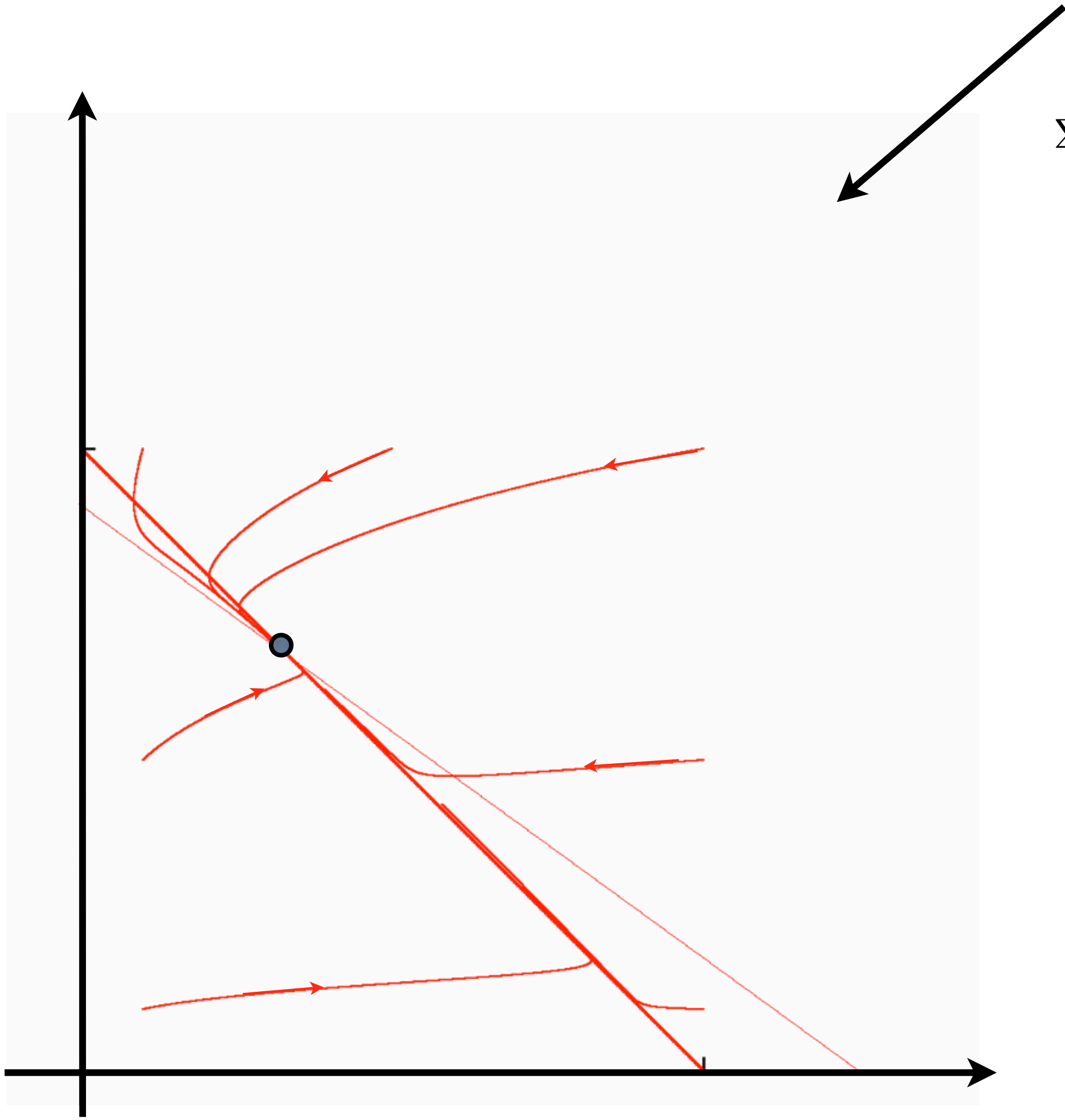


Figure 5 c.

Le cas de la persistance forte

$$\Sigma_1 \left\{ \begin{array}{lcl} \dot{x} & = & 5\,x(1-x-y) \\ \dot{y} & = & 1\,y(1-0.8x-1.1y) \end{array} \right.$$

$$\Sigma_2 \left\{ \begin{array}{lcl} \dot{x} & = & 1\,x(1-1.2x-1.2y) \\ \dot{y} & = & 5\,y(1-1.1x-1.25y) \end{array} \right.$$



Le cas de la persistance forte

$$\Sigma_1 \left\{ \begin{array}{rcl} \dot{x} & = & 5\,x(1-x-y) \\ \dot{y} & = & 1\,y(1-0.8x-1.1y) \end{array} \right.$$

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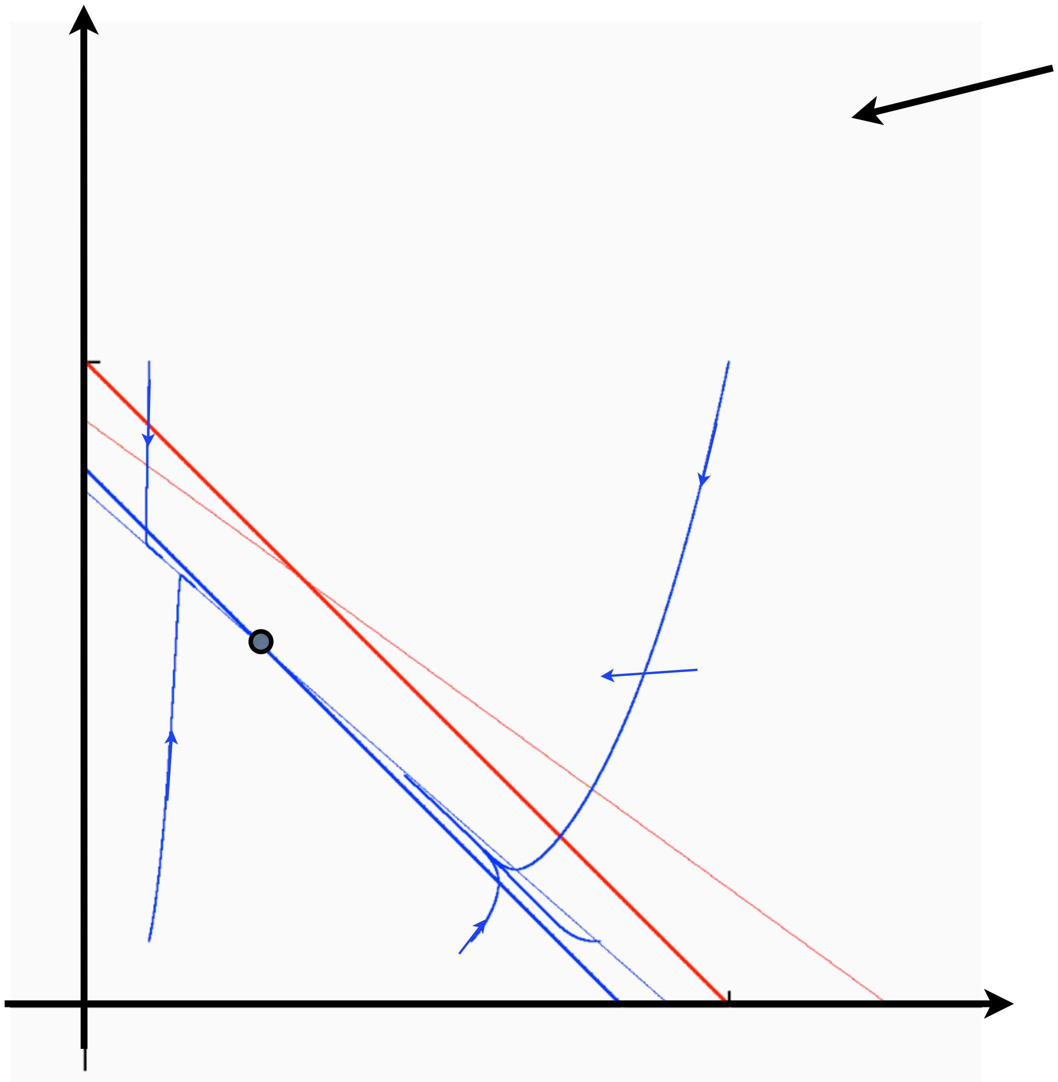
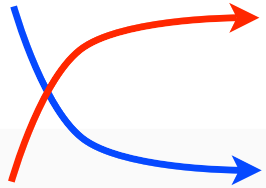


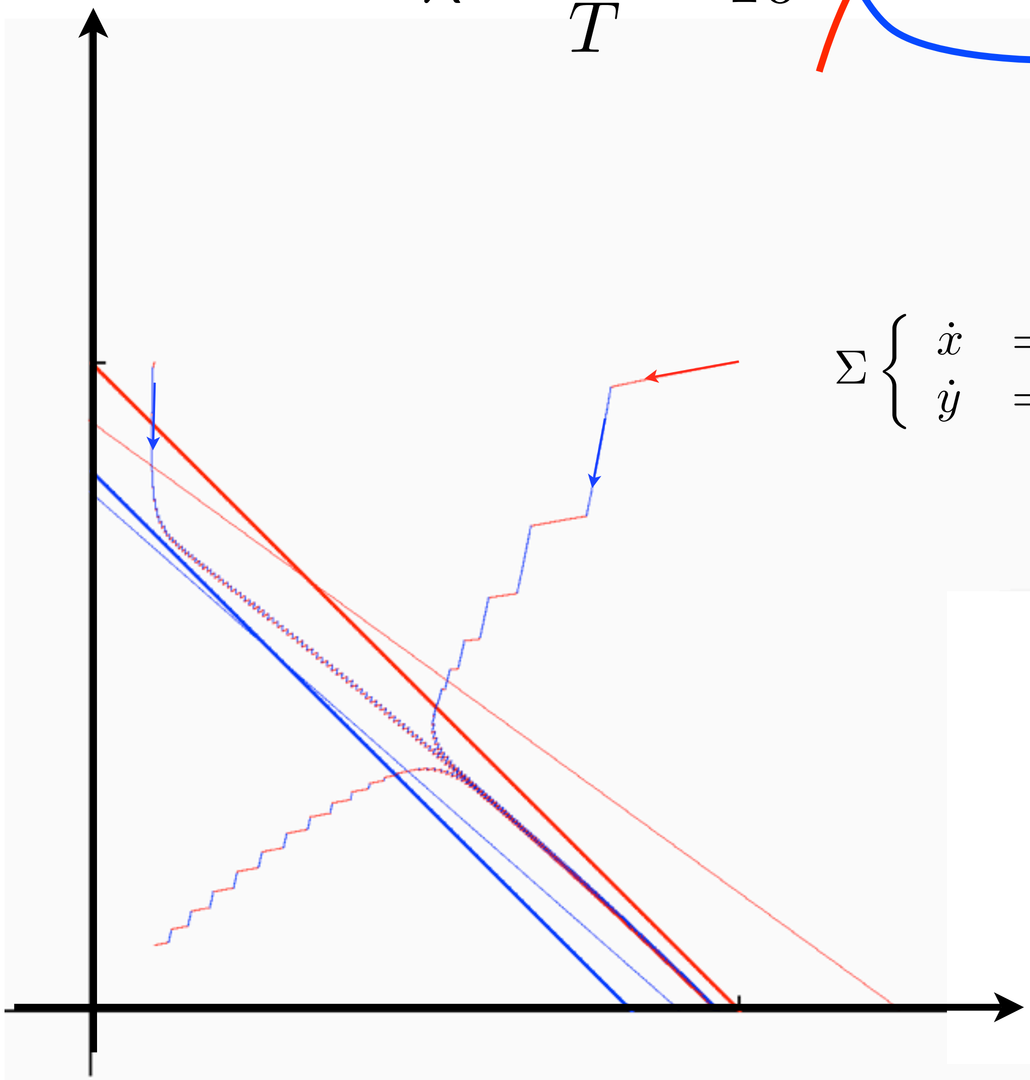
Le cas de la persistance forte

$$\lambda = \frac{1}{T} = 10$$



$$\Sigma_1 \left\{ \begin{array}{lll} \dot{x} & = & 5\,x(1-x-y) \\ \dot{y} & = & 1\,y(1-0.8x-1.1y) \end{array} \right.$$

$$\Sigma_2 \left\{ \begin{array}{lll} \dot{x} & = & 1\,x(1-1.2x-1.2y) \\ \dot{y} & = & 5\,y(1-1.1x-1.25y) \end{array} \right.$$

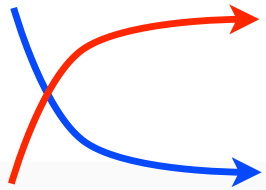


$$\Sigma \left\{ \begin{array}{lll} \dot{x} & = & 1/2\,x(6-6.2x-6.2y) \\ \dot{y} & = & 1/2\,y(6-6.3.8x-7.35y) \end{array} \right.$$

Non persistant

Le cas de la persistance forte

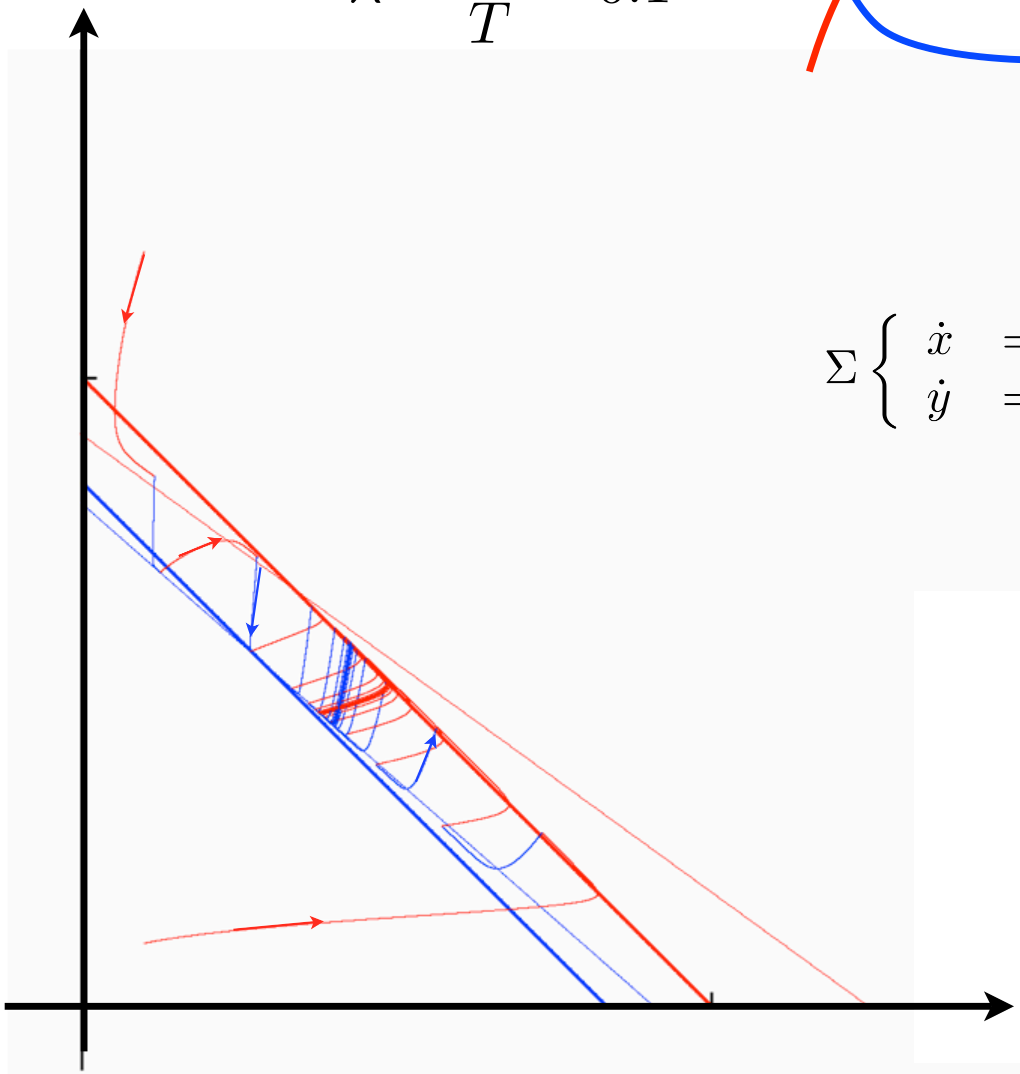
$$\lambda = \frac{1}{T} = 0.1$$



$$\Sigma_1 \begin{cases} \dot{x} &= 5\,x(1 - x - y) \\ \dot{y} &= 1\,y(1 - 0.8x - 1.1y) \end{cases}$$

$$\Sigma_2 \begin{cases} \dot{x} &= 1\,x(1 - 1.2x - 1.2y) \\ \dot{y} &= 5\,y(1 - 1.1x - 1.25y) \end{cases}$$

$$\Sigma \begin{cases} \dot{x} &= 1/2\,x(6 - 6.2x - 6.2y) \\ \dot{y} &= 1/2\,y(6 - 6.3.8x - 7.35y) \end{cases}$$



Persistant

Eléments de contexte :

Systèmes linéaires commutés

$$\dot{x} = u(t)Ax + (1 - u(t))Bx \; ; \; u(t) \in \{0, 1\}$$

Inconditionnellement stable

Boscain, Balde, Sigalotti..........2000

Eléments de contexte :

P.D.M.P.

Piecewise Deterministic Makov Processes

Davis, M. H. A. (1984). "Piecewise-Deterministic Markov Processes: A General Class of Non-Diffusion Stochastic Models". *Journal of the Royal Statistical Society. Series B (Methodological)* **46** (3): 353–388. JSTOR 2345677.

The Annals of Applied Probability

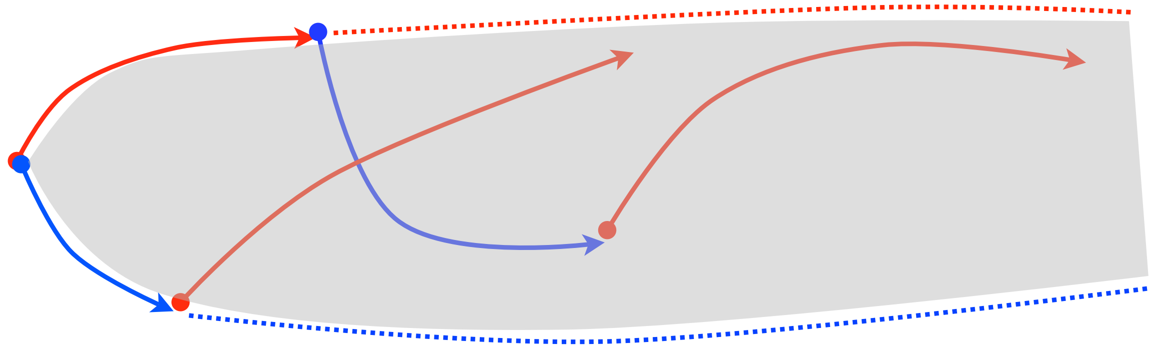
Info | Current issue | All issues | Search

Ann. Appl. Probab.
Volume 24, Number 1 (2014), 292-311.

On the stability of planar randomly switched systems

Michel Benaïm, Stéphane Le Borgne, Florent Malrieu, and Pierre-André Zitt

Contrôlabilité
(1970)

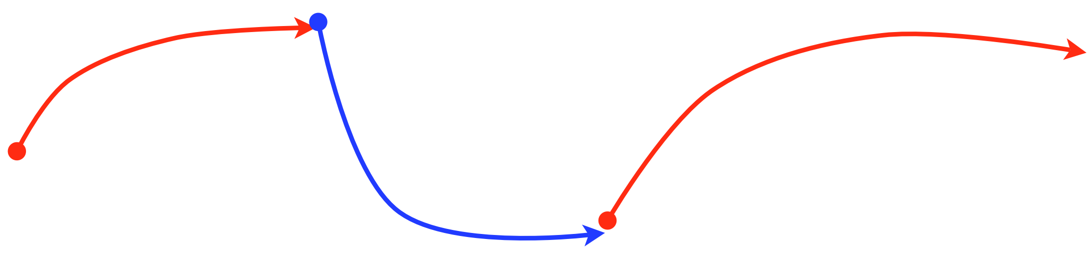


$$X_{t_3} \circ Y_{t_2} \circ X_{t_1}(x_o)$$

Décrire l'ensemble des points accessibles

Hermes, Jurdjevic, Krener, Kupka, Lobry, Sallet, Sussman....

P.D.M.P.
Piecewise Deterministic Makov Processes

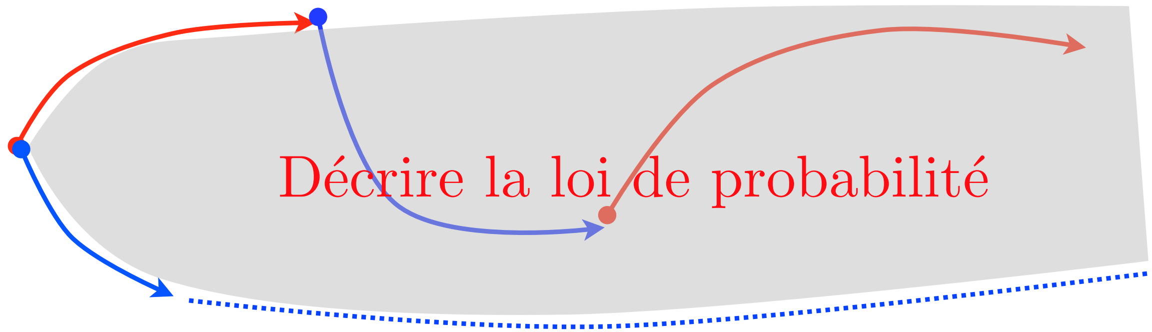


$$X_{t_3} \circ Y_{t_2} \circ X_{t_1}(x_o)$$

- Les t_i sont aléatoires selon une loi exponentielle.
- On tire au sort X ou Y selon une loi de proba fixée.

$$X_{t_3} \circ Y_{t_5+t_4+t_3} \circ X_{t_2+t_1}(x_o)$$

P.D.M.P.
Piecewise Deterministic Makov Processes



$$X_{t_3} \circ Y_{t_2} \circ X_{t_1}(x_o)$$

- Les t_i sont aléatoires selon une loi exponentielle.
- On tire au sort X ou Y selon une loi de proba fixée.

$$X_{t_3} \circ Y_{t_5+t_4+t_3} \circ X_{t_2+t_1}(x_o)$$

Michel Benaim
Persistance stochastique

BERNOULLI LECTURE III

Part the role of mathematics and computer science in ecological theory

CIB/EPFL

Stochastic Persistence

An important issue in ecology is to understand under which conditions a group of interacting species - whether they are plants, animals, or viral particles - can coexist over long periods of time. A fruitful approach to this question has been the development of nonlinear models of deterministic interactions, leading to what is now known as *the Mathematical theory of persistence*. *Persistence* amounts to saying that the dynamical system describing the species interactions admits an *attractor* bounded away from *extinction* (i.e. the subset of the state-space where the abundance of one or more species vanishes).

Beside biotic interactions, environmental fluctuations play a key role in population dynamics. In order to take into account these fluctuations and to understand how they may affect persistence, deterministic models need to be replaced by stochastic ones and the theory needs to be revisited.

This talk will survey recent results in this direction laying the groundwork for a mathematical theory of *stochastic persistence*.

Part of this work stems from a close collaboration between Neuchatel's research group in probability and UC Davis department of Evolution and Ecology.

Michel Benaïm

Université de Neuchâtel

Thursday 16 October, 2014
16H15 - Room BI A0 448

Two (canonical) Models

Example (Ecological SDEs)

$$dx_i = x_i[F_i(x)dt + \sum_{j=1}^{m} \sigma_i^j(x)dB_t^j],\ i = 1 \dots n$$

- $x_i \geq 0 =$ abundance of species i.
- *State space* $M = \mathbb{R}_+^n$
- *Extinction set* $M_0 = \{x \in M \ : \prod_i x_i = 0\}$

Two (canonical) Models

Example (Ecological random ODE)

$$\frac{dx_i}{dt} = x_i(t)F_i(x(t), u(t)),\ i = 1 \dots n$$

where

$$u(t) \in \{1, \dots, m\}$$

is a Markov process controlled by x

Stochastic Persistence

- $\Pi_t(.) = \frac{1}{t}\int_0^t \delta_{x(s)} ds$ = empirical occupation measure

$\Pi_t(A) =$ proportion of time spent in A up to t

Definition

We call the process *stochastically persistent* if for all $\epsilon > 0$ there exists a compact $K \subset M_+$ such that

$$\liminf_{t\to\infty} \Pi_t(K) \geq 1 - \epsilon$$

whenever $x = x(0) \in M_+$

Théorèmes de persistance stochastique
(plutôt complexes)

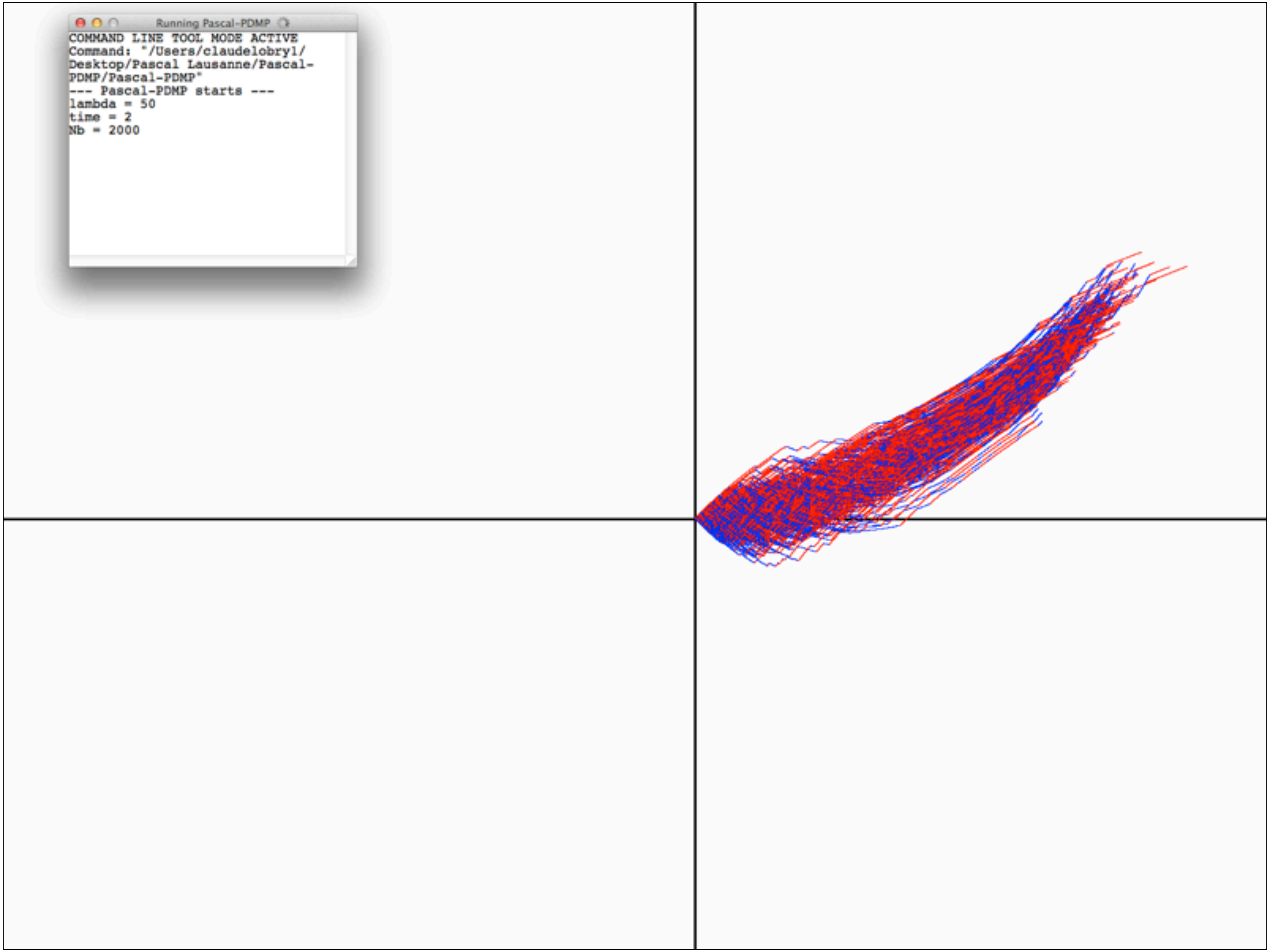
La persistance inconditionnelle entraine évidemment la persistance stochastique

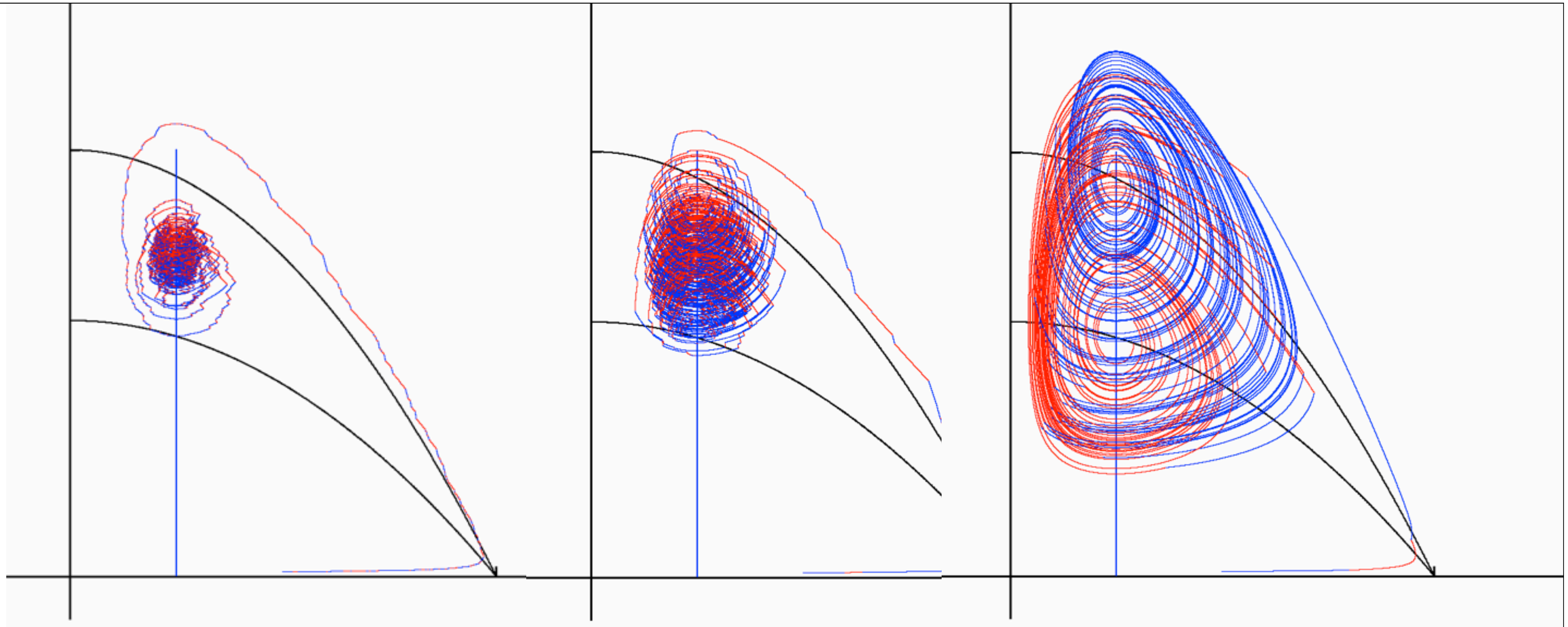
P.D.M.P.
Piecewise Deterministic Makov Processes

Un outil de simulation
des ensembles d’états accessibles

```
procedure Tempsexpo;
begin
    w:= random/32770;
    w:= 0.5*(1+w);
    DeltaT := -ln(w)/lambda;
end;

procedure PDMP ;
Var
thePort : GrafPtr;
begin
xa := xo;
ya := yo;
t := 0;
repeat
    tempsexpo;
    w:= random/32770;w:= 0.5*(1+w);
    if w < 0.5 then
    begin;
      tt := 0;
      repeat
        tt := tt+dt;
        t := t+dt;
        xn:= xa +dt*( f1(xa,ya));
        yn:= ya +dt*( g1(xa,ya)) ;
        xa := xn;
        ya := yn;
        GProuge(xa,ya,1);
      until (tt > DeltaT) or (t > Time);
    GetPort(thePort);
    QDFlushPortBuffer(thePort, nil);
    end else ..............................
```


Running Pascal-PDMP
COMMAND LINE TOOL MODE ACTIVE
Command: "/Users/claudelobry1/Desktop/Pascal Lausanne/Pascal-PDMP/Pascal-PDMP"
--- Pascal-PDMP starts ---
lambda = 50
time = 2
Nb = 2000



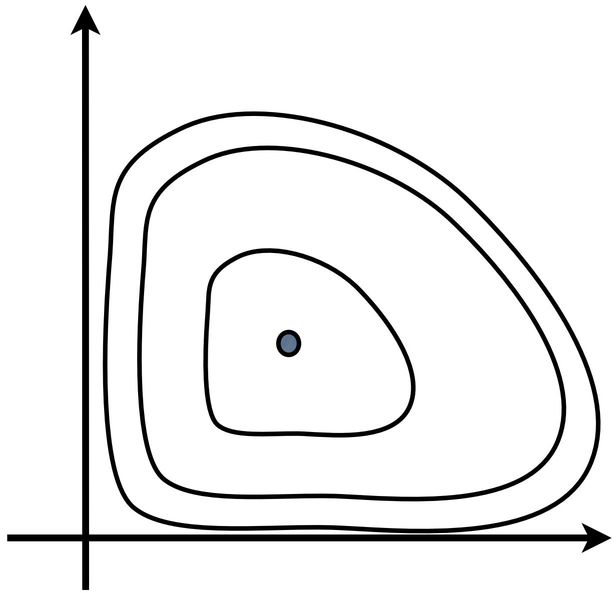
$\lambda = 20$ $\lambda = 5$ $\lambda = 0.2$

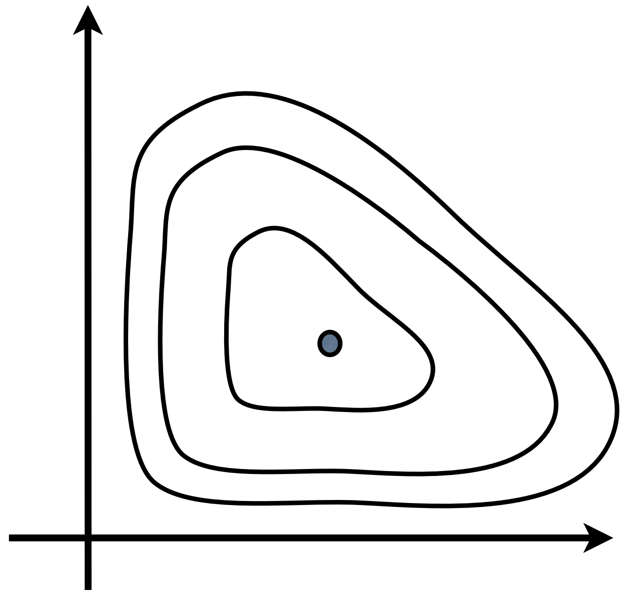
Retour sur la persistence inconditionnelle

Le cas de la persistance faible

$$\frac{dx}{dt} = a_1 x - b_1 xy \qquad \frac{dx}{dt} = a_2 x - b_2 xy$$

$$\frac{dy}{dt} = c_1 xy - d_1 y \qquad \frac{dy}{dt} = c_2 xy - d_2 y$$



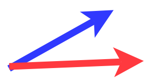


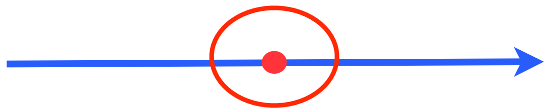
Le cas de la persistance faible

Un vieux résultat de 74 :

- rang $\mathcal{L}\{f(x,1), f(x,2)\} = n$
- les trajectoires de $f(x,i))$ sont récurrentes

alors tout point est accessible de tout point.



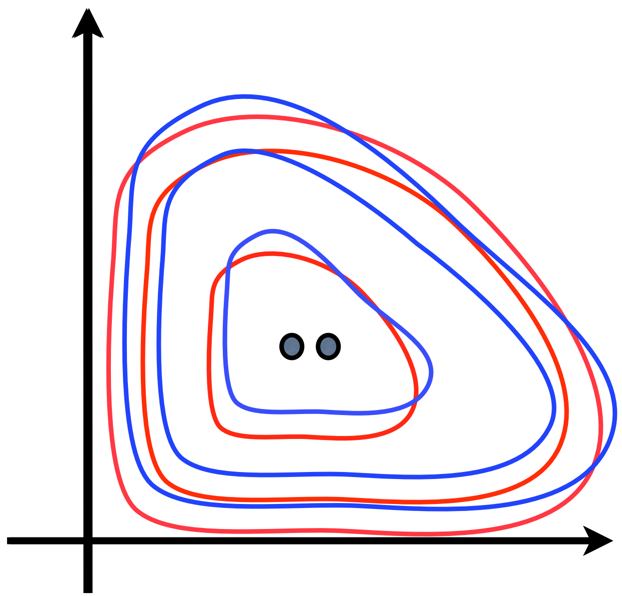


Le cas de la persistance faible

$$\frac{dx}{dt} = a_1 x - b_1 xy \qquad \frac{dx}{dt} = a_2 x - b_2 xy$$

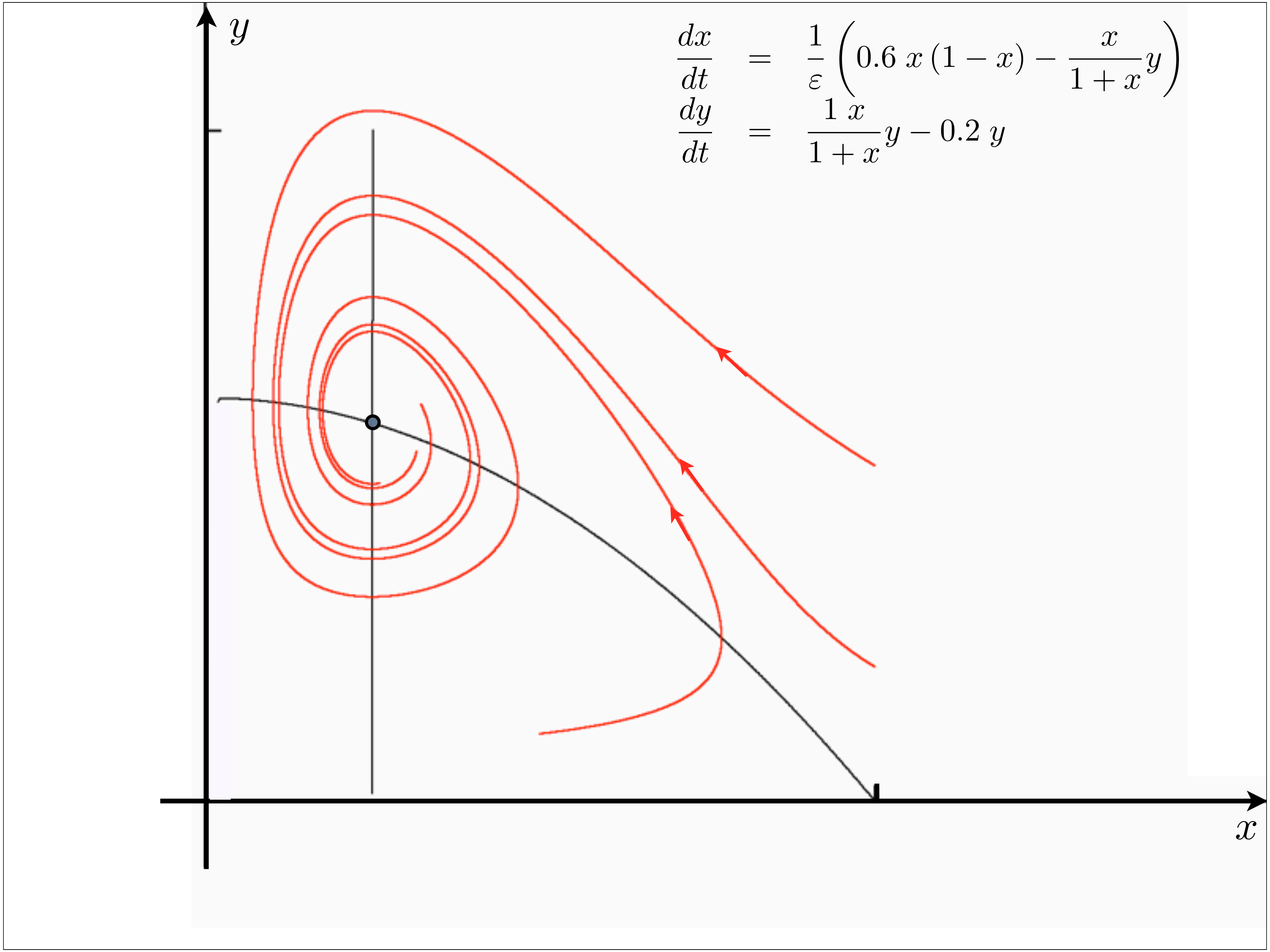
$$\frac{dy}{dt} = c_1 xy - d_1 y \qquad \frac{dy}{dt} = c_2 xy - d_2 y$$

En dim 2 il suffit de regarder les trajectoires

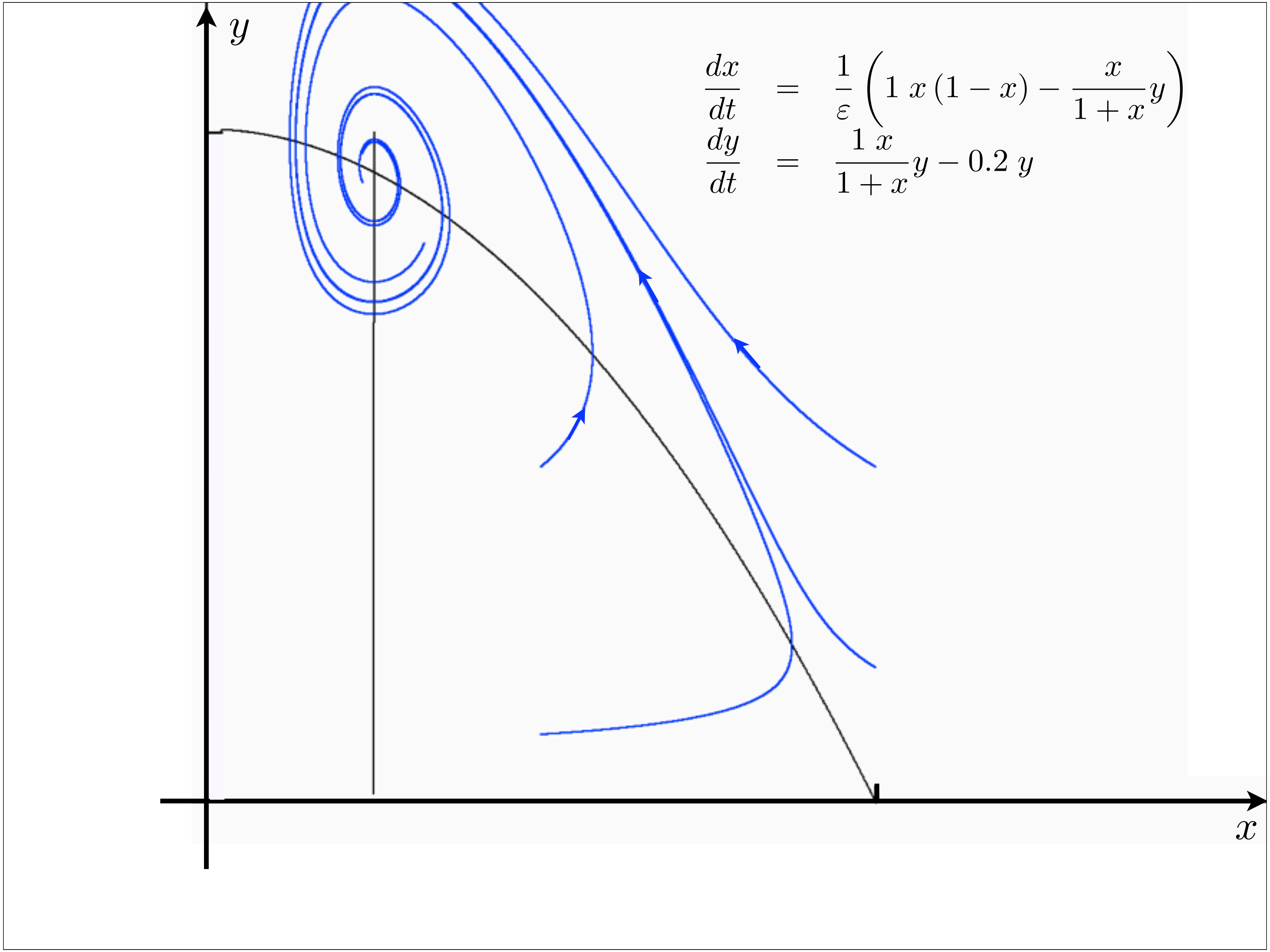


Le cas de la persistance forte

Retour sur Gause-Rozsenzweig-MacArthur

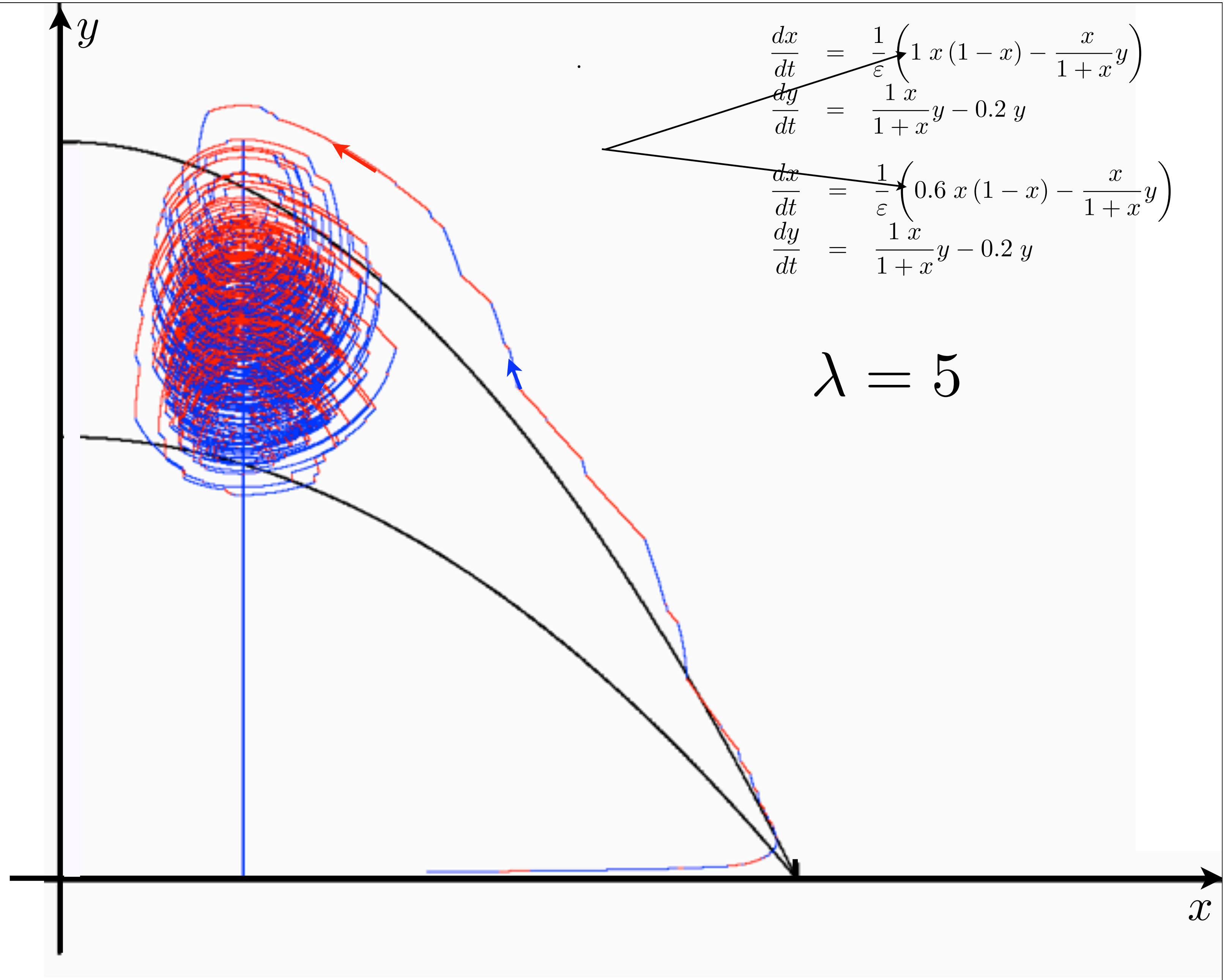


y
x
$$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(0.6\,x\,(1-x) - \frac{x}{1+x}y\right)$$
$$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$$

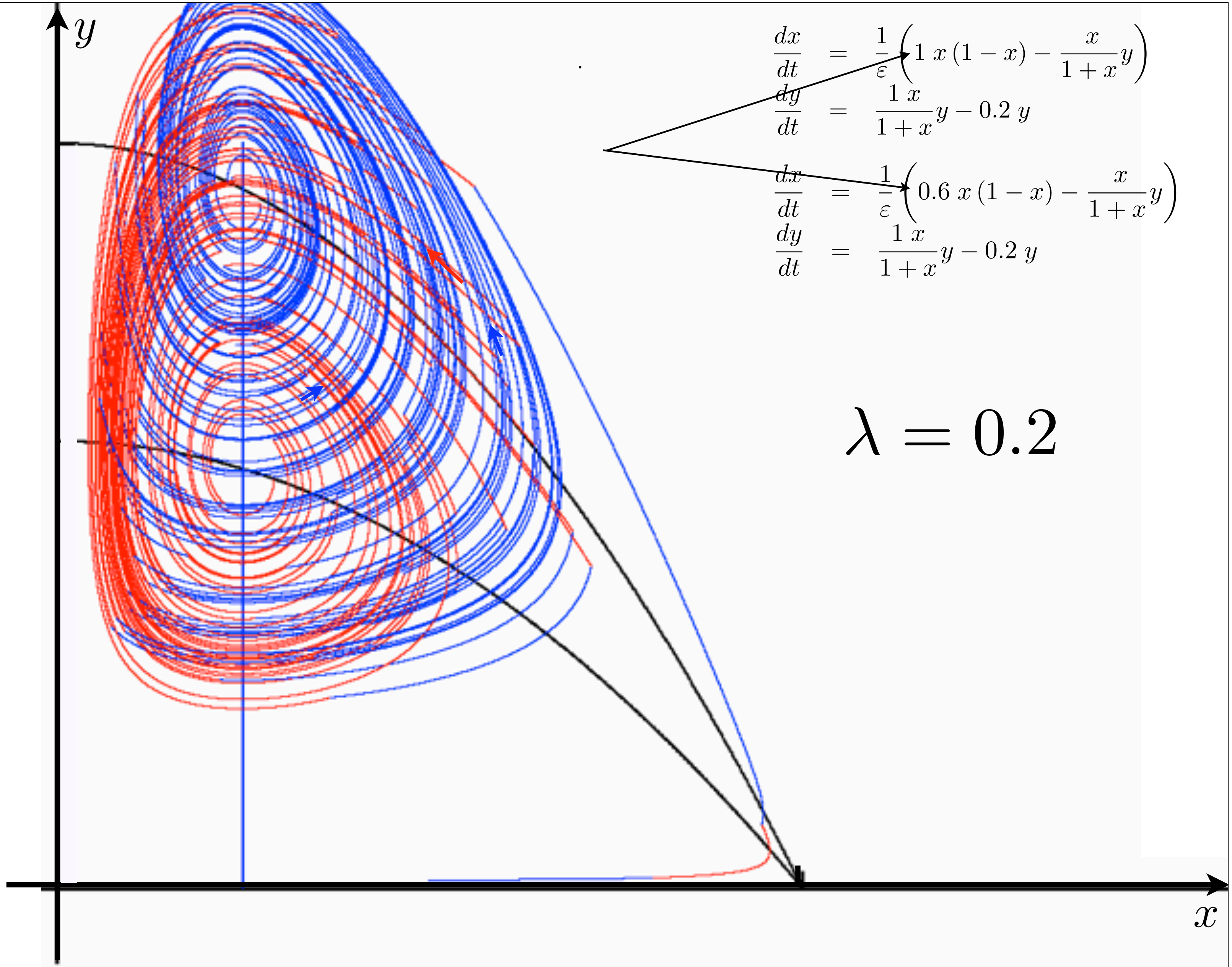


y
$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(1\,x\,(1-x) - \frac{x}{1+x}y\right)$
$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$
x

PDMP
Associé



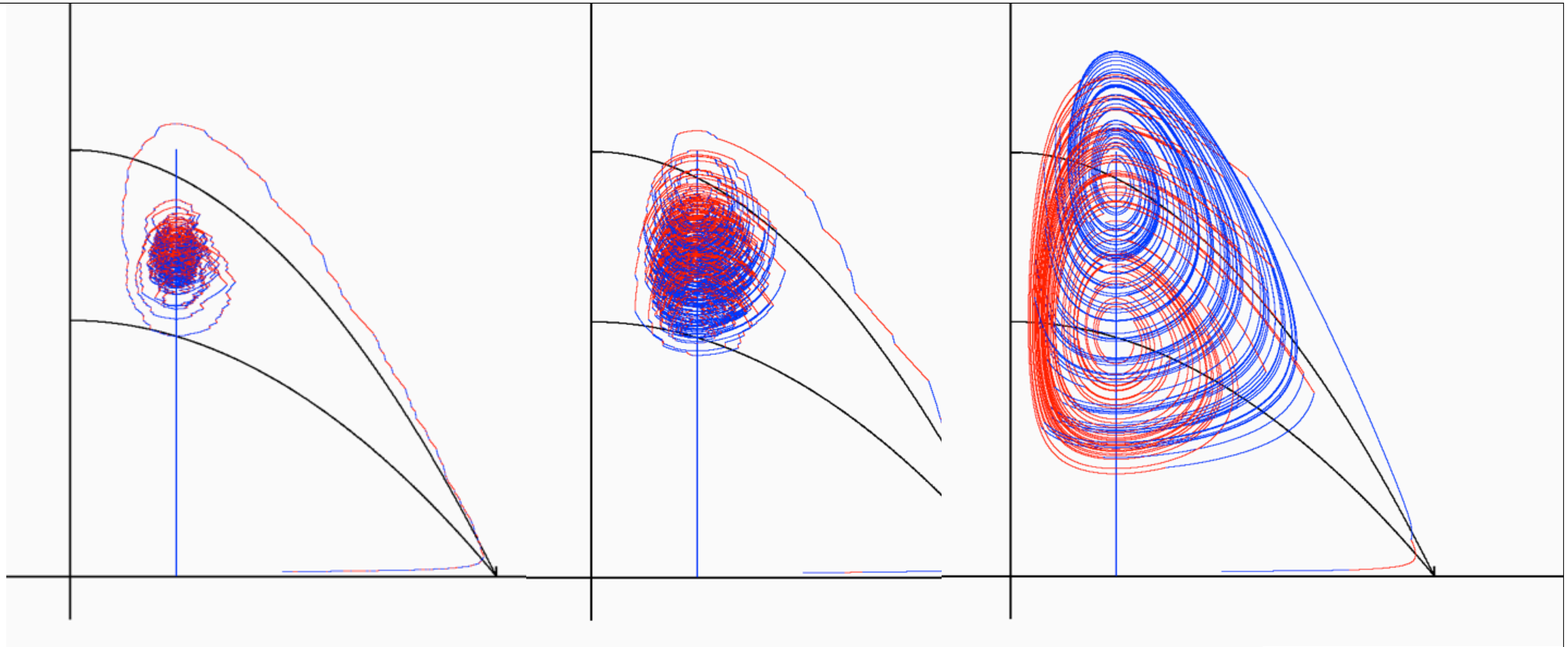
y
$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(1\,x\,(1-x) - \frac{x}{1+x}y\right)$
$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$
$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(0.6\,x\,(1-x) - \frac{x}{1+x}y\right)$
$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$
$\lambda = 5$
x



$$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(1\,x\,(1-x) - \frac{x}{1+x}y\right)$$
$$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$$

$$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(0.6\,x\,(1-x) - \frac{x}{1+x}y\right)$$
$$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$$

$$\lambda = 0.2$$



$\lambda = 20$ $\lambda = 5$ $\lambda = 0.2$

PREPRINTS

IFAC 75

VI

USA

INTERNATIONAL FEDERATION OF AUTOMATIC CONTROL
6TH TRIENNIAL WORLD CONGRESS
August 24-30, 1975

Boston/Cambridge
Massachusetts, U.S.A.

PART IB

THEORY:
Linear Control Systems
Algebraic Methods in Control
Computational Methods in Control

ON THE STRUCTURAL STABILITY OF DYNAMICAL CONTROL SYSTEMS

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INTRODUCTION.

Consider a non linear control system on $\mathbb{R}^n$ (or on some manifold M) of the following form :

$$(1) \quad \frac{dx}{dt} = Y(x) + \sum_{i=1}^{p} u_i X^i(x), \quad x \in \mathbb{R}^n \; ; \; u_i \in U_i \subset \mathbb{R},$$

where the set U_i may be $[-1, +1]$ or $\{-1 ; +1\}$ depending on the fact that we are interested by Bang Bang properties or not ; Y and X^i are non linear vector fields.

The fact that the inputs enters linearly in system (1) is not an important loss of generality ; the main problems are in the nonlinearities in the space. This approach to non linear control problems based on geometric arguments seems to have been very useful in the last few years in various directions:

Controllability : See ref. (1-3), (6), (8-18), (20-22), (28-30), (35-40), (46-47).
Optimality : See ref. (15), (24), (25), (26).
Realization theory: See ref. (1), (2), (23), (26-27), (41-44).

One problem seems to be of interest now. Try to make a classification of systems of type (1), but with respect to what equivalence relation ? What are the systems whose properties are stable under small perturbations of the datas, but what kind of properties are of interest to us ?

The case p=0 (i.e. no input) in system (1) reduce to the well known theory of dynamical systems. To some extend our control system (1) can be considered as a collection of dynamical systems and we are tempted to try to extend the known concepts and results of the theory of dynamical systems to control systems. The objective of this paper is to discuss about this question. We first show (section 1) why one cannot adapt in a too naive way the theory of topological and differentiable dynamics. By some examples we show how the classification problem is difficult even in very simple cases. In section 2 we propose a definition for "structurally stable control systems". We show in section 3 ho "structural stability" can be obtained from the "continuity" of a certain mapping. We conclude by a theorem (which actually is with minor changes a theorem of (6)) on the density of structurally stabl systems.

I - CLASSIFICATION PROBLEM

I. 1. - The case of dynamical systems

On fig. (1) we have drawn the phase portrait of typical systems and made some comments about them. It turns out that in every cases the qualitative feature are expressed in terms of the pology of the underlying space on one hand and in terms of "trajectories" or "orbits" on the other hand. For this reason the :

I. 1. 1. - DEFINITION

Two dynamical systems s_1 and s_2 on a manifol M are topologically equivalent if there exists an homeomorphism of M which maps every sensed trajectory of s_1 onto a sensed trajectory of s_2.

will preserve the above qualitative properties. It turns out that the classification of dynamical systems with respect to the above definition is known in the plane (19), (31), (32), and to large extend o surfaces. For higher dimensions the present know lege is very far to be complete. See (32).

I.2. - The case of control systems

I. 2. 1. - Let us consider the celebradet linear sys

$$\left.\begin{aligned} \frac{dx}{dt} &= y \\ \frac{dy}{dt} &= 0 + u \qquad -1 \leqslant u \leqslant 1 \end{aligned}\right\}$$

i)- controlable to th origin

ii)- unique time optimal control for return to the orig

iii)- optimal controls are Bang-Bang

iiii)- existence of a r gular stable sy thesis

C. R. Acad. Sc. Paris, t. 280 (20 janvier 1975) Série A — 109

ÉQUATIONS DIFFÉRENTIELLES. — *Classification de certains systèmes dynamiques contrôlés du plan.* Note (*) de M. **Yves Gerbier**, présentée par M. Jacques-Louis Lions.

Soit $\mathscr{D}$ une famille de champs de vecteurs sur une variété. On note $(t, x) \mapsto X_t(x)$ le système dynamique associé au champ X. Des articles récents ont montré que l'étude des propriétés de l'ensemble :

$$A(x) = \{ X^1_{t_1} \circ X^2_{t_2} \circ \ldots \circ X^i_{t_i} \circ \ldots \circ X^p_{t_p}(x);\ p \in \mathbf{N},\ t_i \geq 0,\ X^i \in \mathscr{D} \}$$

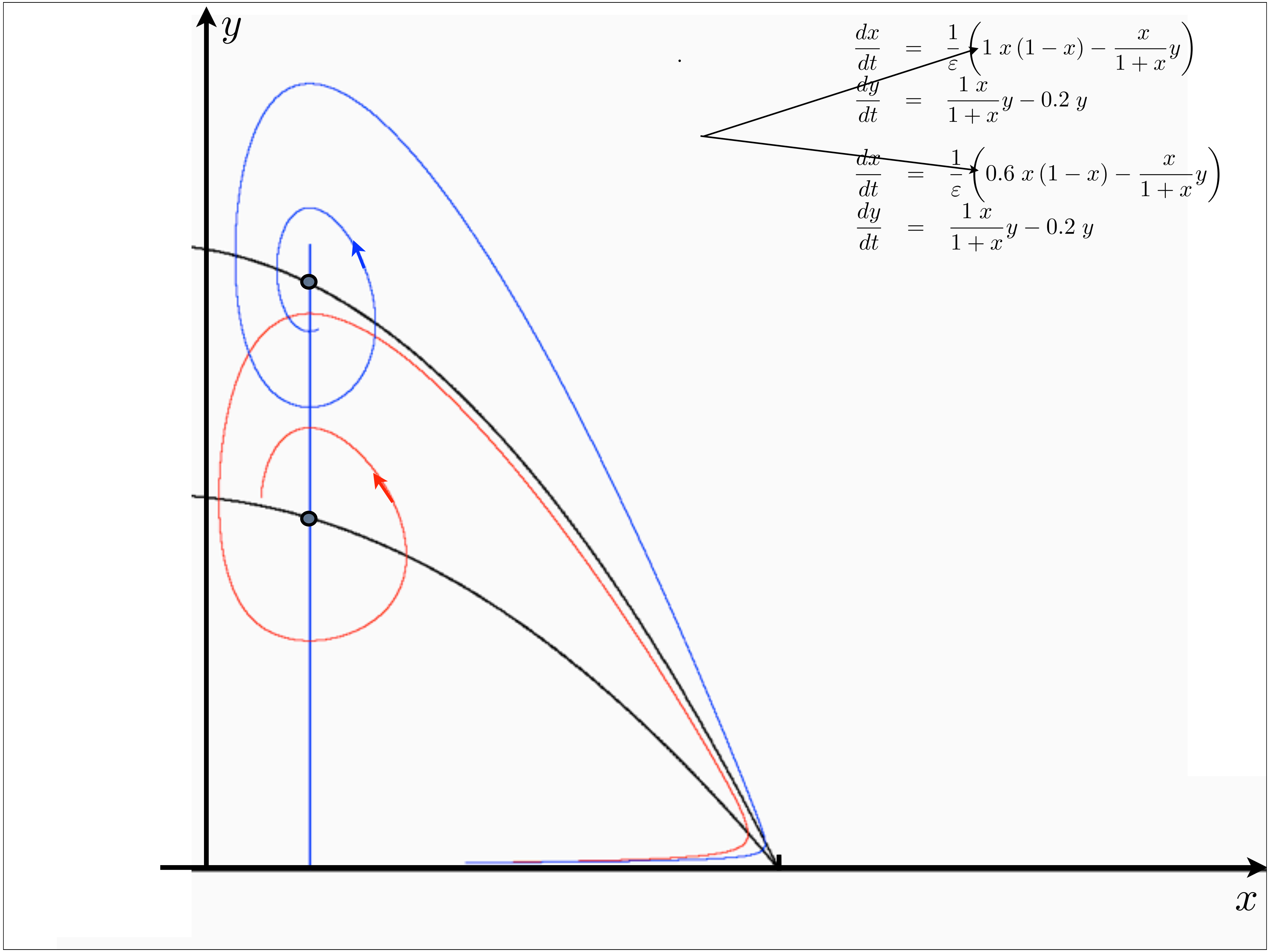
Gerbier (1975)

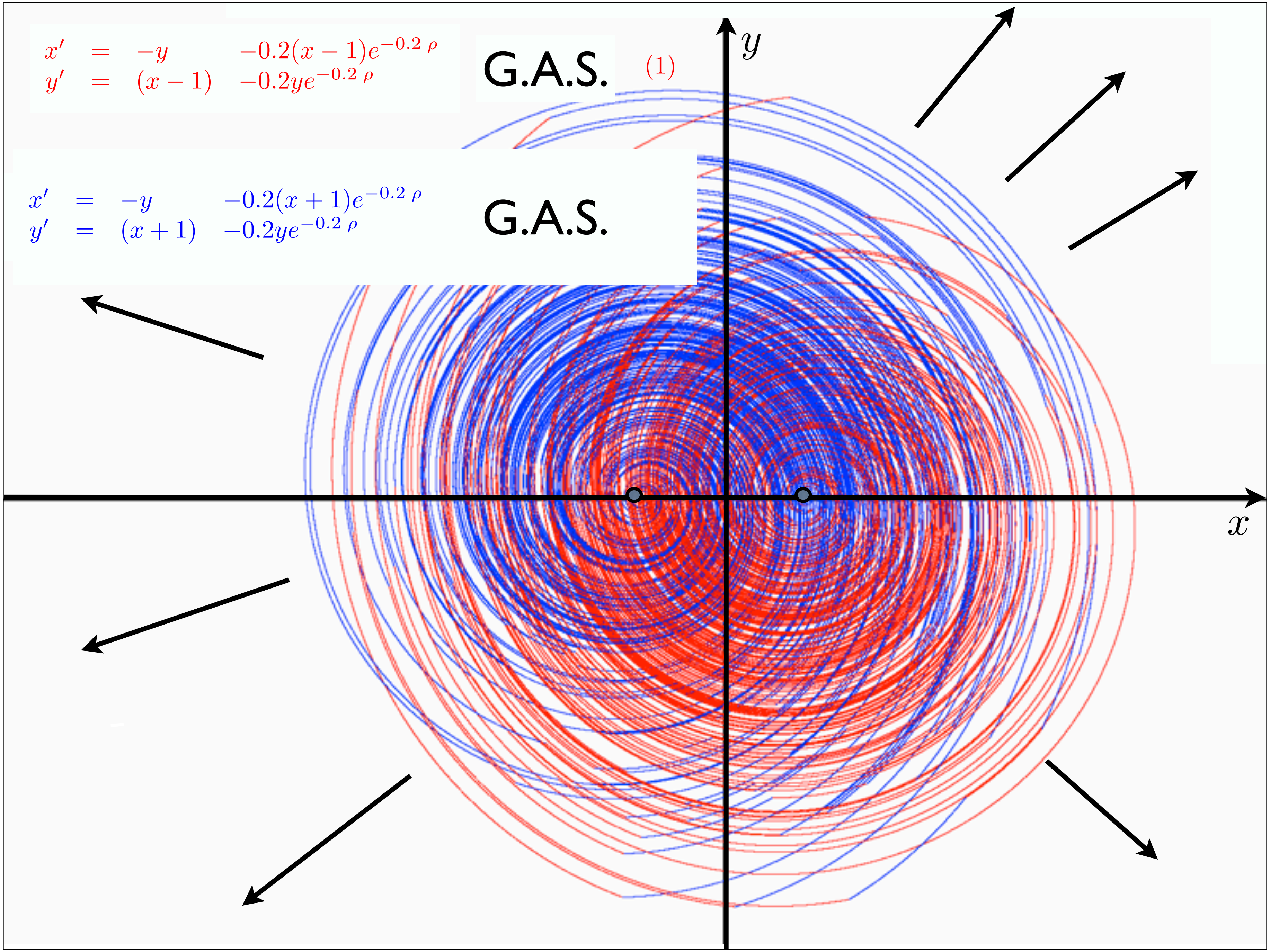
The case of strong persistance

Rosenzweig - MacArthur model

$$\begin{aligned}\frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right)-\frac{a\,x}{e+x}y\\ \frac{dy}{dt} &= \varepsilon\frac{a\,x}{e+x}y-\varepsilon my\end{aligned}$$

$$\begin{aligned}\frac{dx}{dt} &= \frac{1}{\varepsilon}\left(rx\left(1-\frac{x}{K}\right)-\frac{a\,x}{e+x}y\right)\\ \frac{dy}{dt} &= \frac{a\,x}{e+x}y-my\end{aligned}$$


y
x
$$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(1\,x\,(1-x) - \frac{x}{1+x}y\right)$$
$$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$$
$$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(0.6\,x\,(1-x) - \frac{x}{1+x}y\right)$$
$$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$$



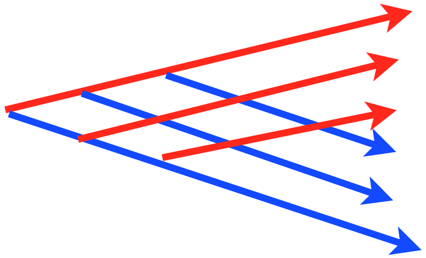
$x' = -y \quad -0.2(x-1)e^{-0.2\,\rho}$
$y' = (x-1) \quad -0.2ye^{-0.2\,\rho}$
G.A.S.
(1)
$x' = -y \quad -0.2(x+1)e^{-0.2\,\rho}$
$y' = (x+1) \quad -0.2ye^{-0.2\,\rho}$
G.A.S.
y
x

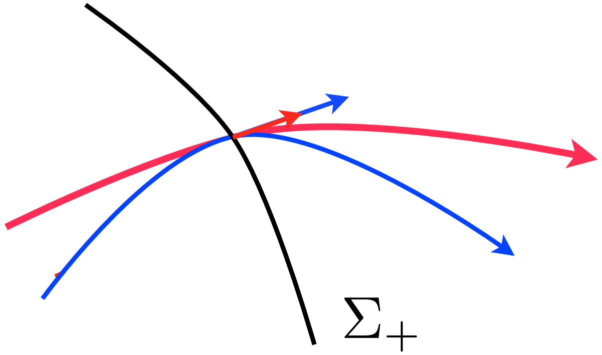
Etats accessibles de 2 champs dans le plan

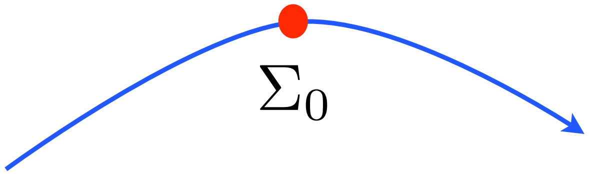
$$\mathcal{U} = \{t \mapsto u(t) \in \{1;2\}\}$$

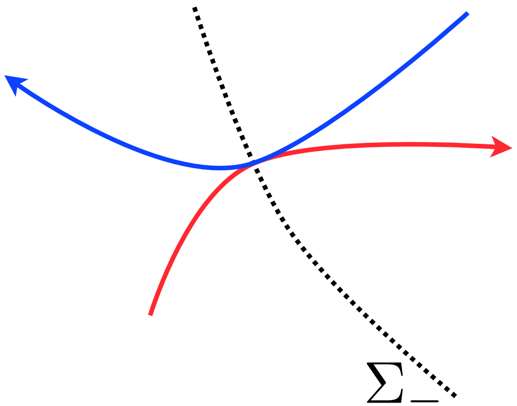
$$\mathcal{A}(S) = \{x(t, x_o, u(.)) \ : \ x_0 \in S \ ; \ u(.) \in \mathcal{U}\}$$

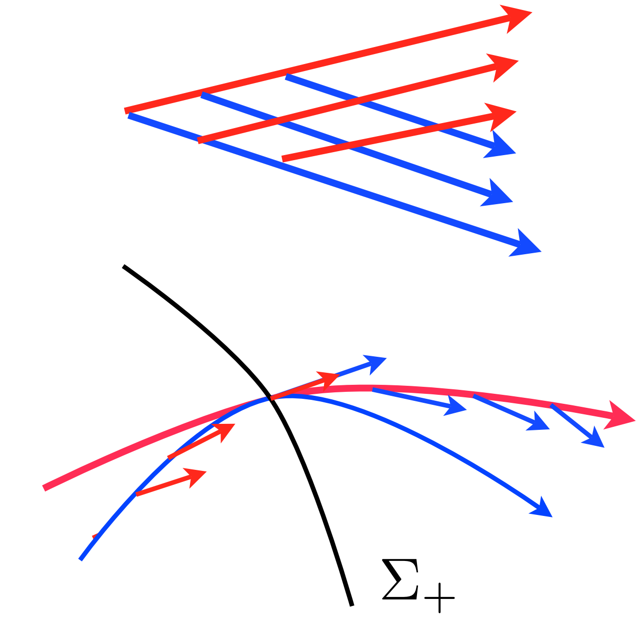
Point régulier



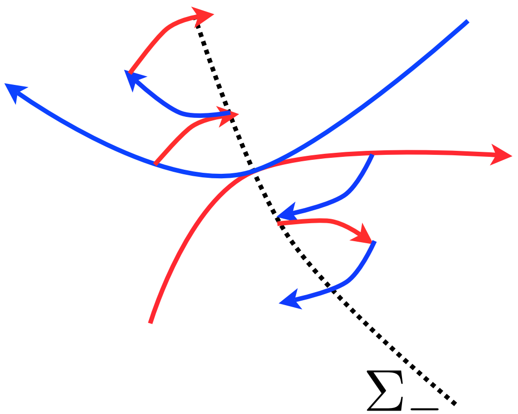




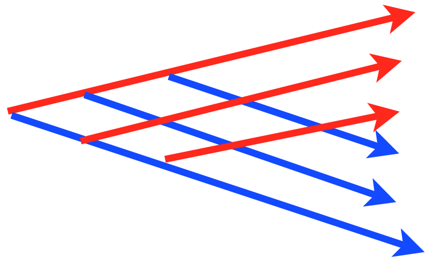


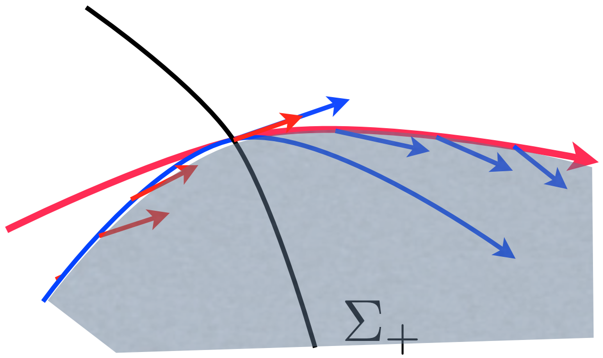


Frontière locale

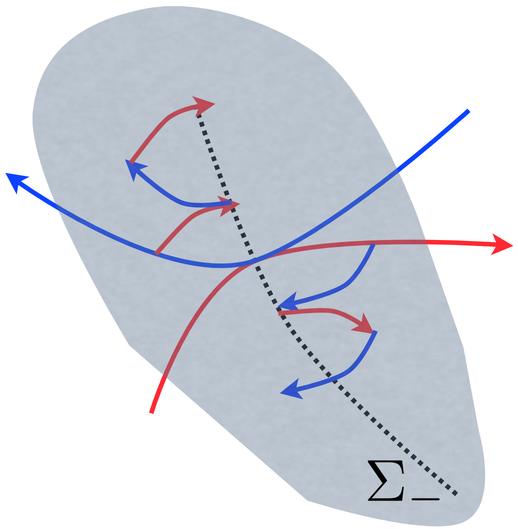


Points intérieurs





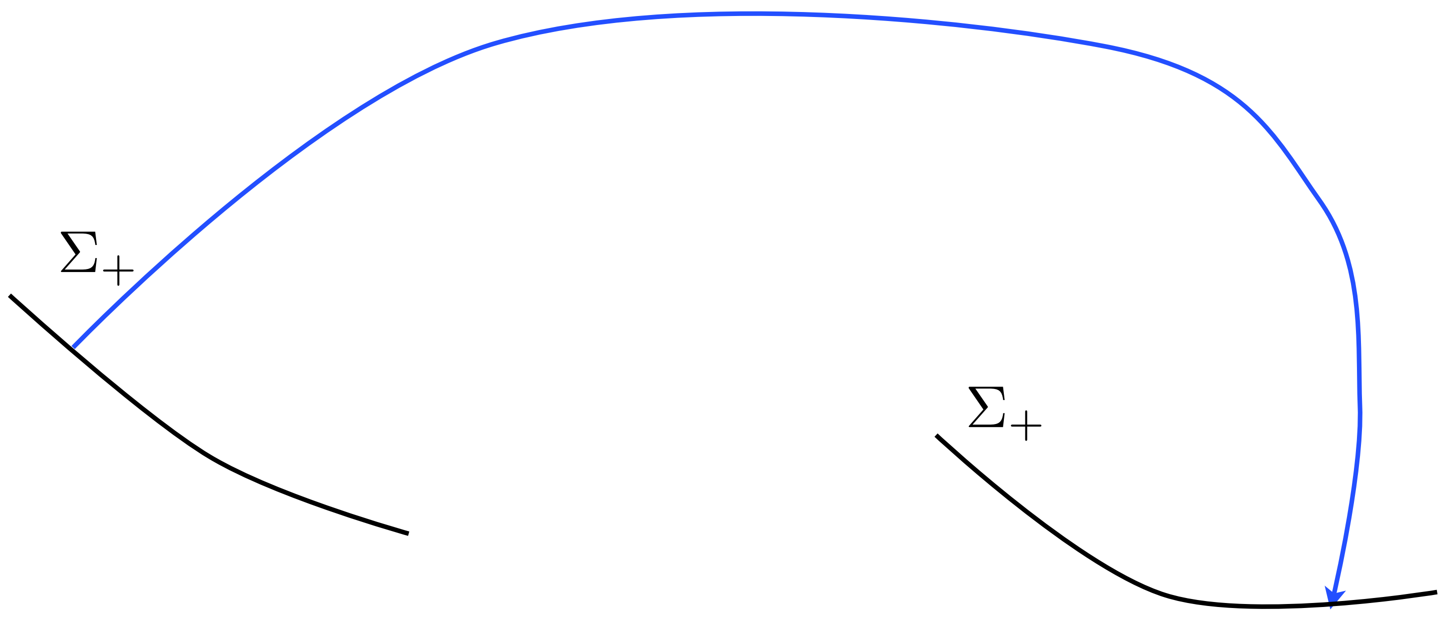
Frontière locale



Points intérieurs

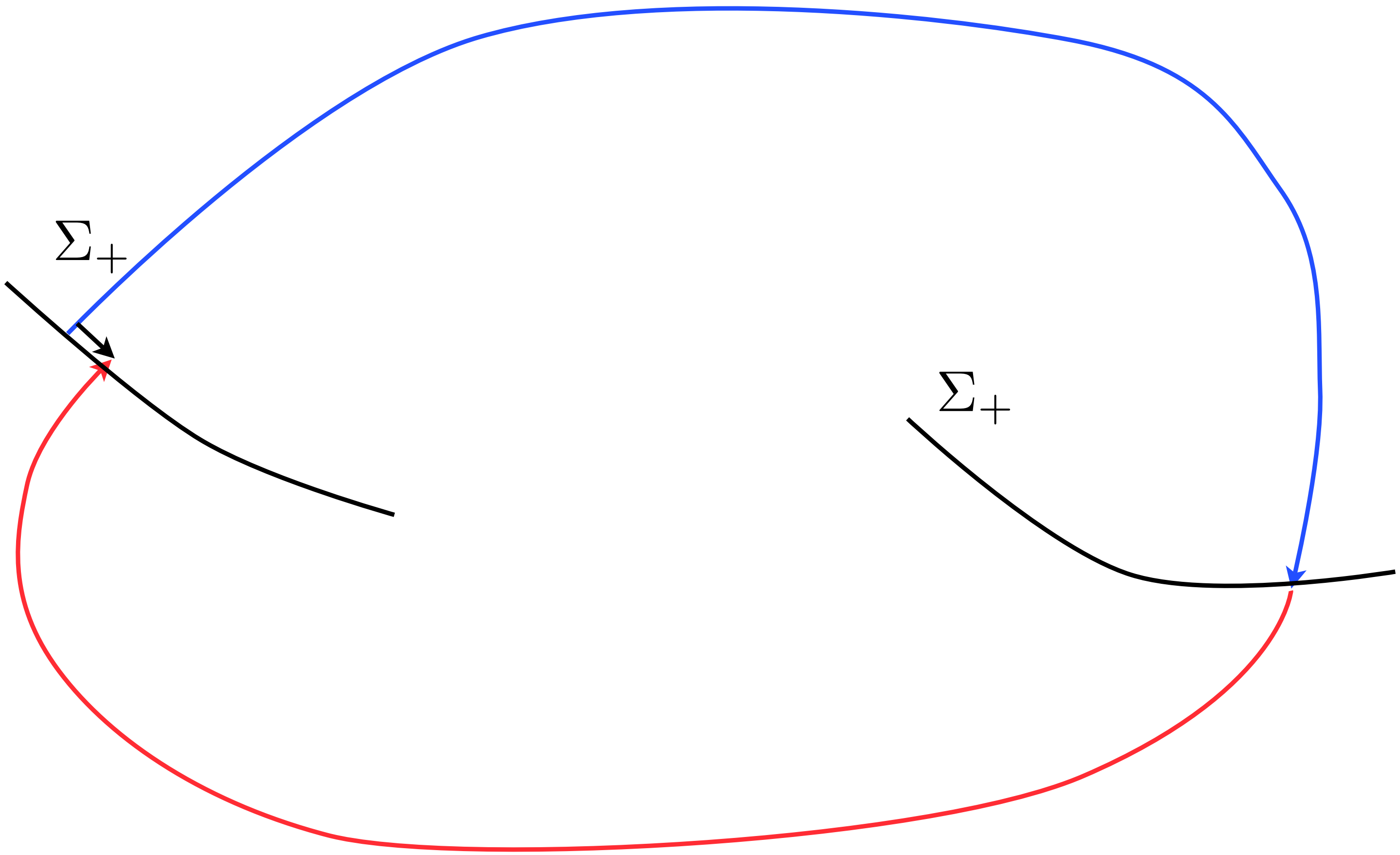
Application de "retour"

Application de “retour”



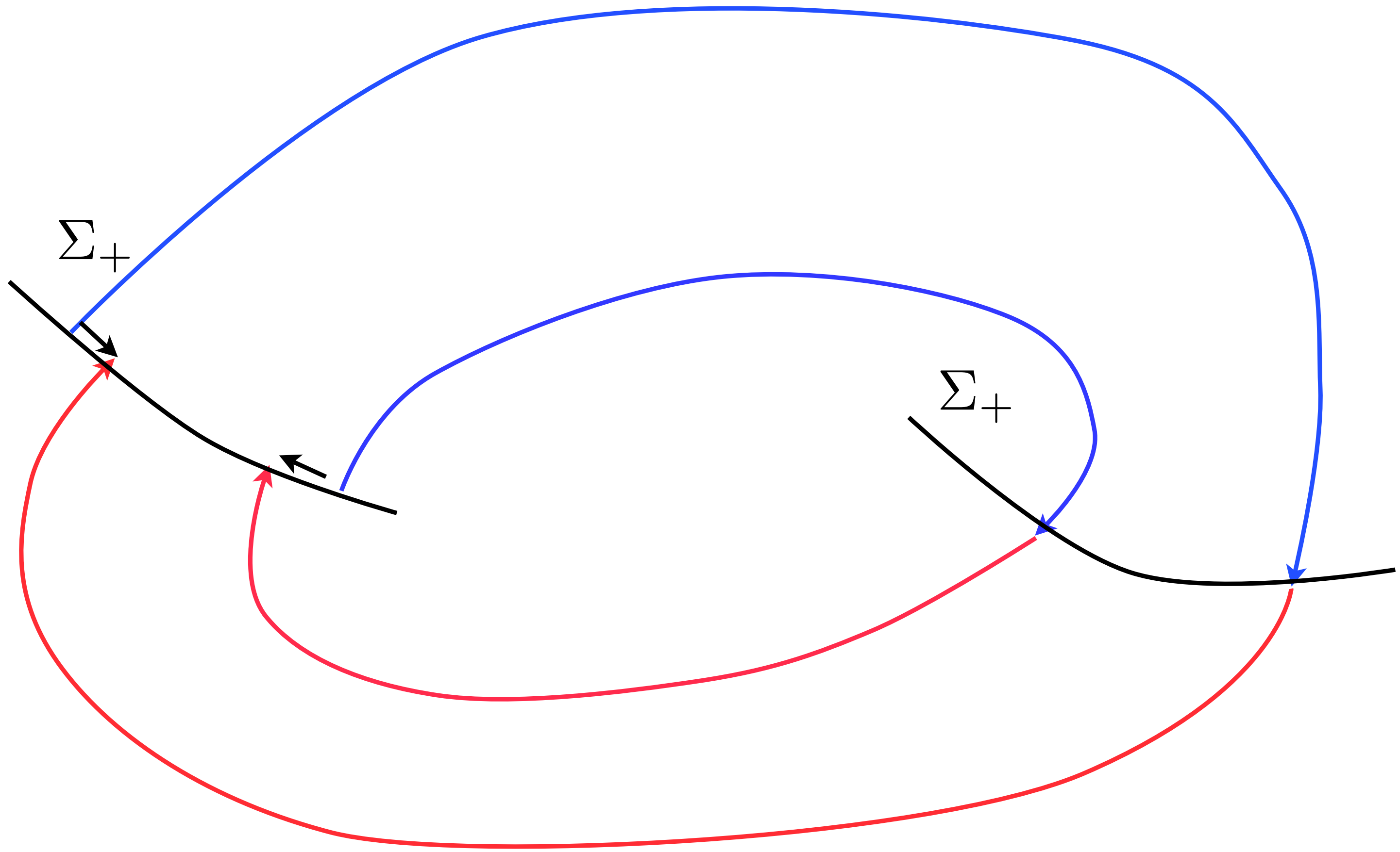
Condition suffisante d'invariance

Application de "retour"



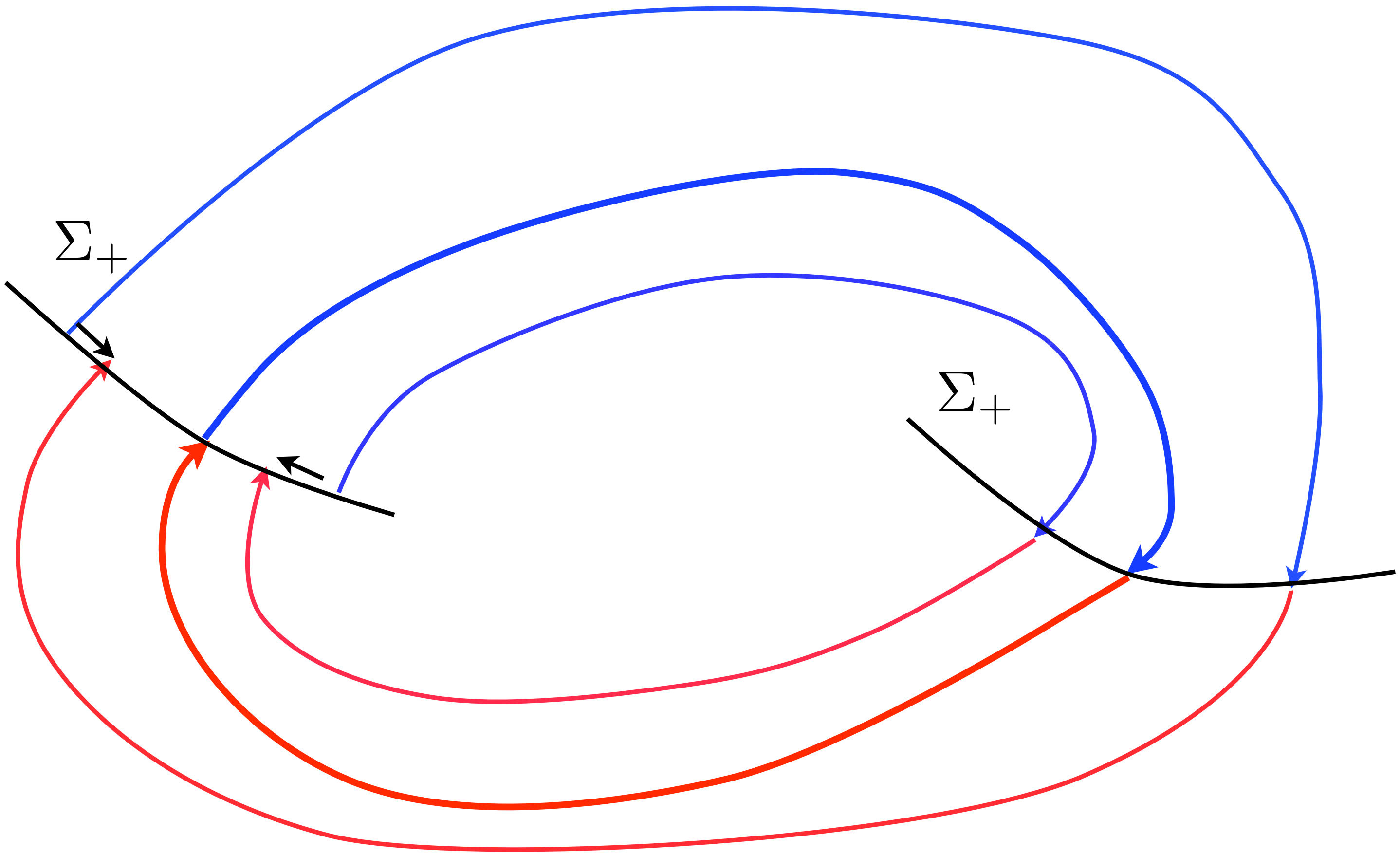
Condition suffisante d'invariance

Application de "retour"

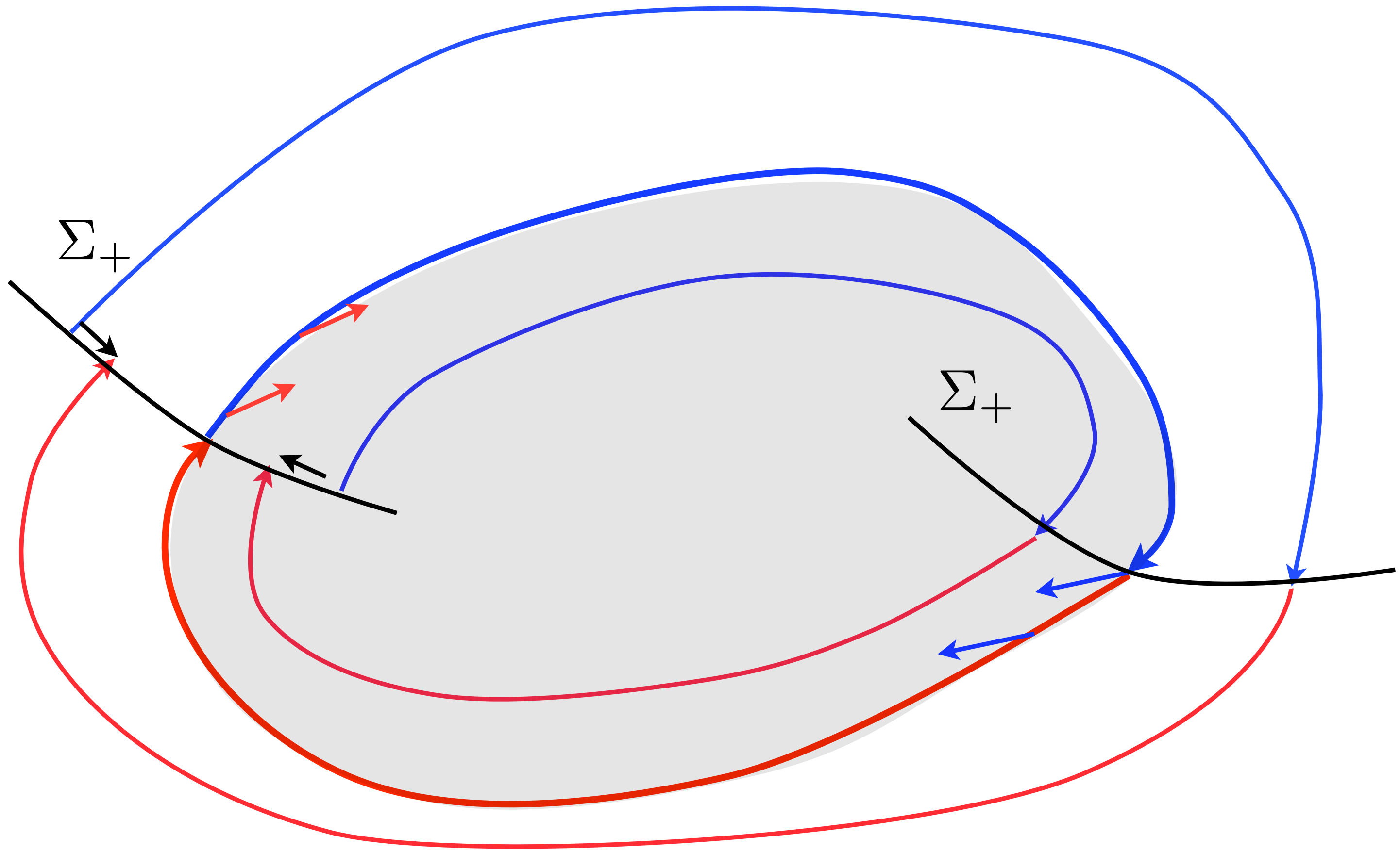


Condition suffisante d'invariance

Application de "retour"



Poincaré return map



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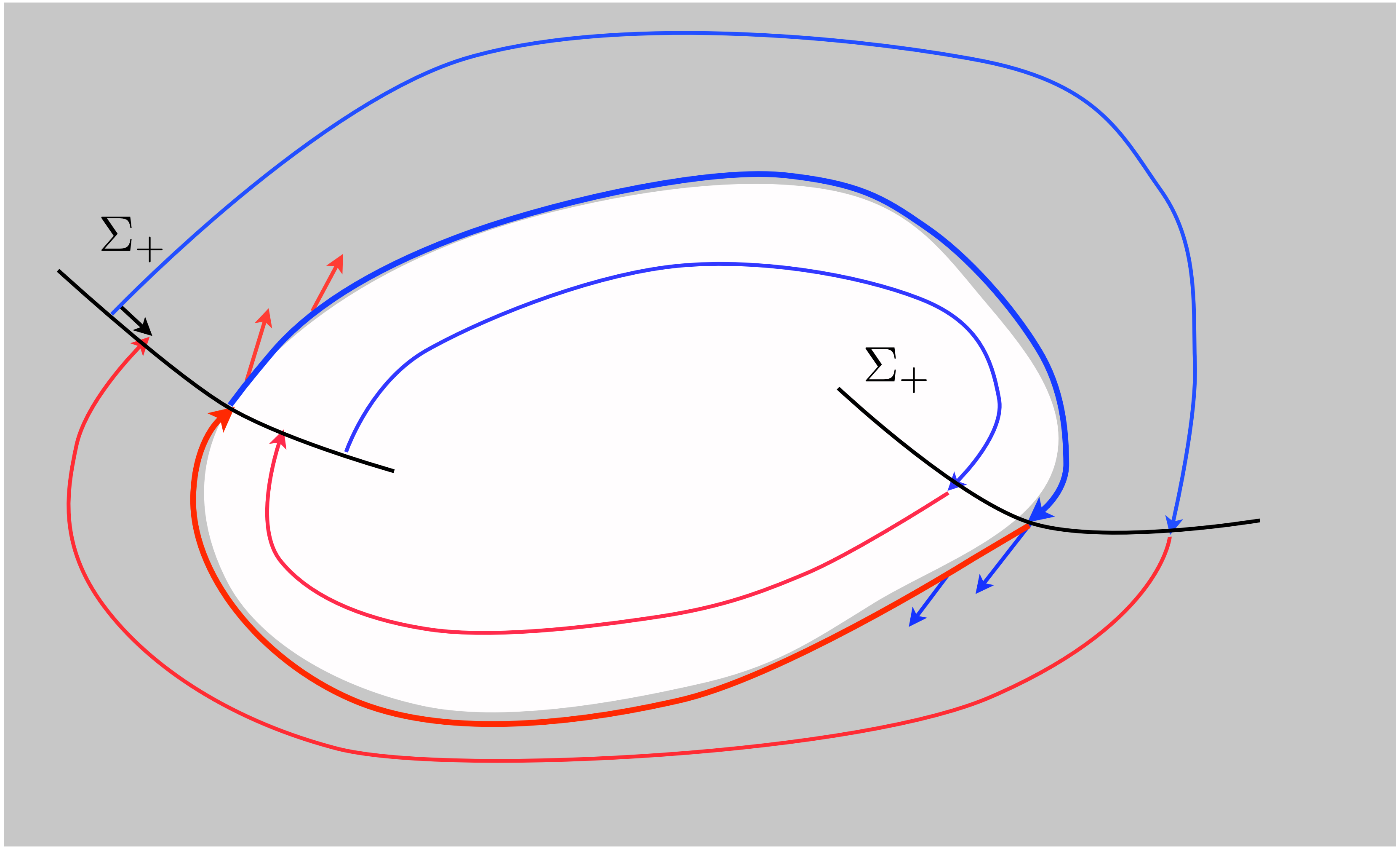
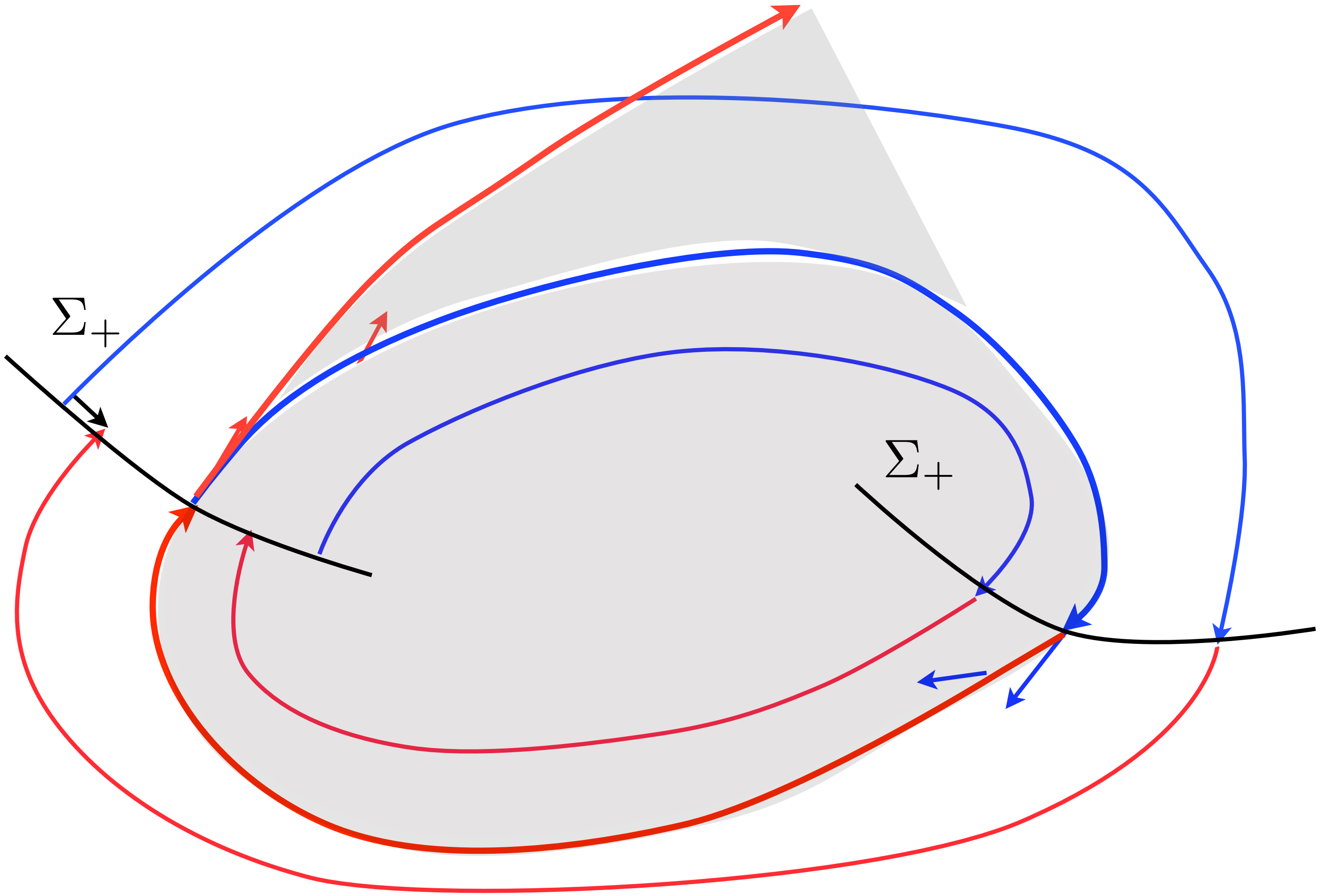


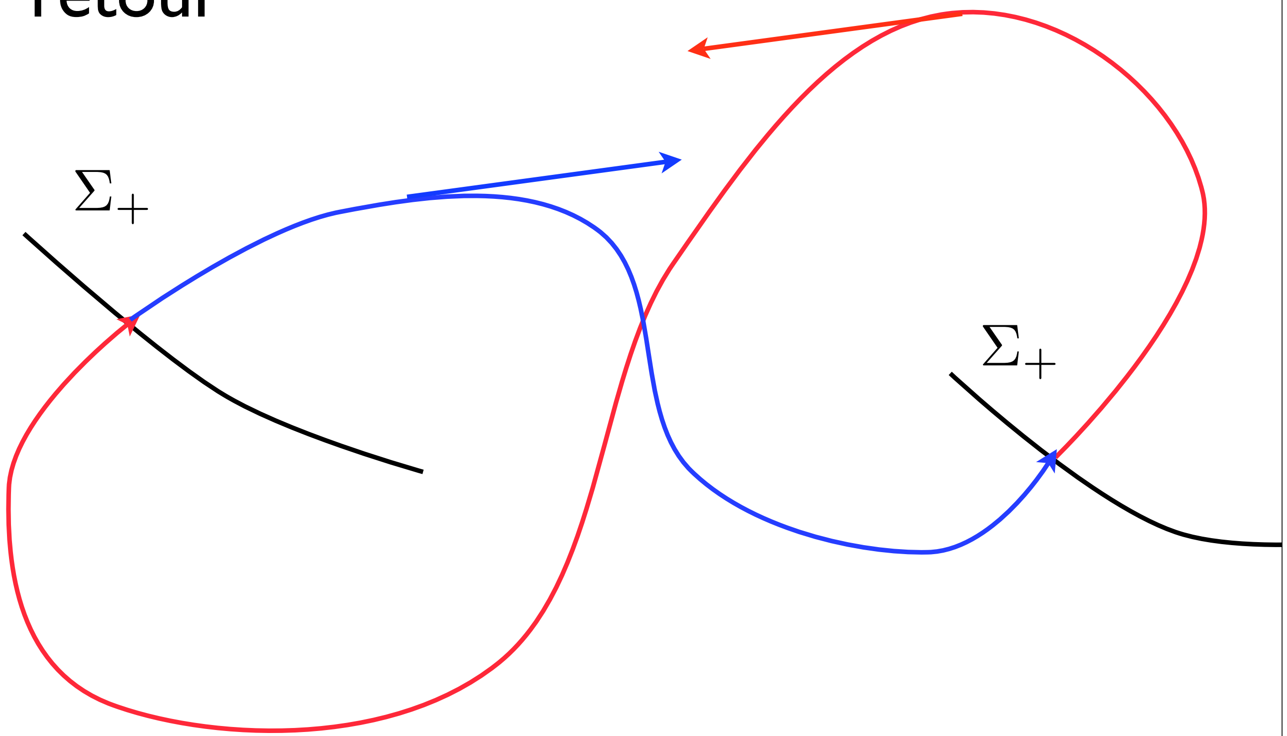
Condition suffisante d'invariance

Application de "retour"

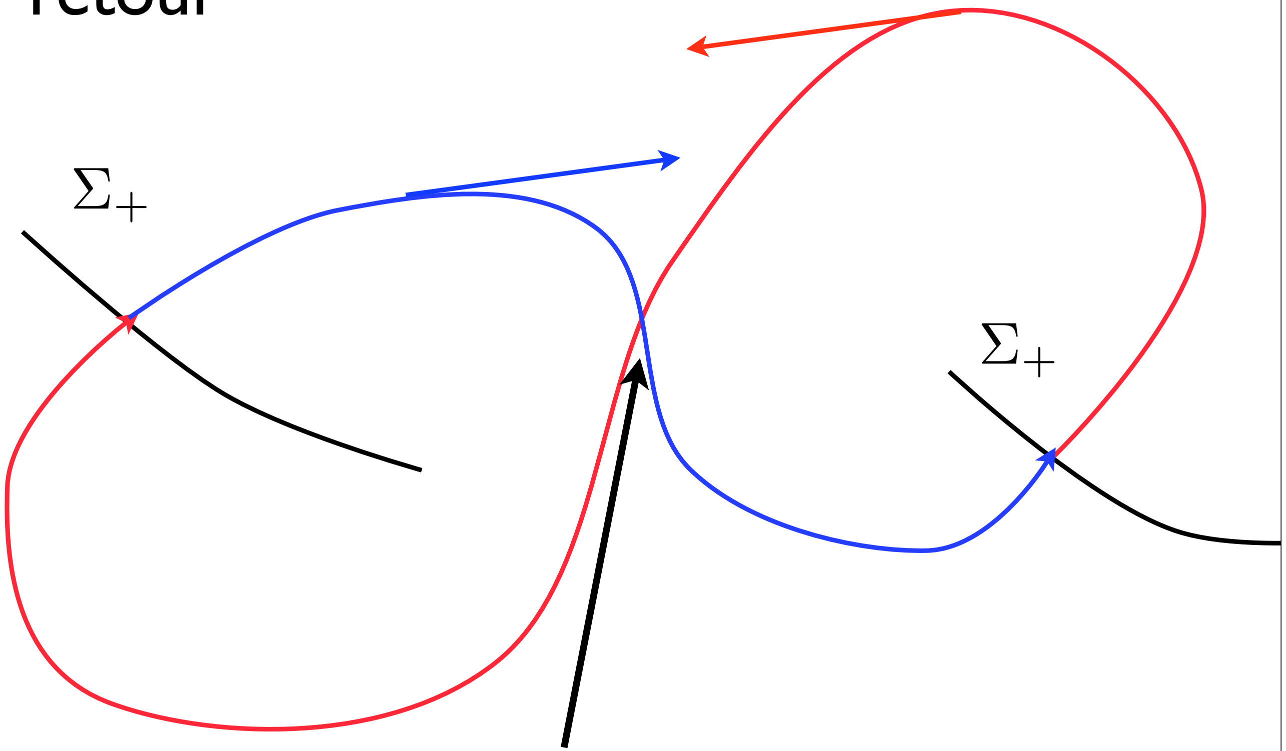


Condition nécessaire d’invariance

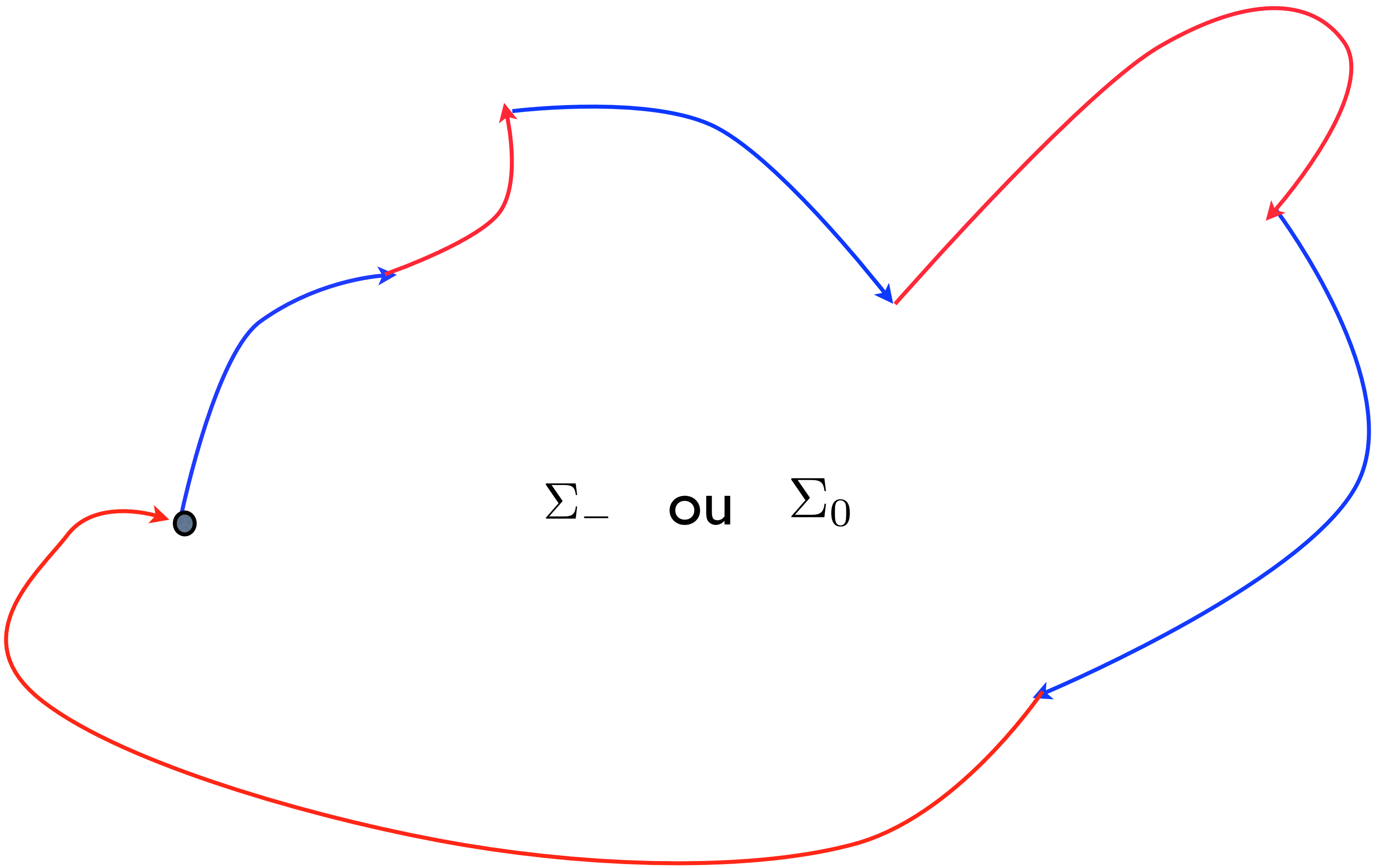
Application de “retour”



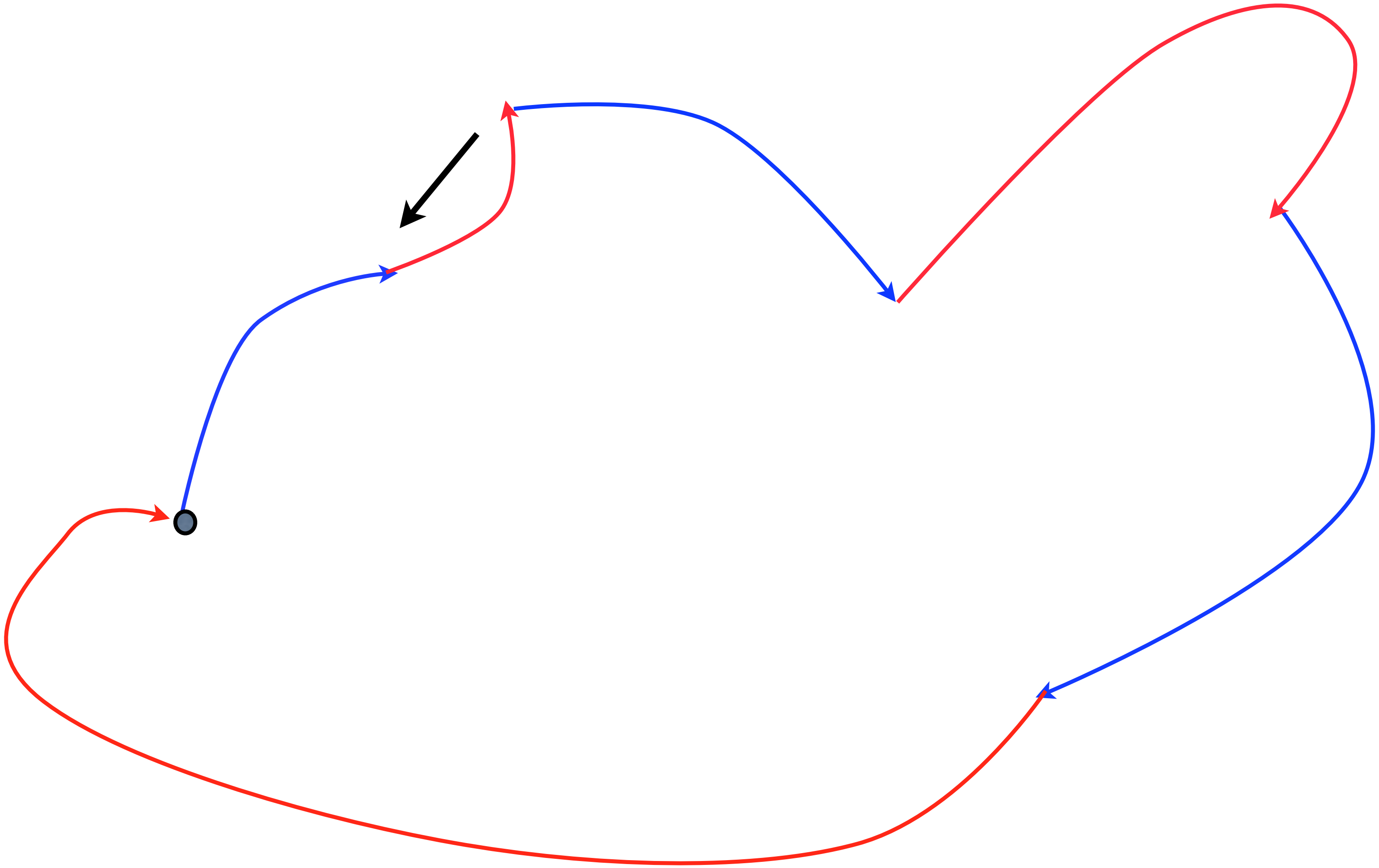
Application de "retour"



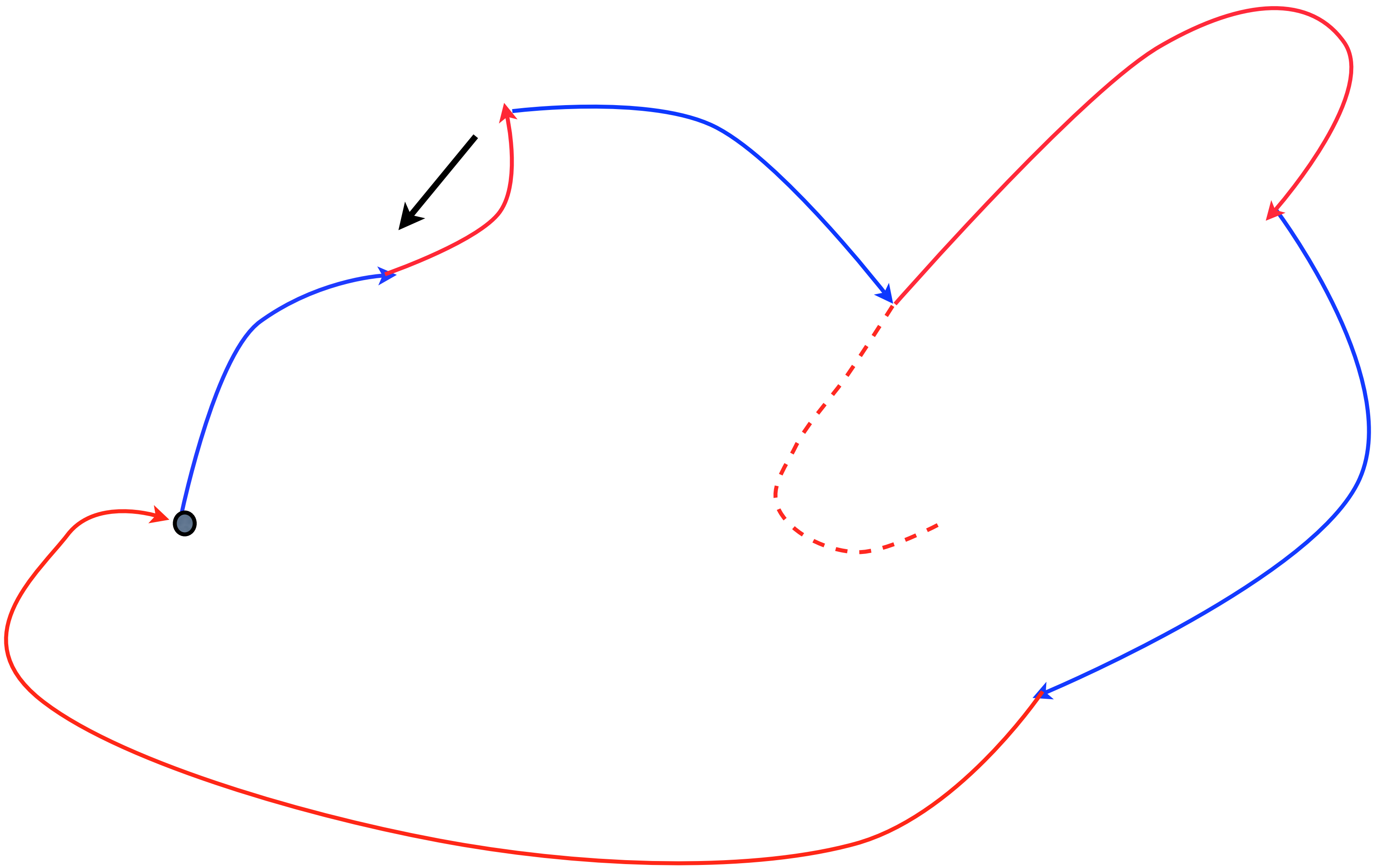
Condition nécessaire d'invariance



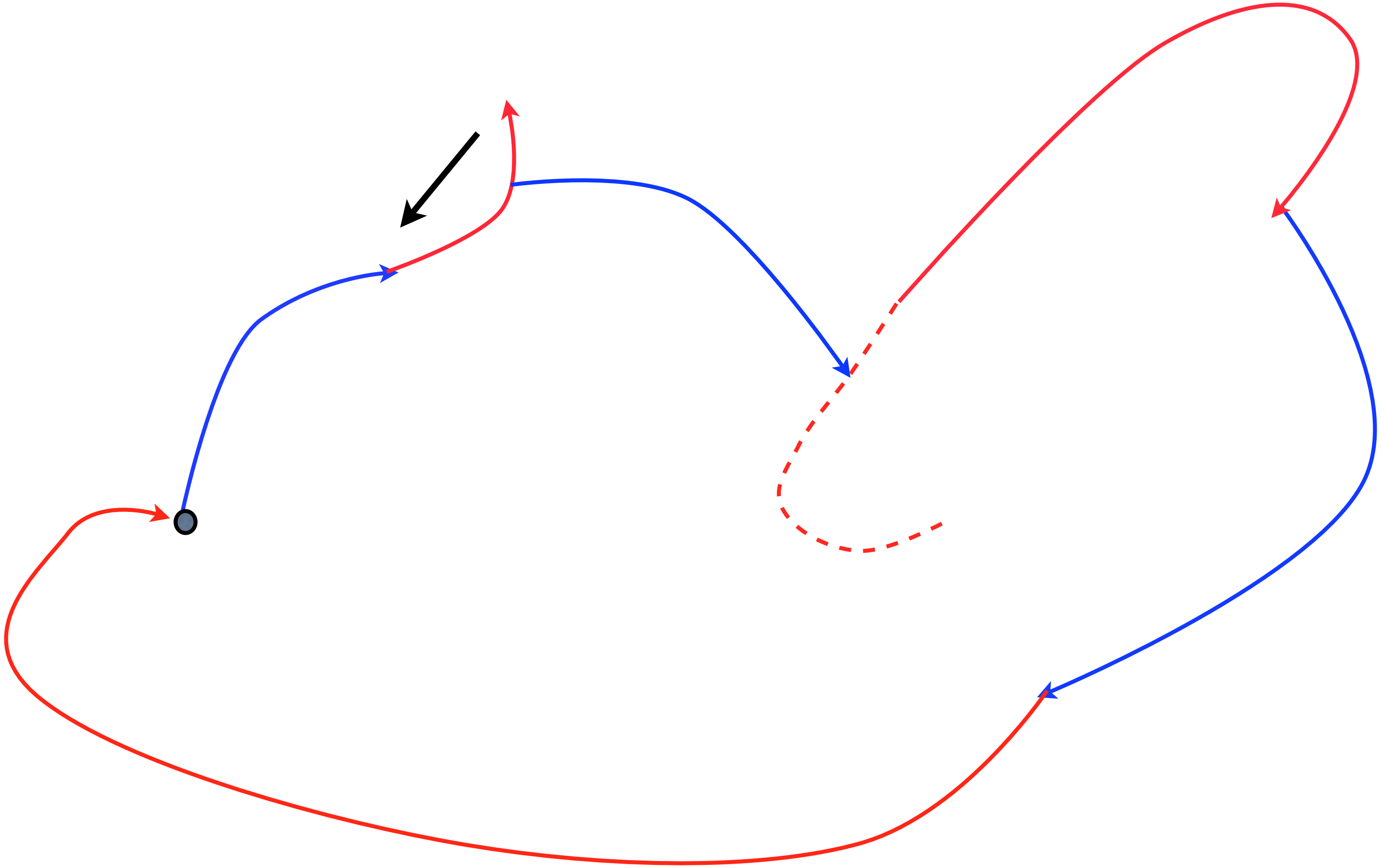
Condition nécessaire d'invariance



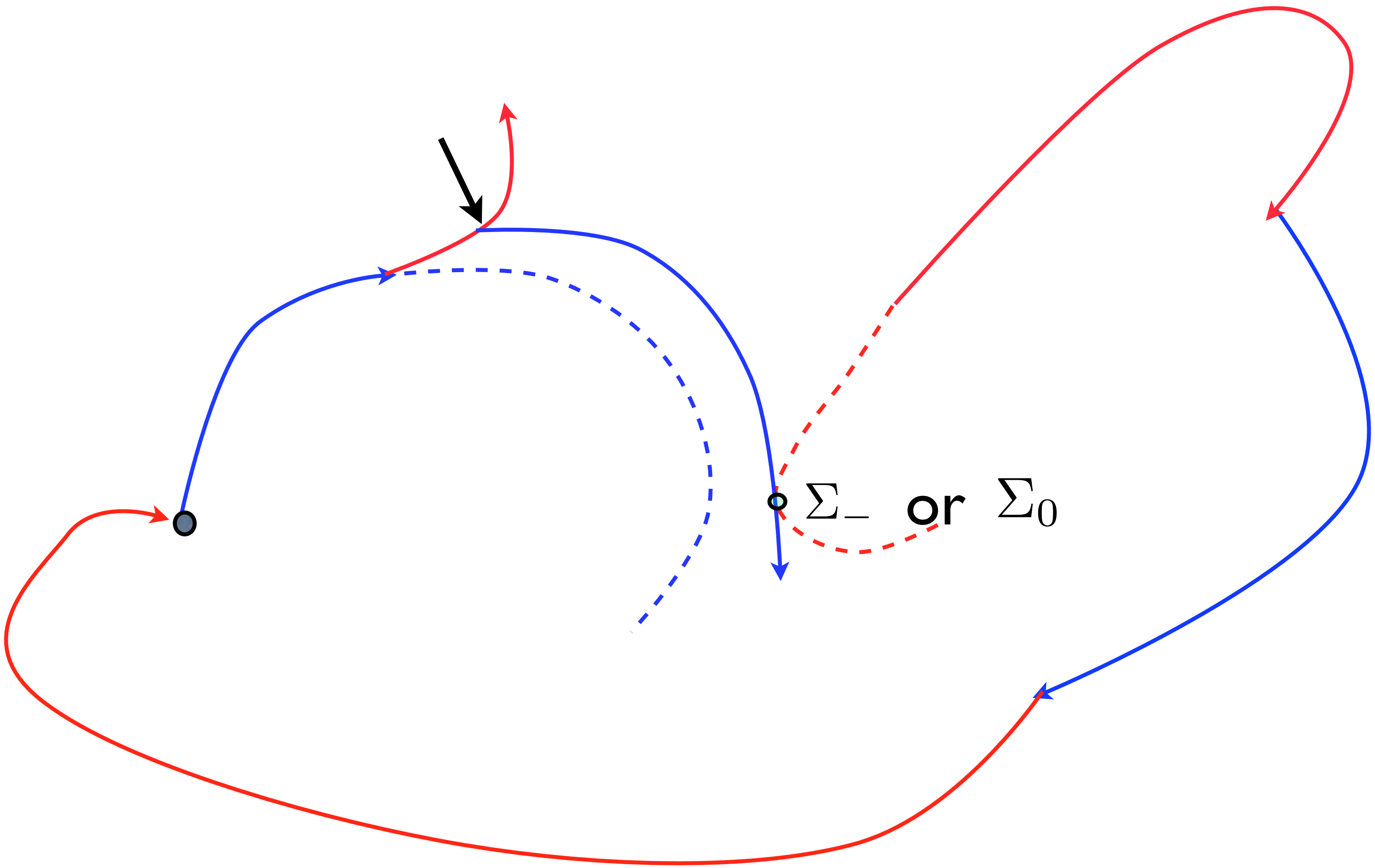
Condition nécessaire d'invariance



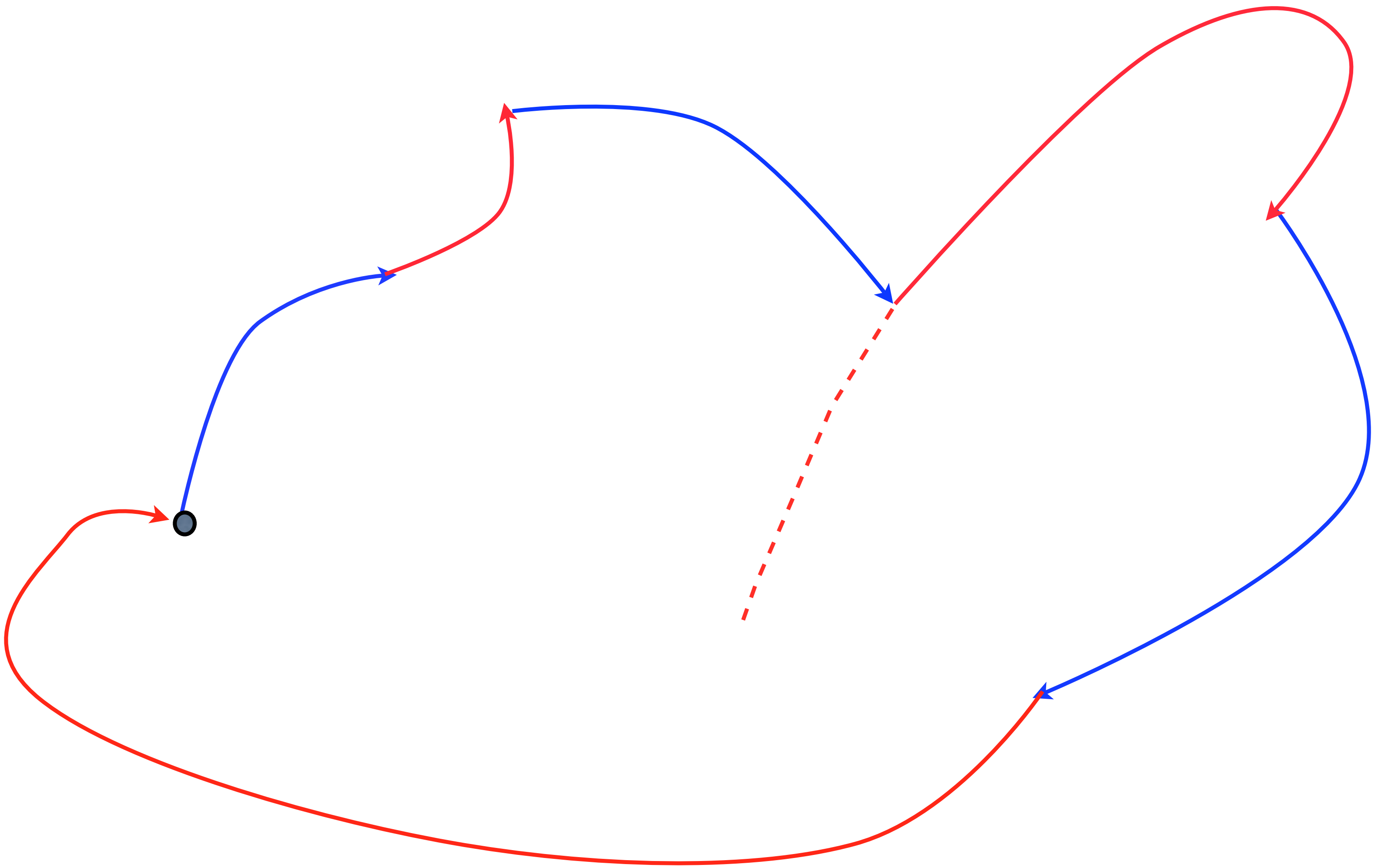
Condition nécessaire d'invariance



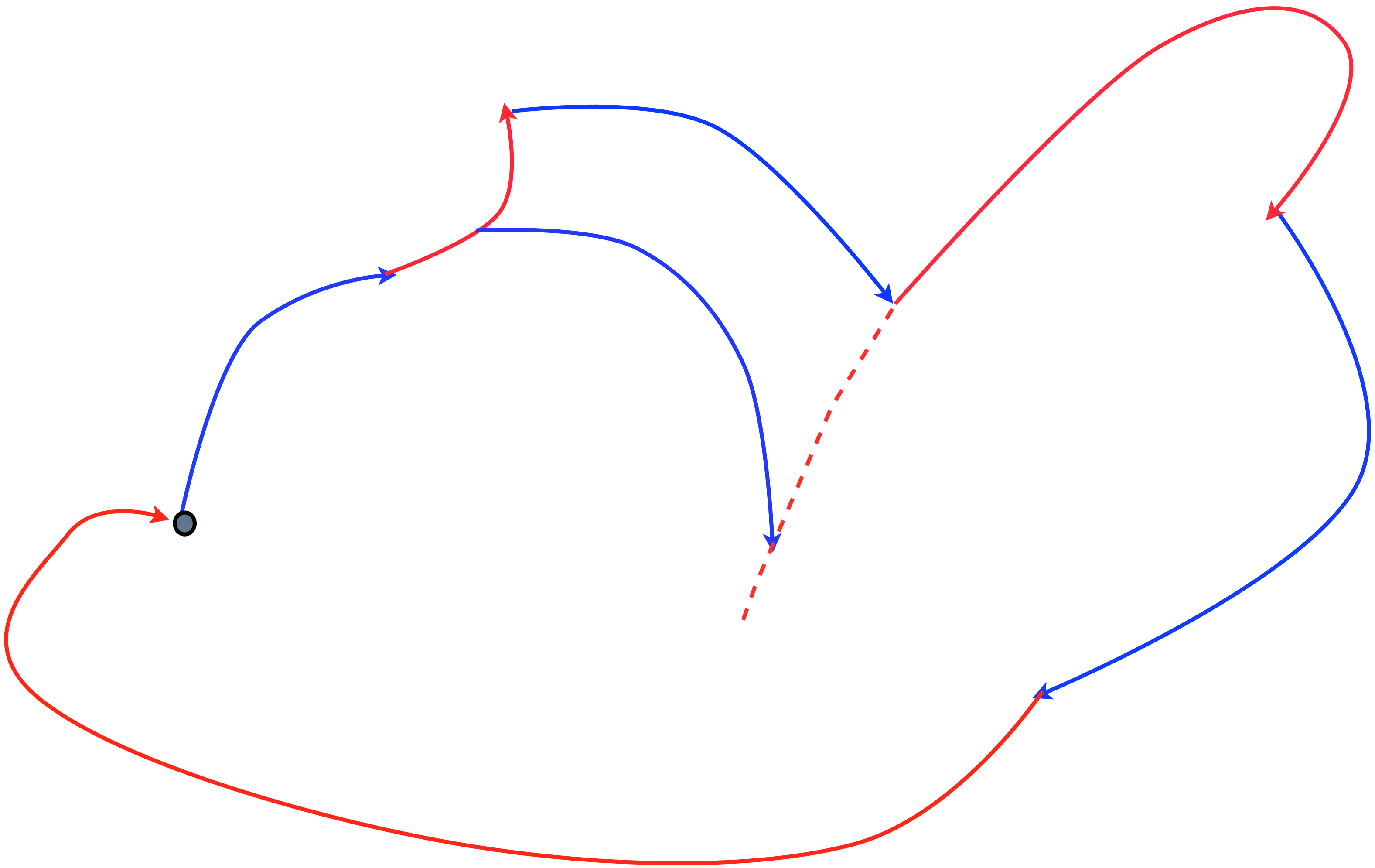
Condition nécessaire d'invariance



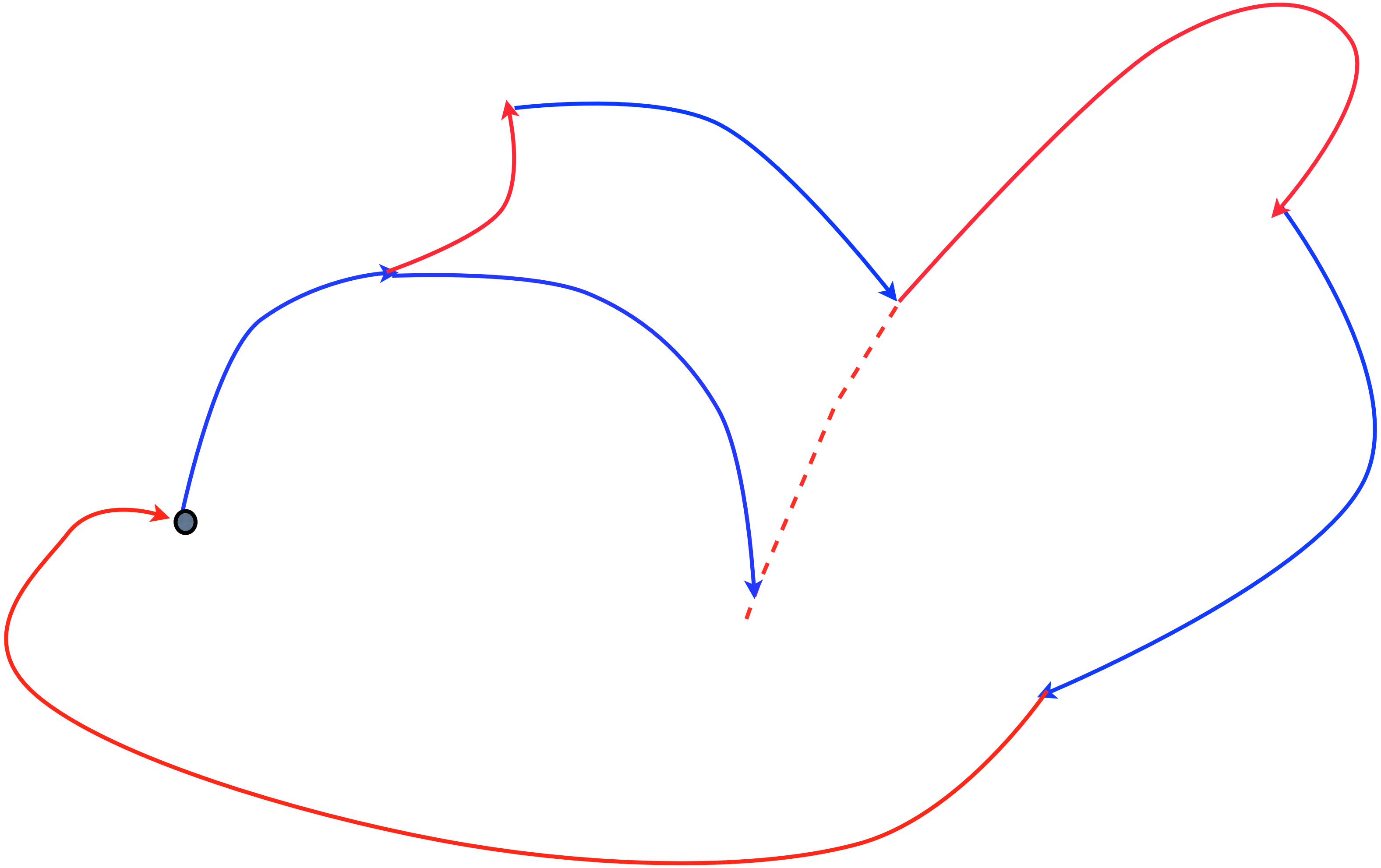
Condition nécessaire d'invariance



Condition nécessaire d'invariance



Condition nécessaire d'invariance



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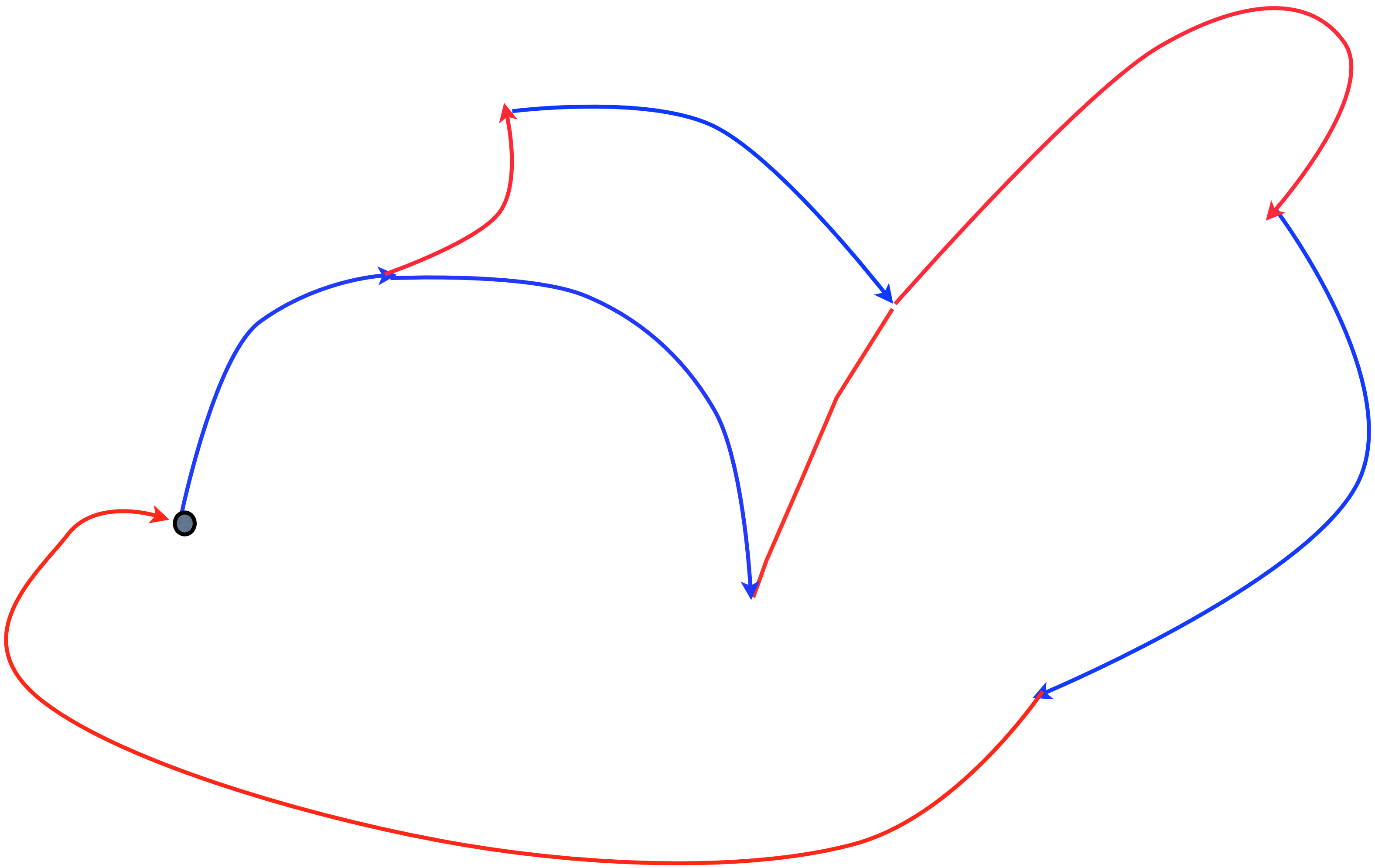

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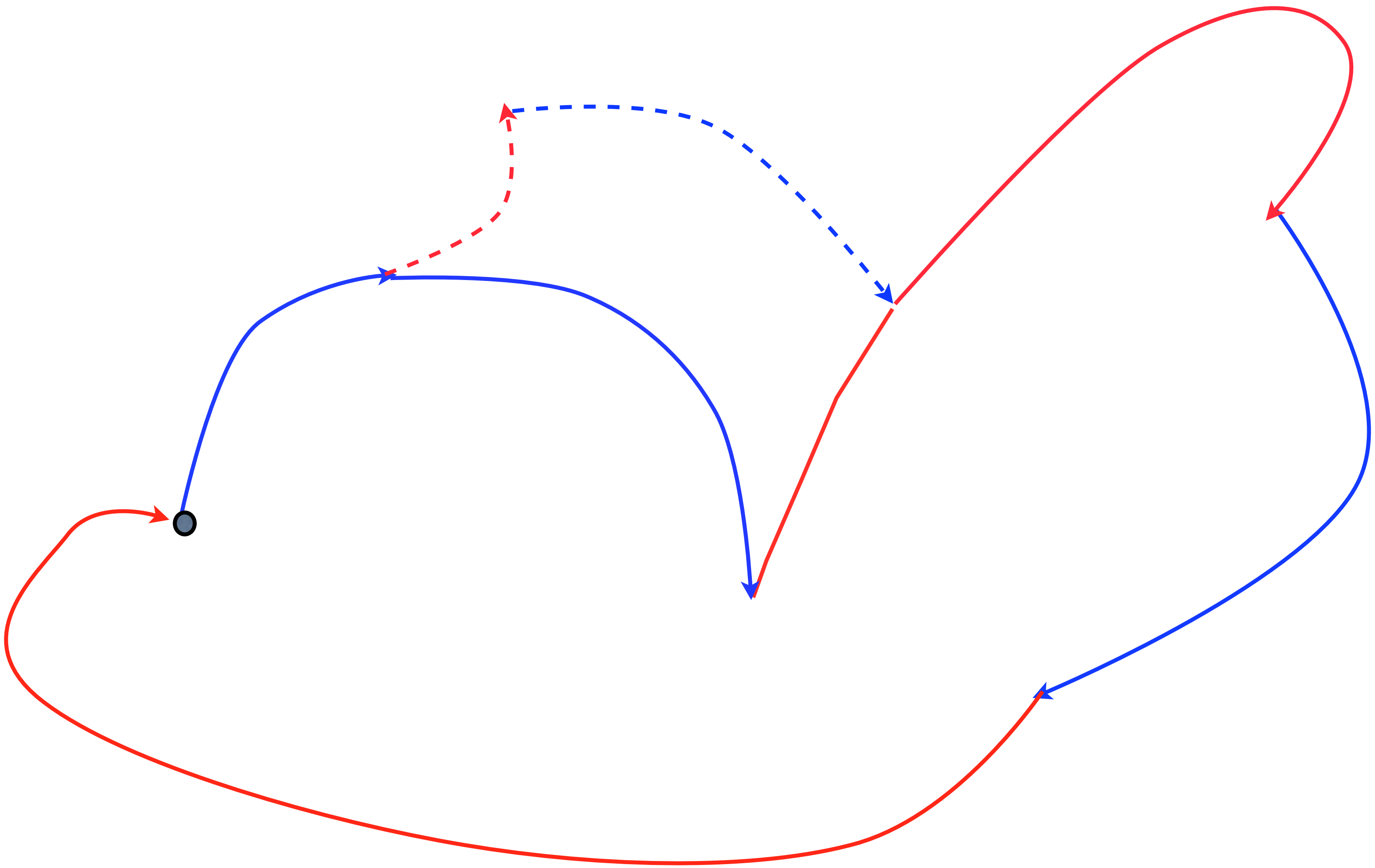
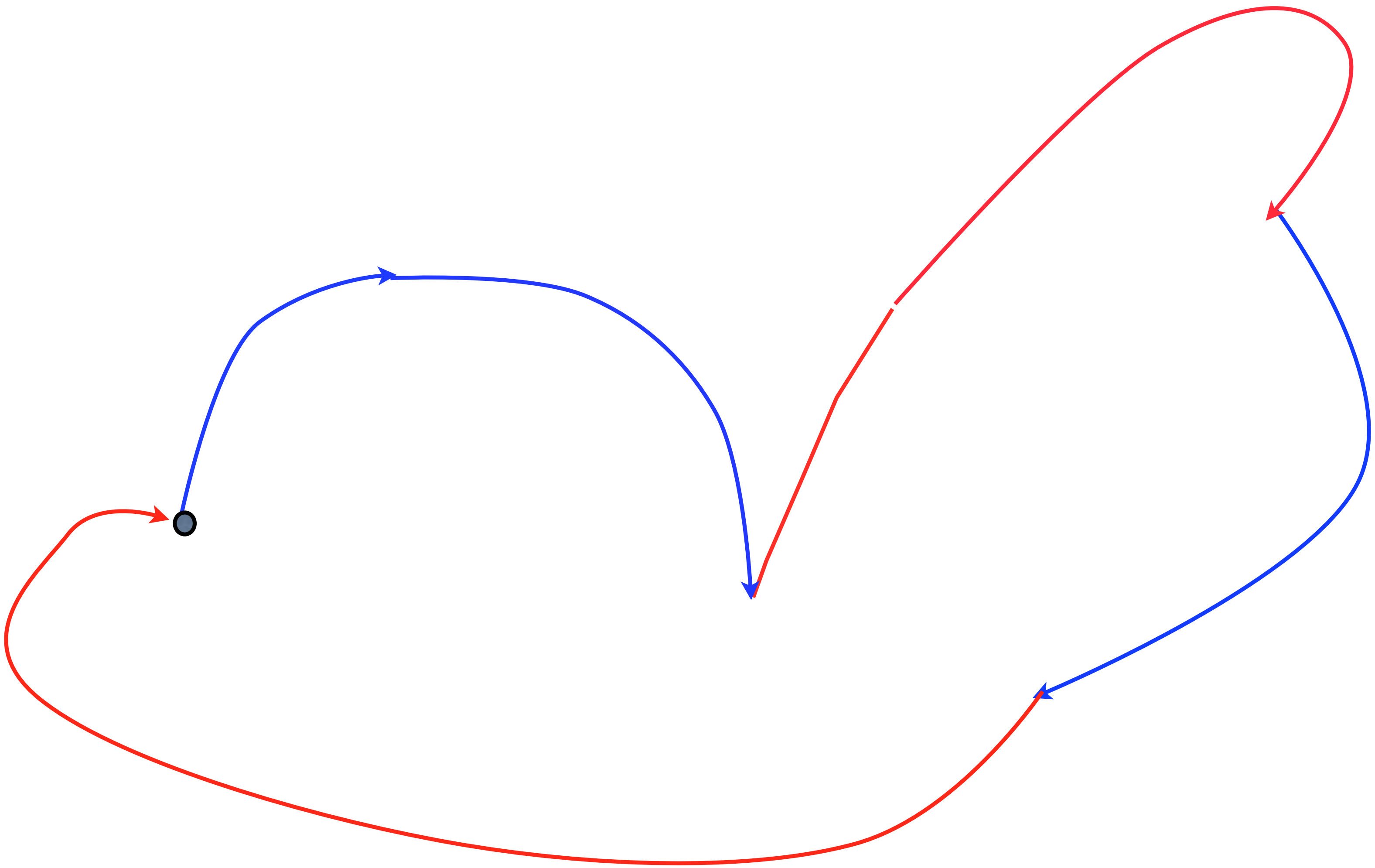
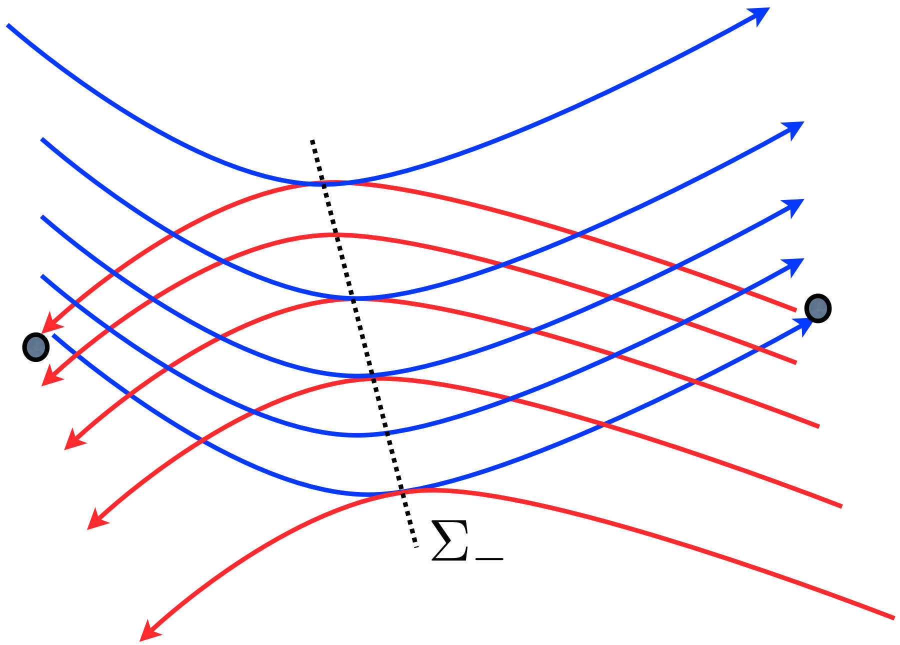


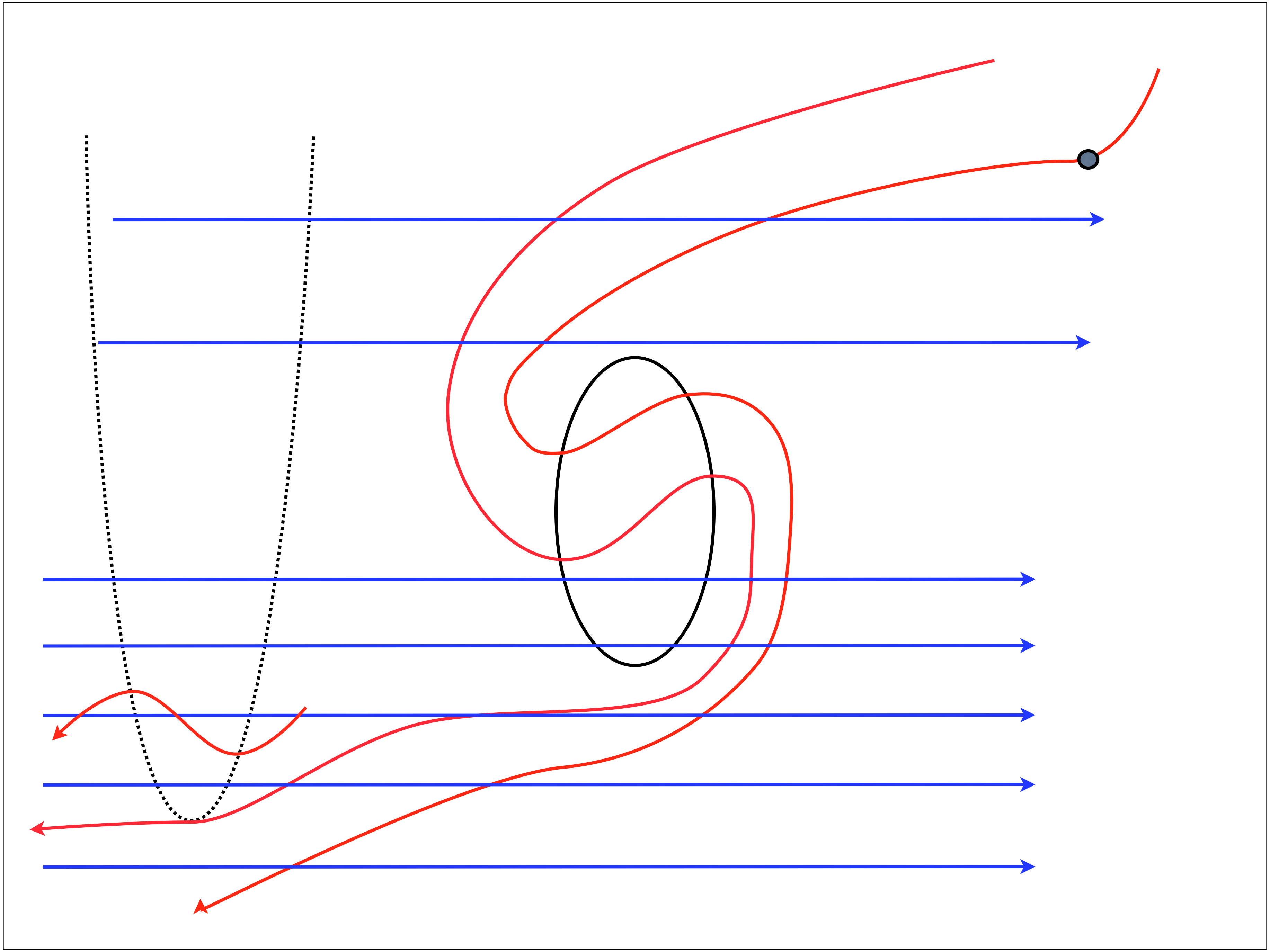
Condition nécessaire d'invariance

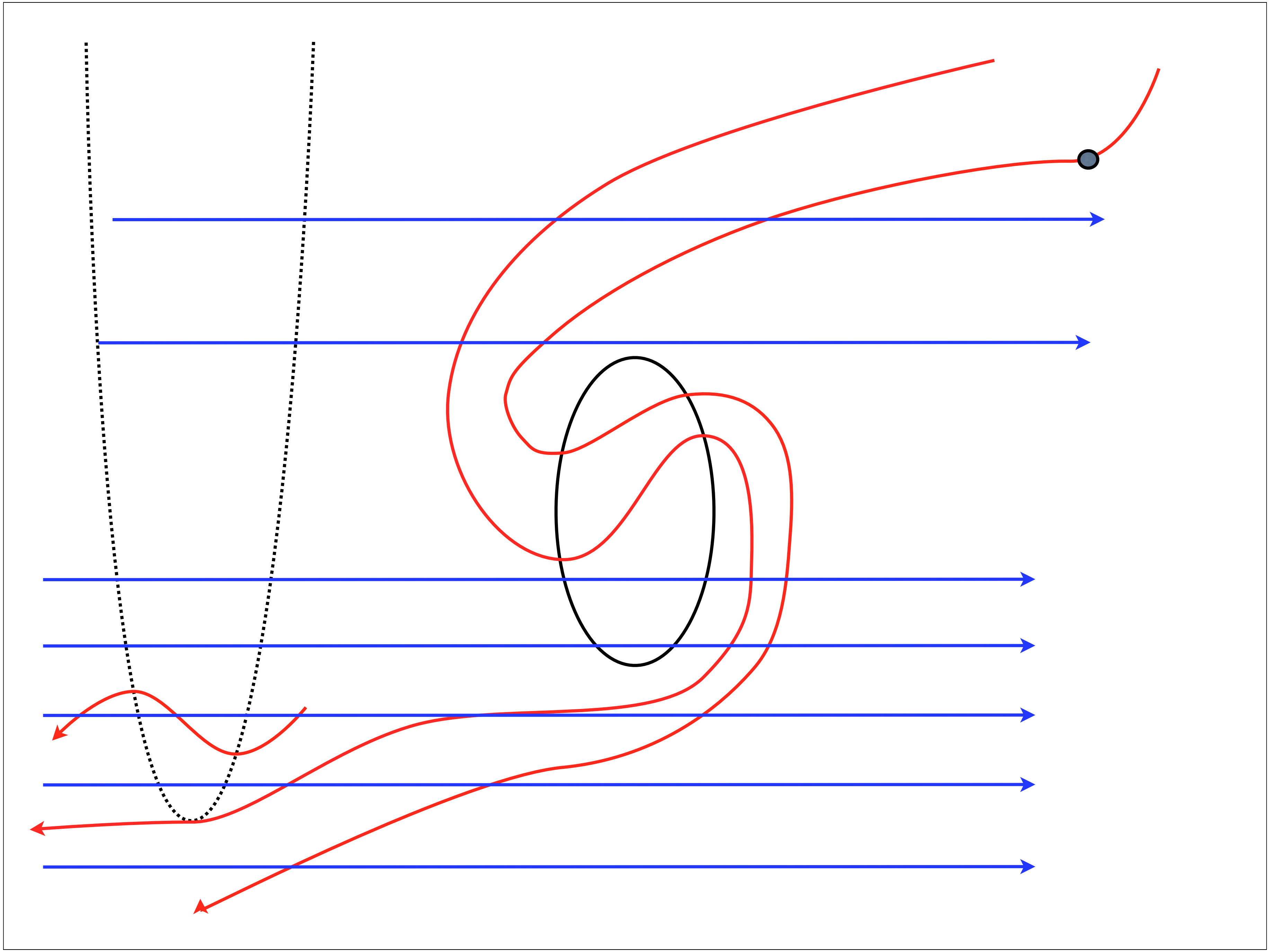


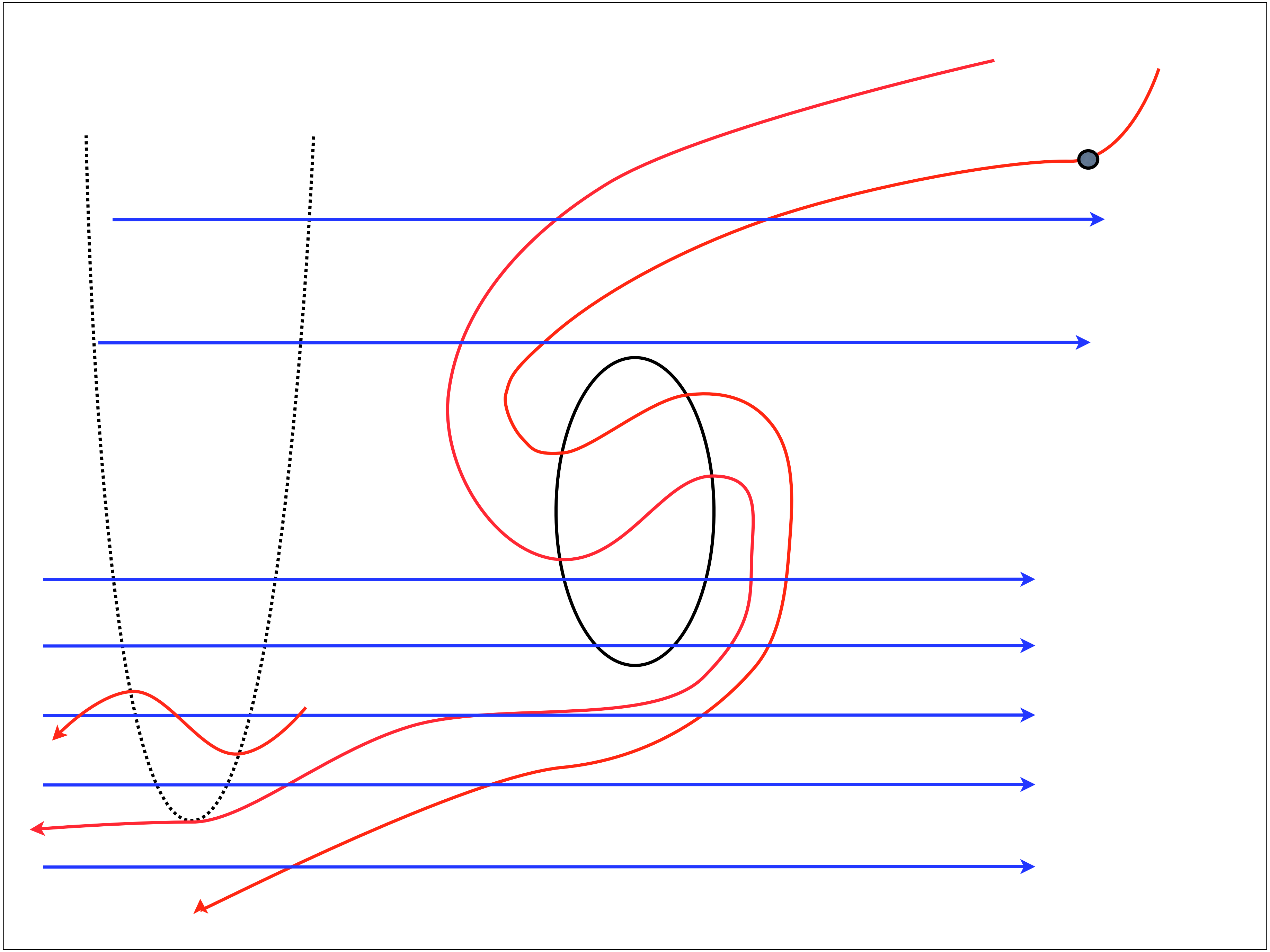
Necessary condition for transitivity

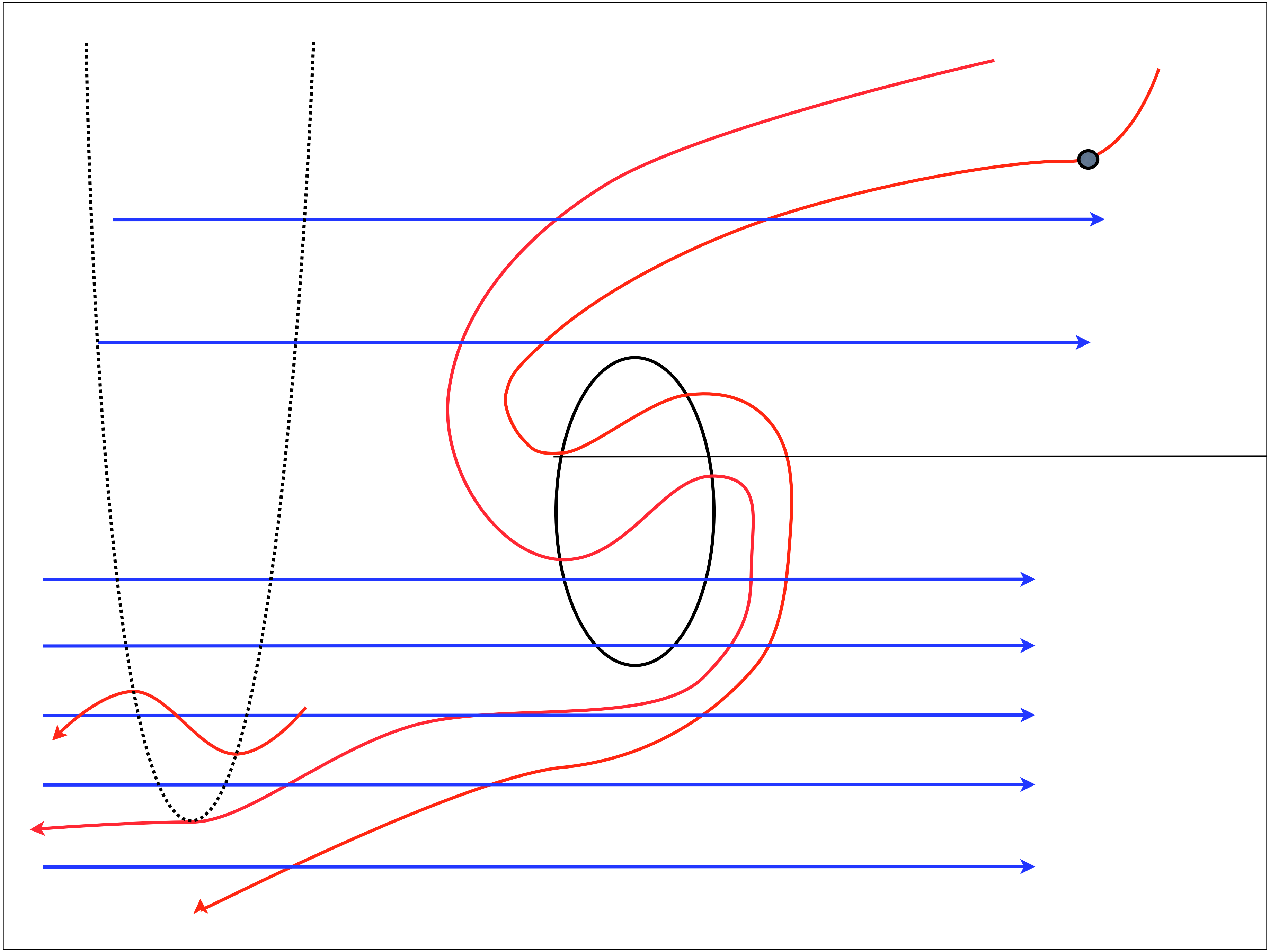
$$\Sigma_- \cup \Sigma_o \neq \emptyset$$

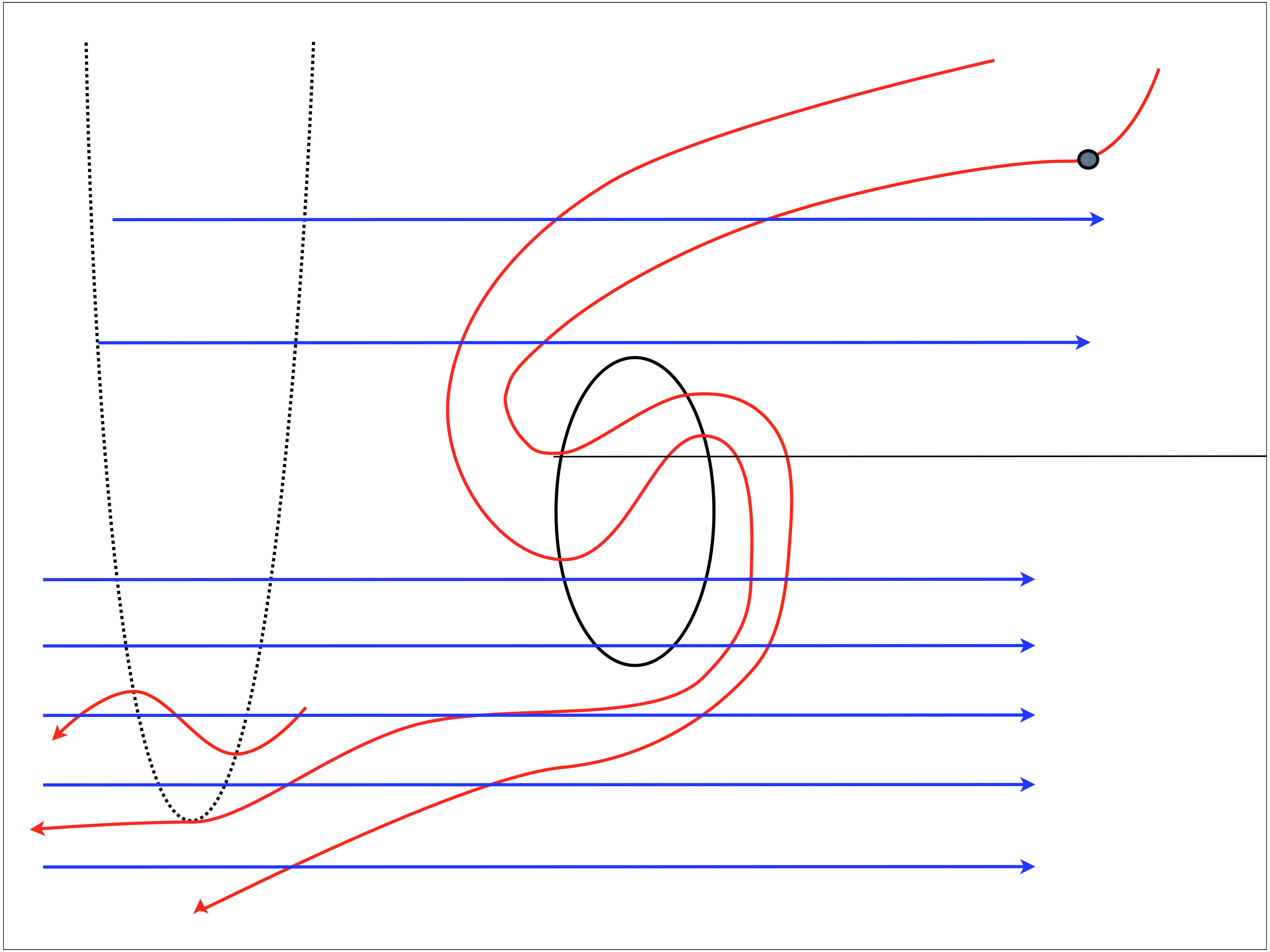


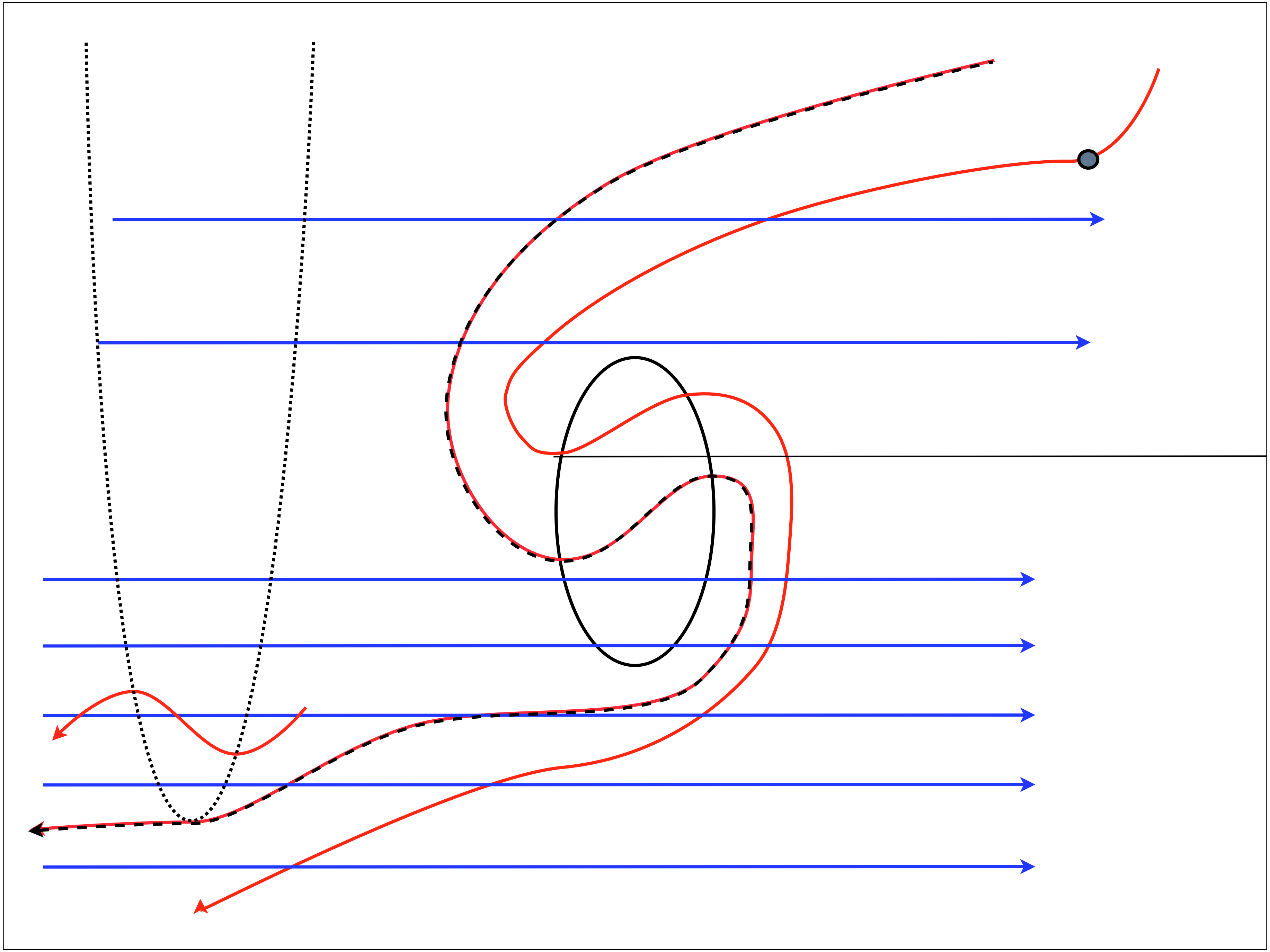


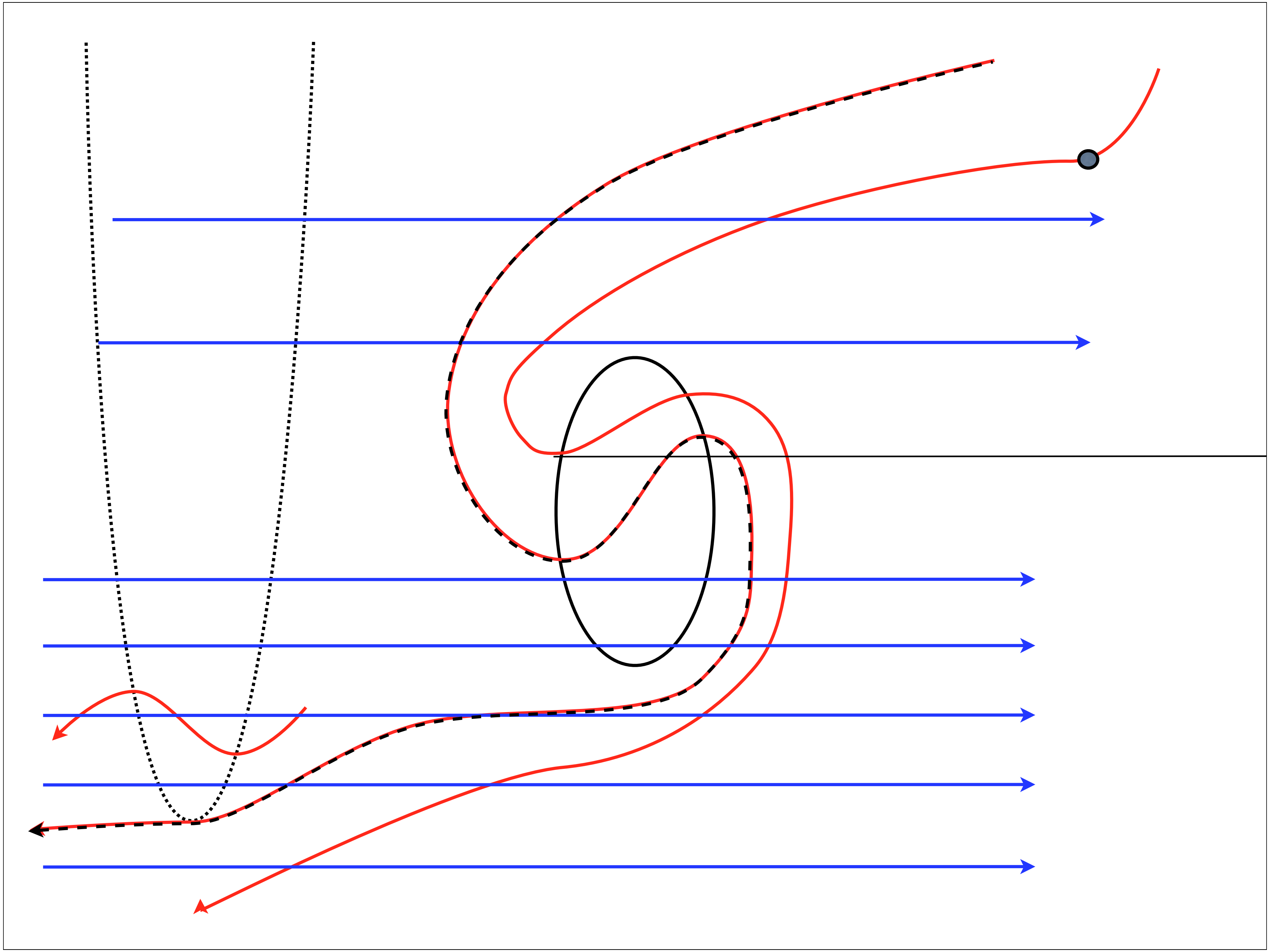


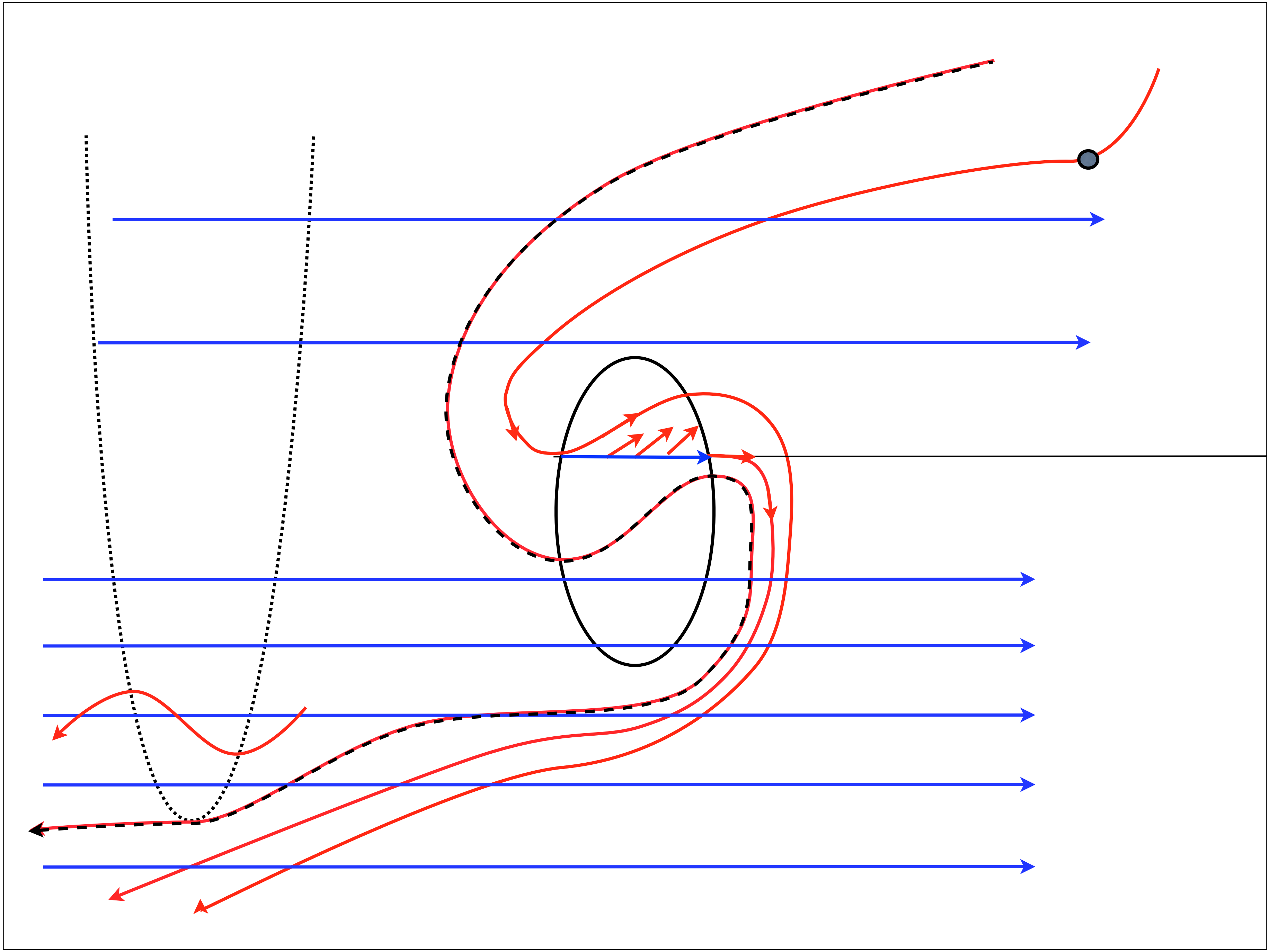


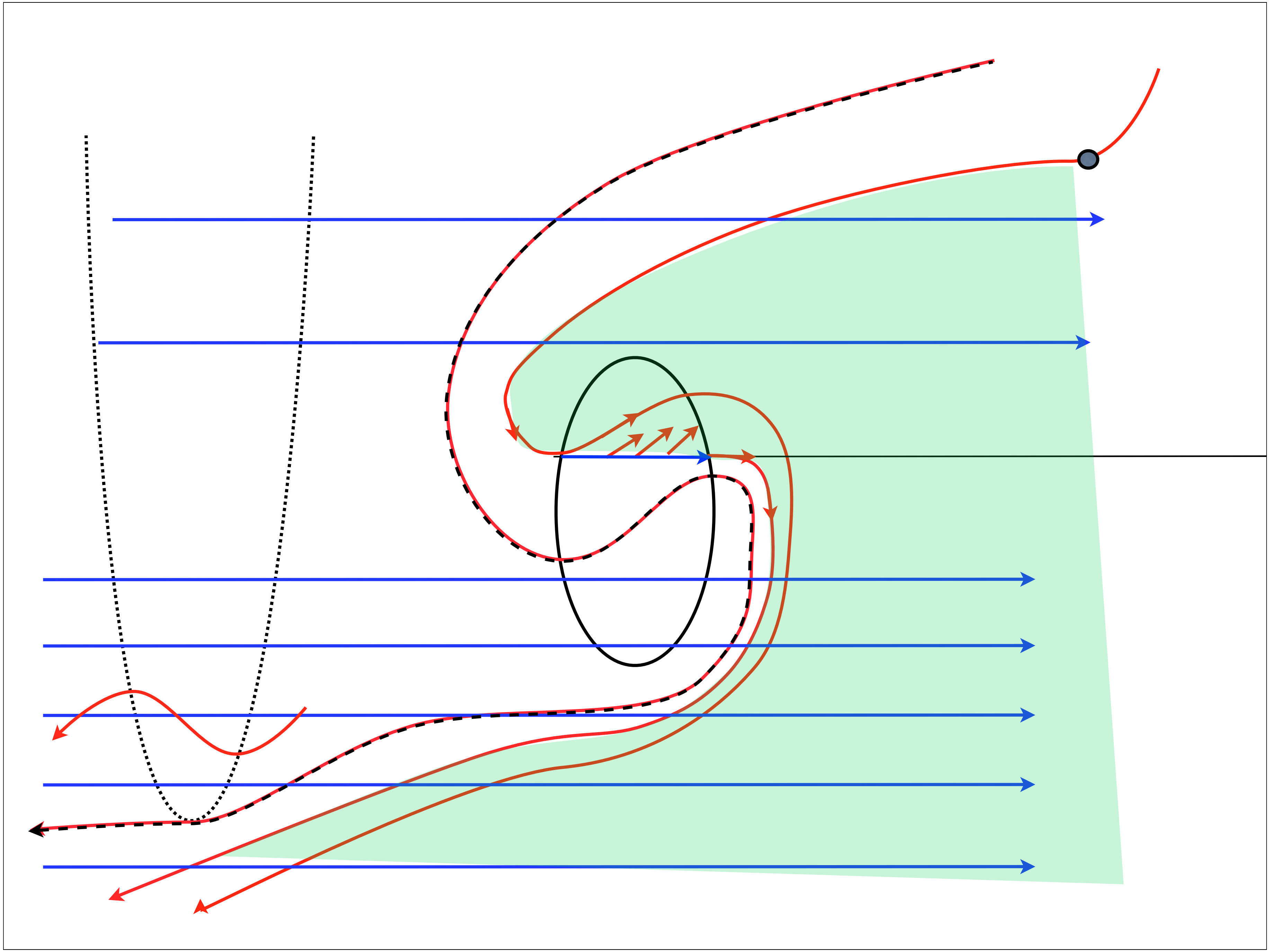


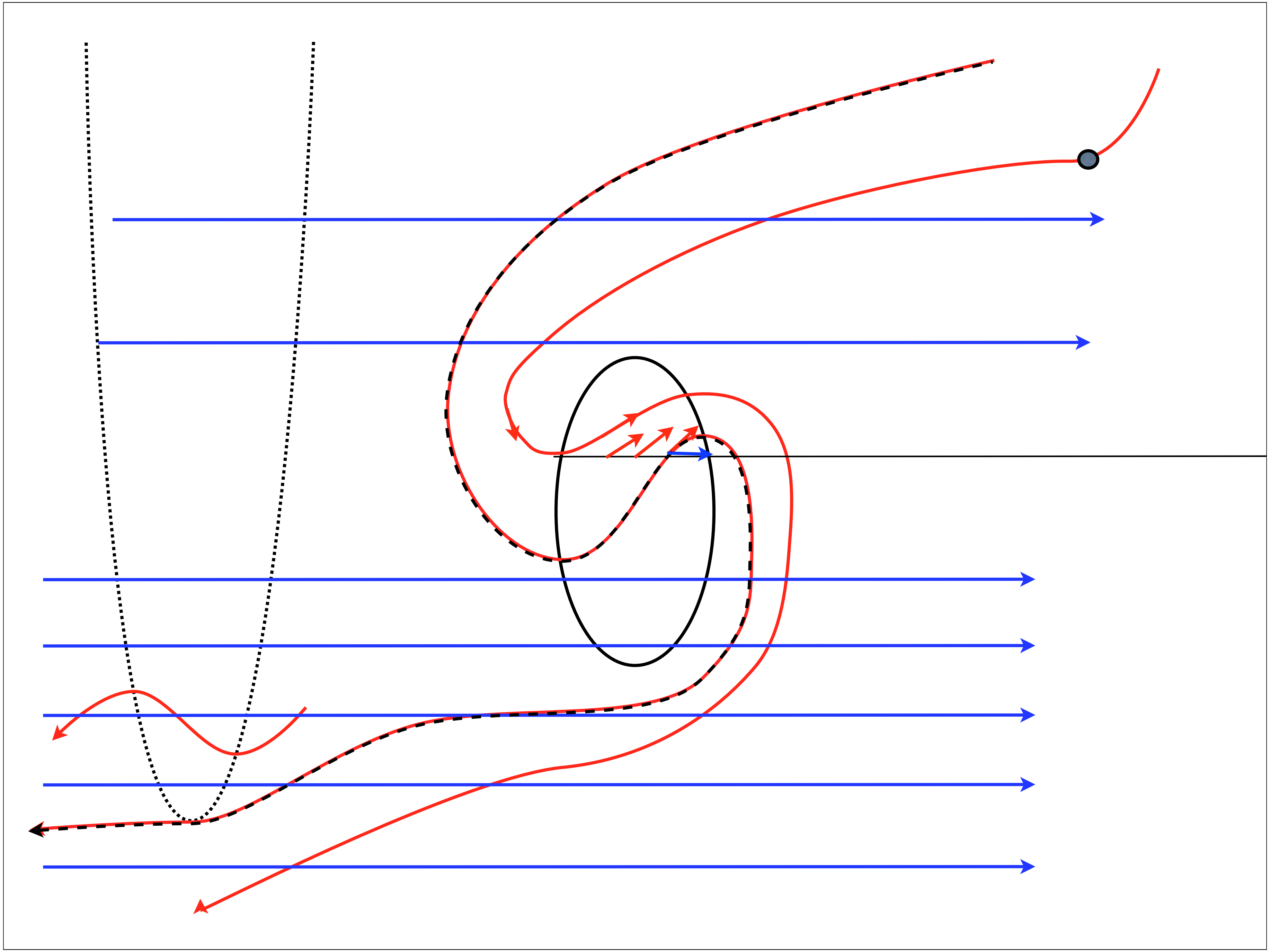


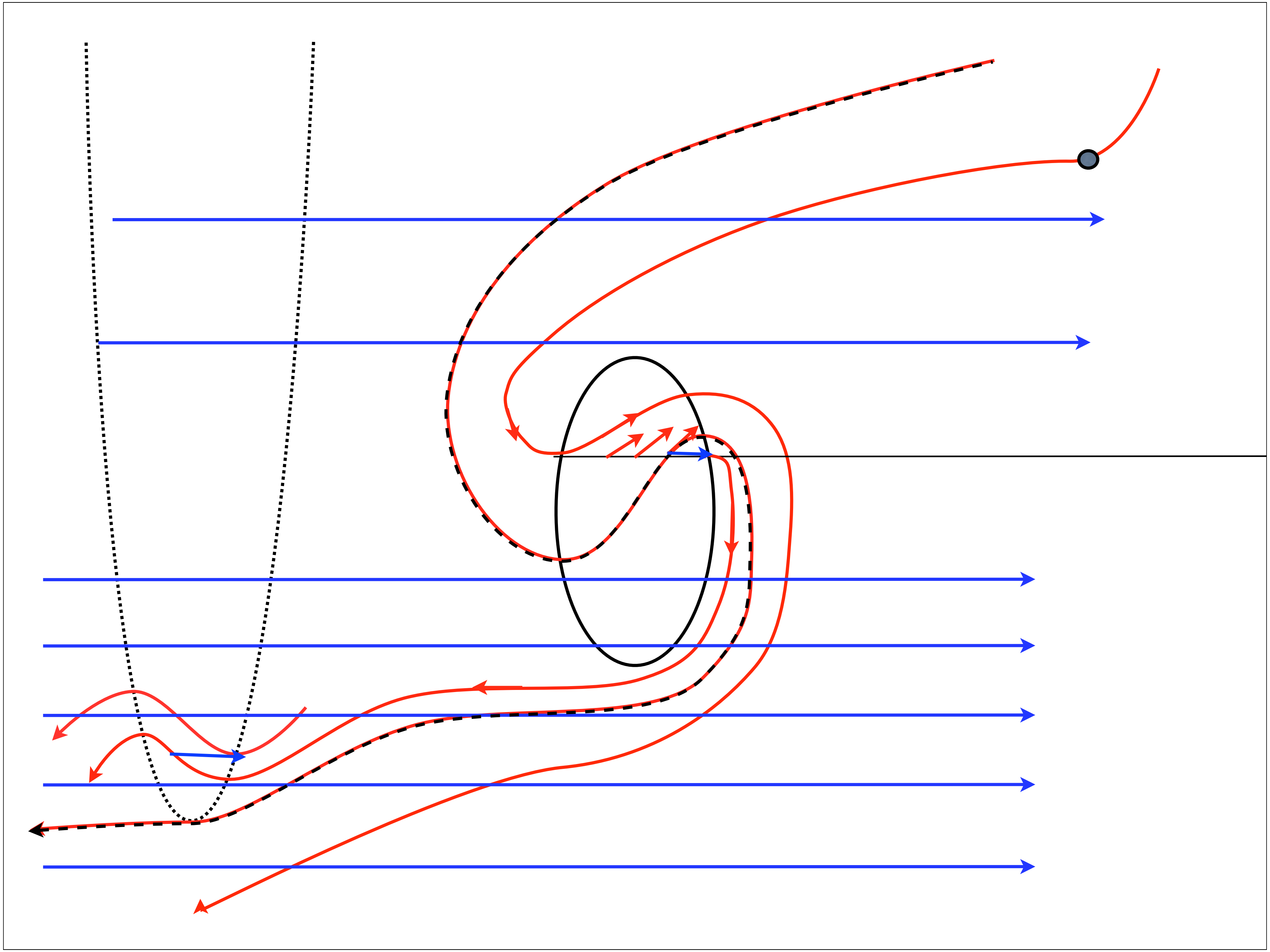


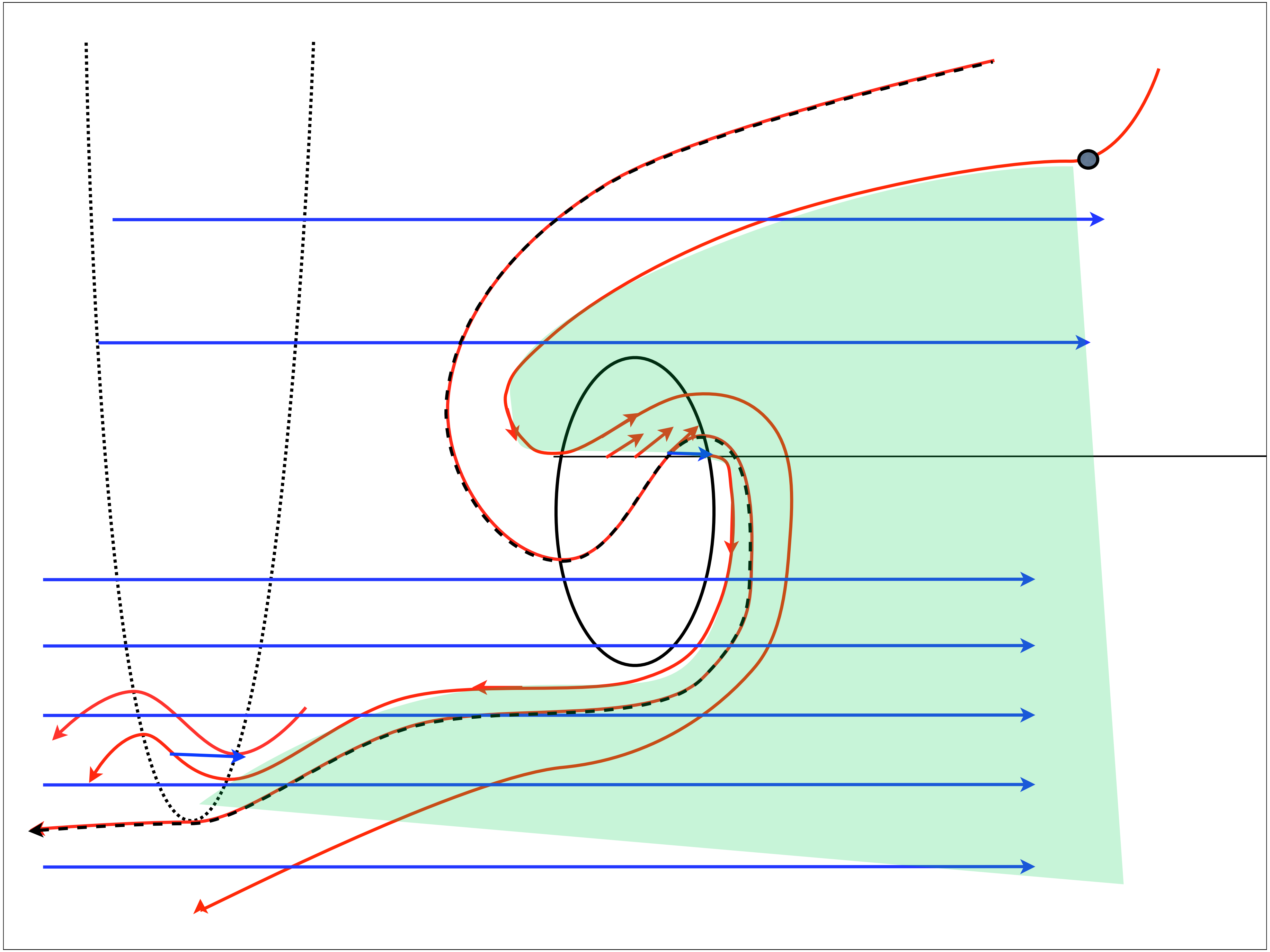


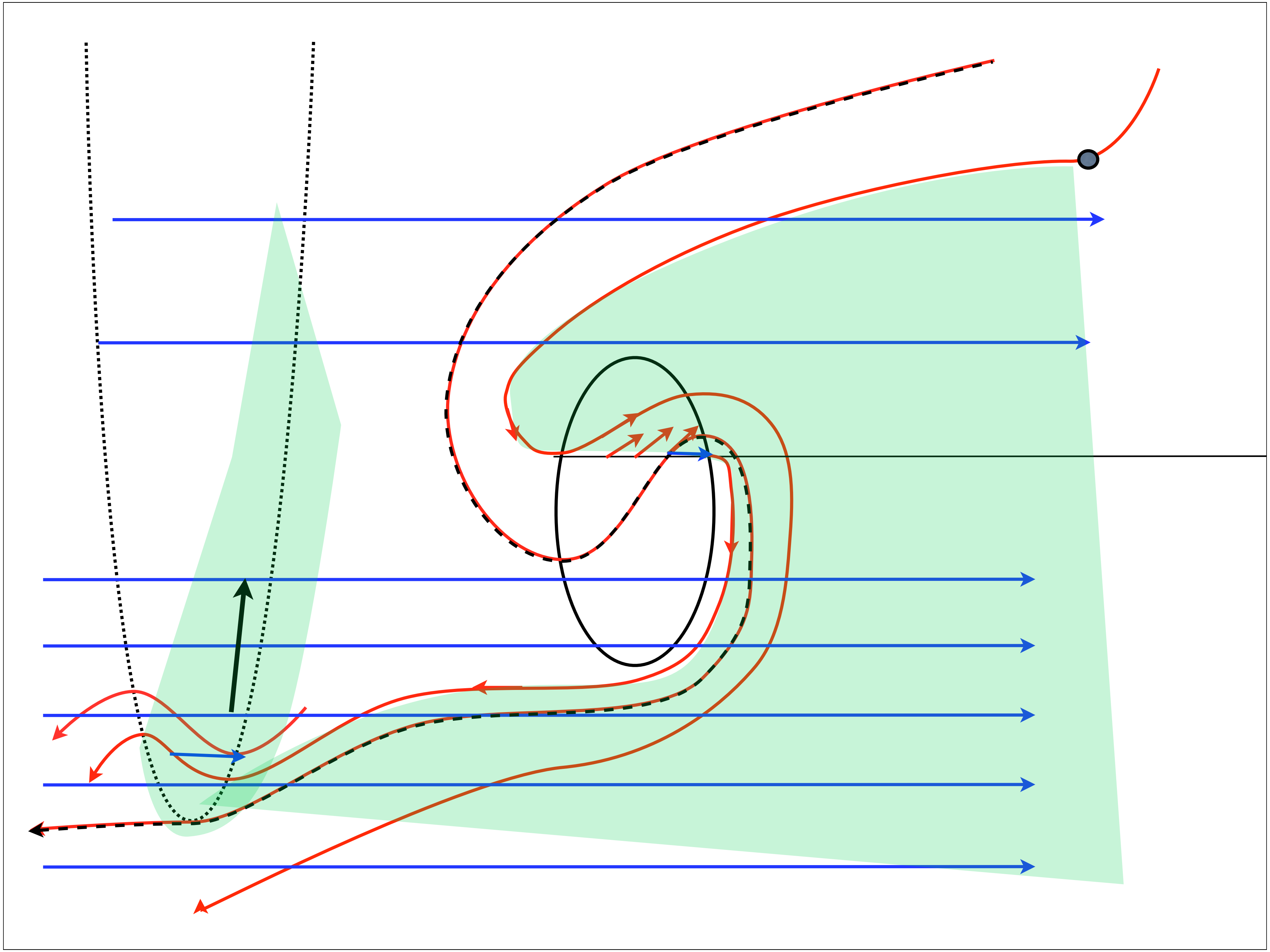


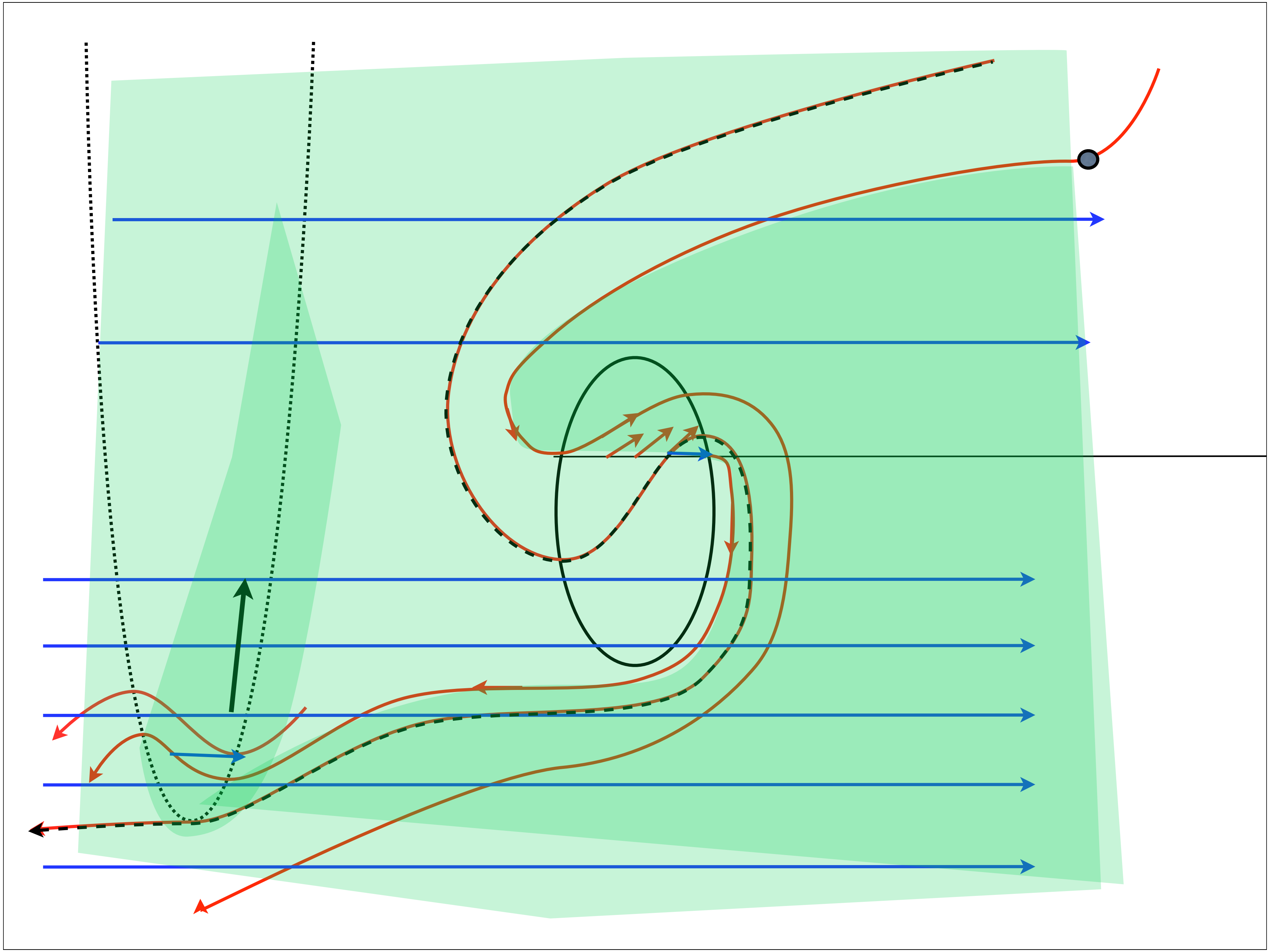




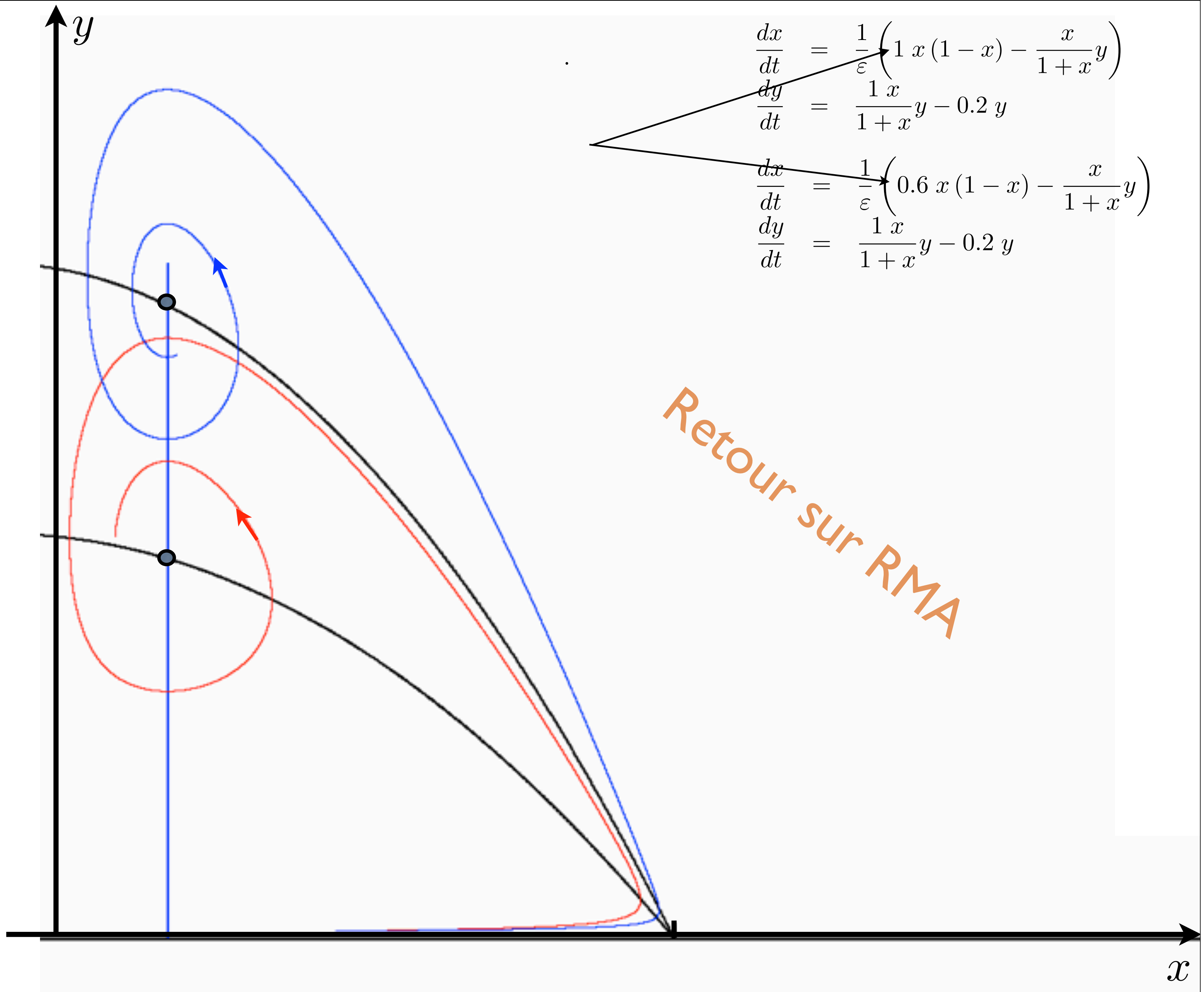








Retour sur RMA

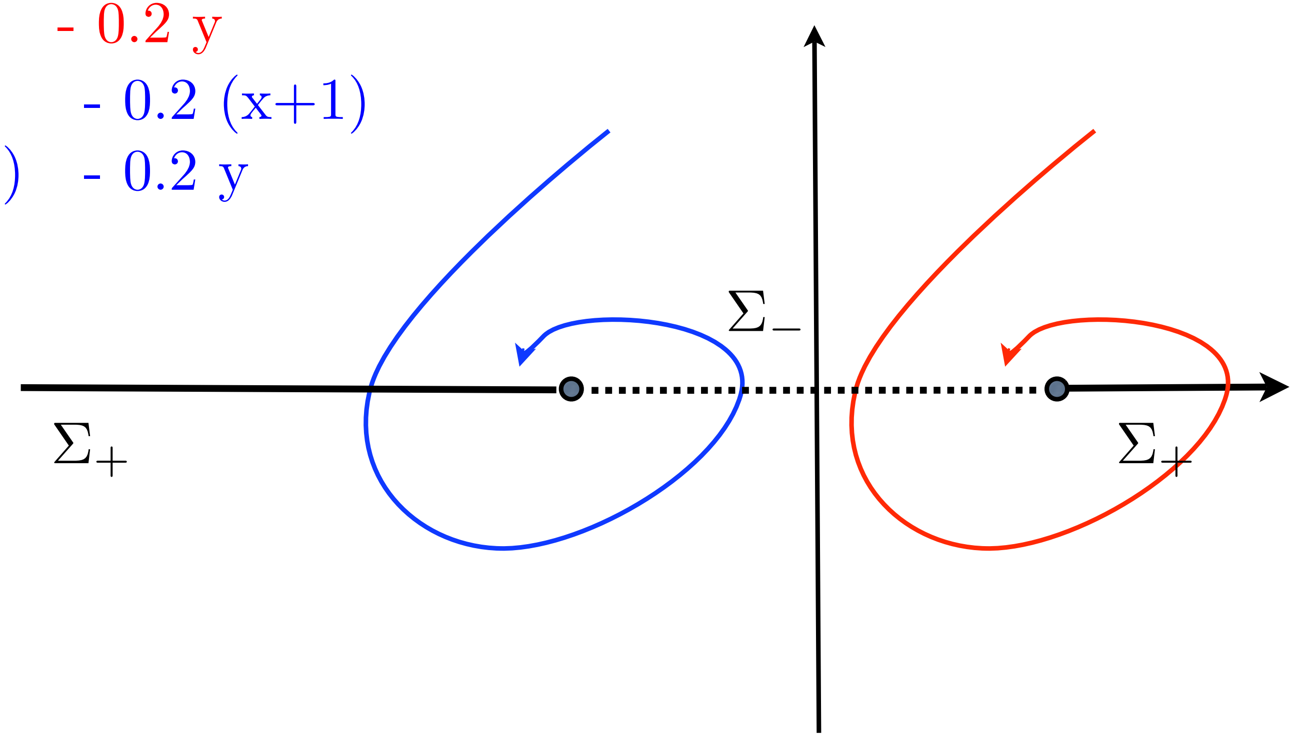


$$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(1\,x\,(1-x) - \frac{x}{1+x}y\right)$$
$$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$$

$$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(0.6\,x\,(1-x) - \frac{x}{1+x}y\right)$$
$$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$$

Deux foyers stable du plan

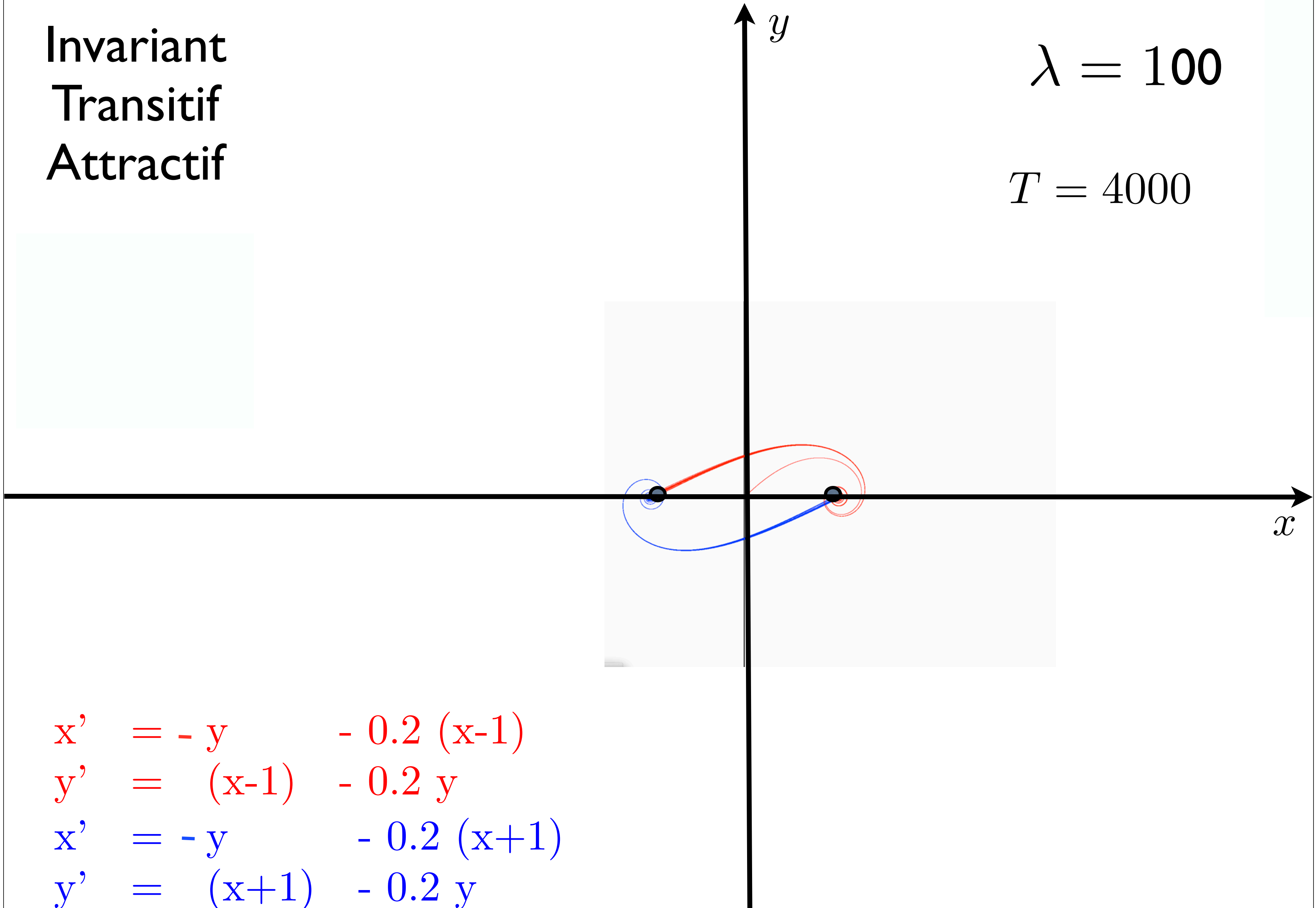
$$x' = -y - 0.2\,(x-1)$$
$$y' = (x-1) - 0.2\,y$$
$$x' = -y - 0.2\,(x+1)$$
$$y' = (x+1) - 0.2\,y$$



Invariant
Transitif
Attractif

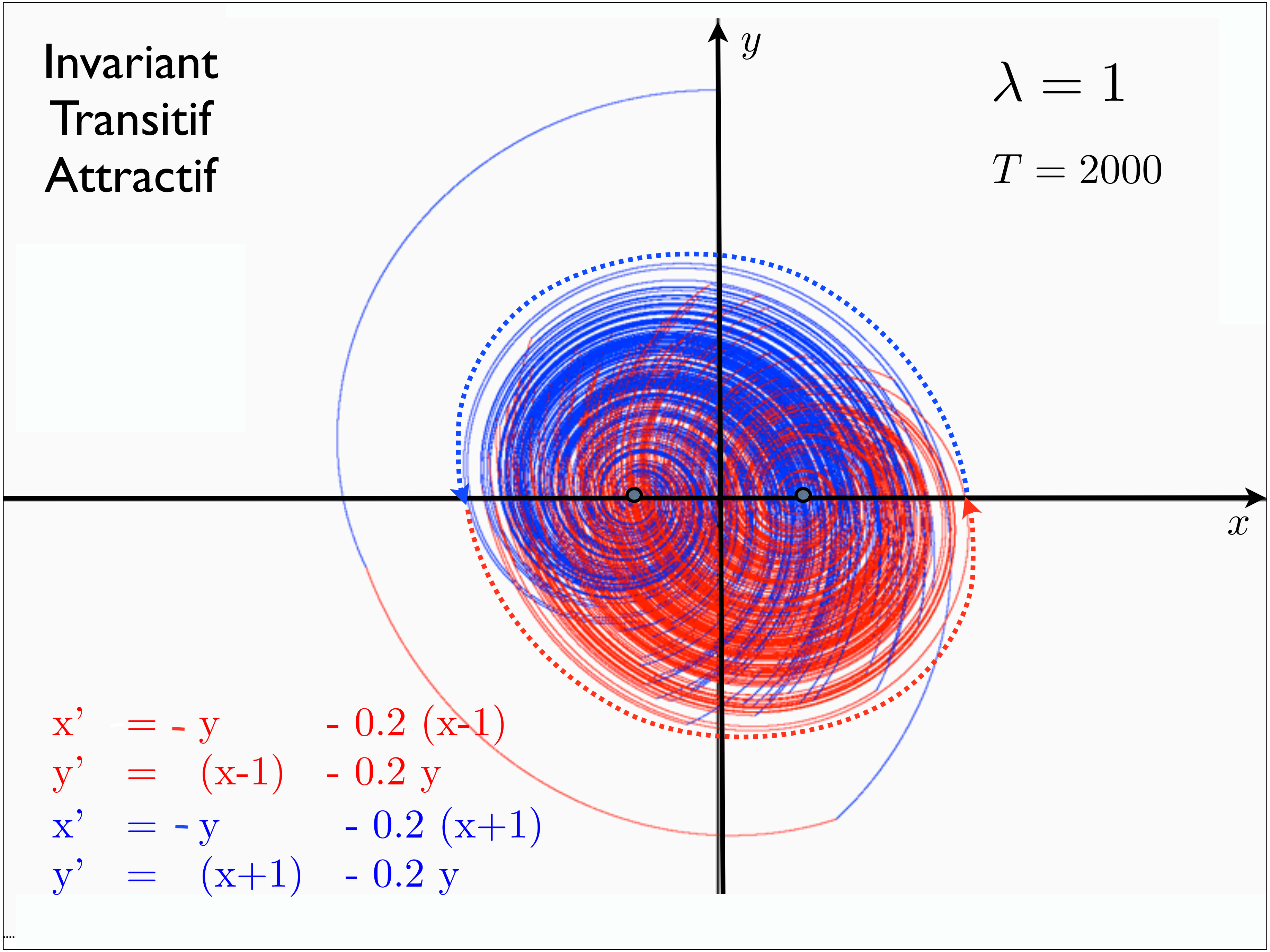
$\lambda = 100$

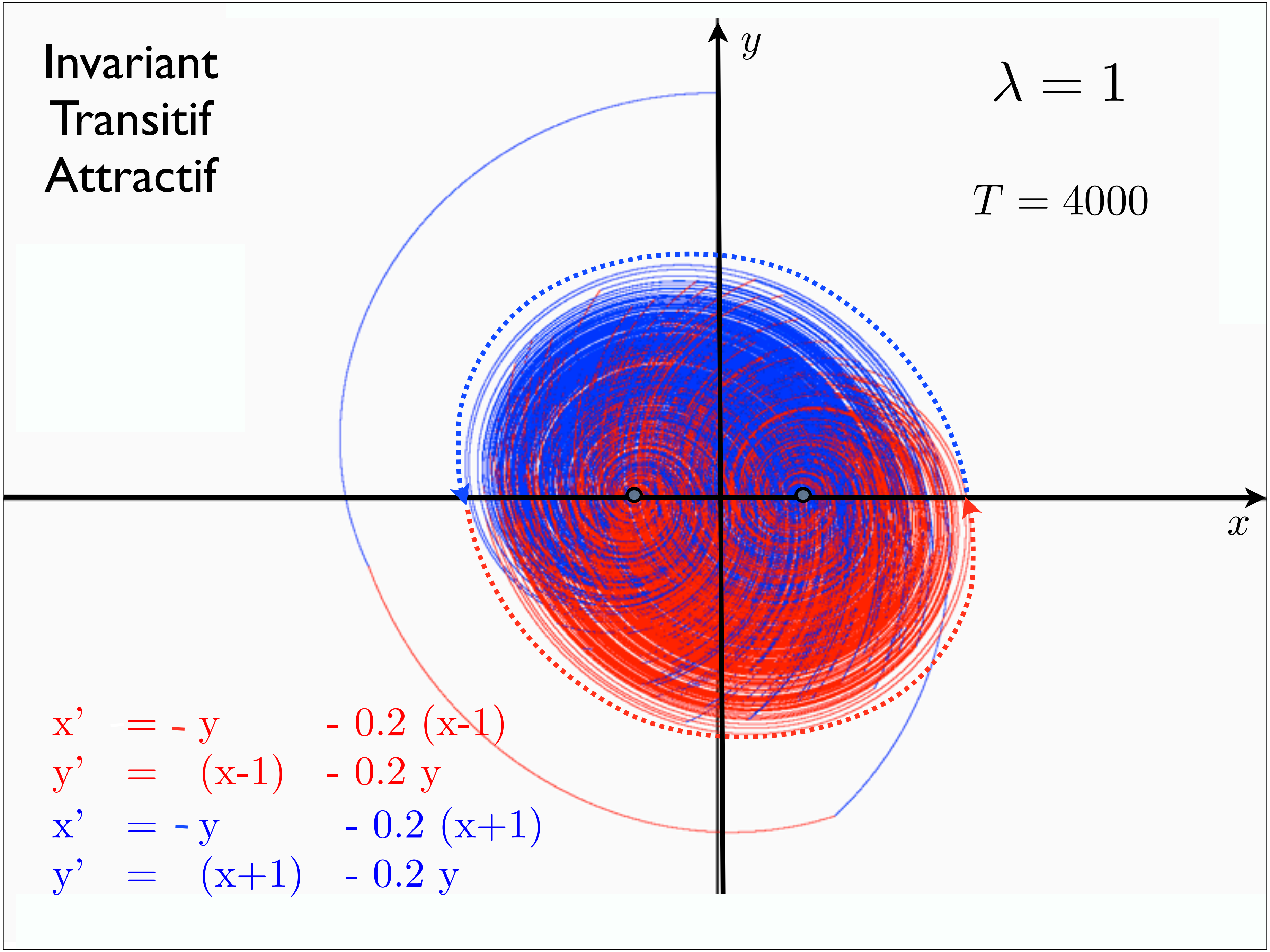
$T = 4000$



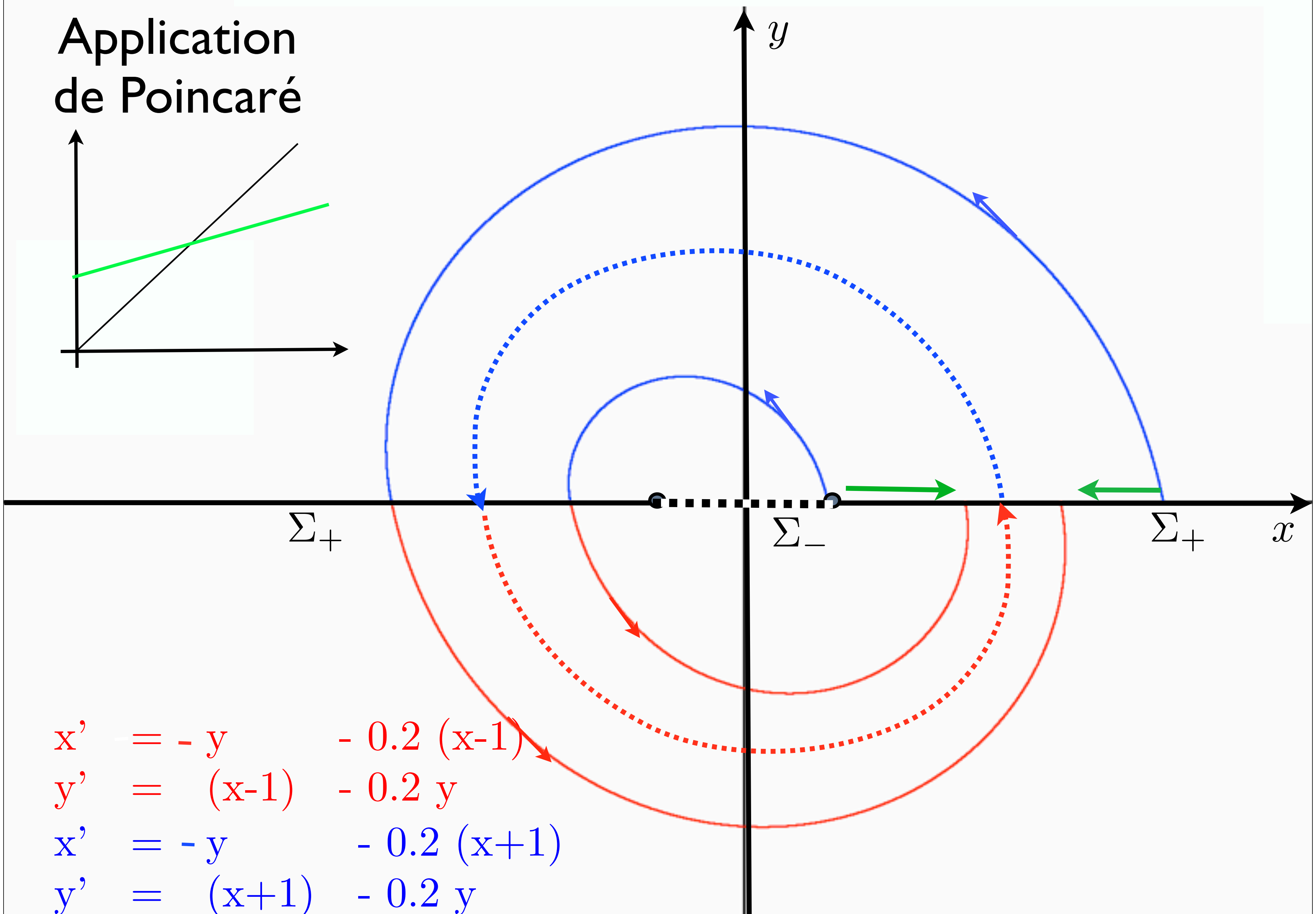
$x' = -y - 0.2\,(x-1)$
$y' = (x-1) - 0.2\,y$

$x' = -y - 0.2\,(x+1)$
$y' = (x+1) - 0.2\,y$


Invariant
Transitif
Attractif
$\lambda = 1$
$T = 2000$
y
x
x' = - y - 0.2 (x-1)
y' = (x-1) - 0.2 y
x' = - y - 0.2 (x+1)
y' = (x+1) - 0.2 y


Invariant
Transitif
Attractif
λ = 1
T = 4000
y
x
x' = - y - 0.2 (x-1)
y' = (x-1) - 0.2 y
x' = - y - 0.2 (x+1)
y' = (x+1) - 0.2 y

Application de Poincaré

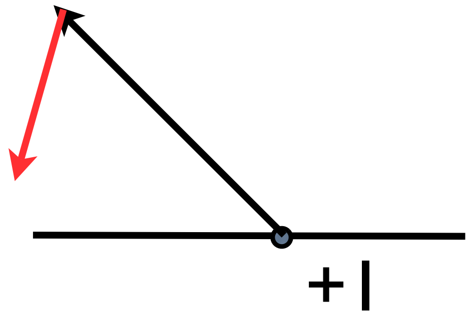


$$x' = -y - 0.2\,(x-1)$$
$$y' = (x-1) - 0.2\,y$$

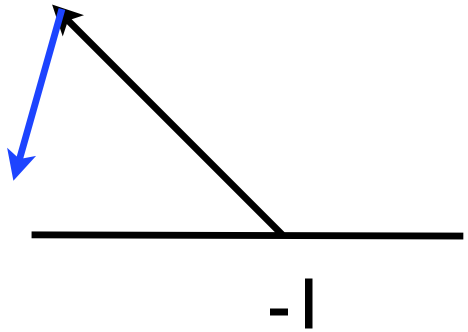
$$x' = -y - 0.2\,(x+1)$$
$$y' = (x+1) - 0.2\,y$$

Application de poincaré sans point fixe

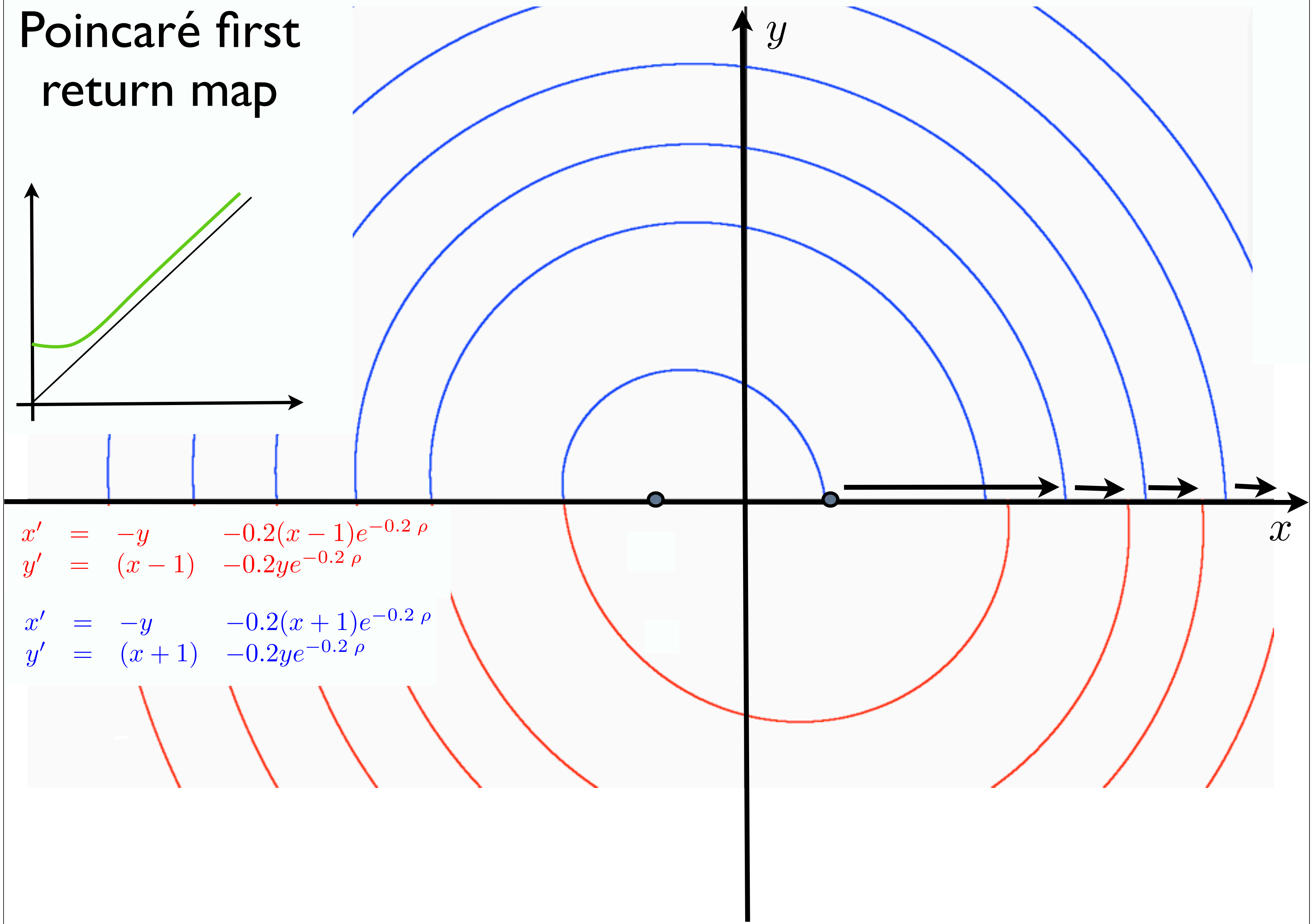
$$
\begin{aligned}
x' &= -y \quad -0.2(x-1)e^{-0.2\,\rho} \\
y' &= (x-1) \quad -0.2ye^{-0.2\,\rho}
\end{aligned}
$$



$$
\begin{aligned}
x' &= -y \quad -0.2(x+1)e^{-0.2\,\rho} \\
y' &= (x+1) \quad -0.2ye^{-0.2\,\rho}
\end{aligned}
$$

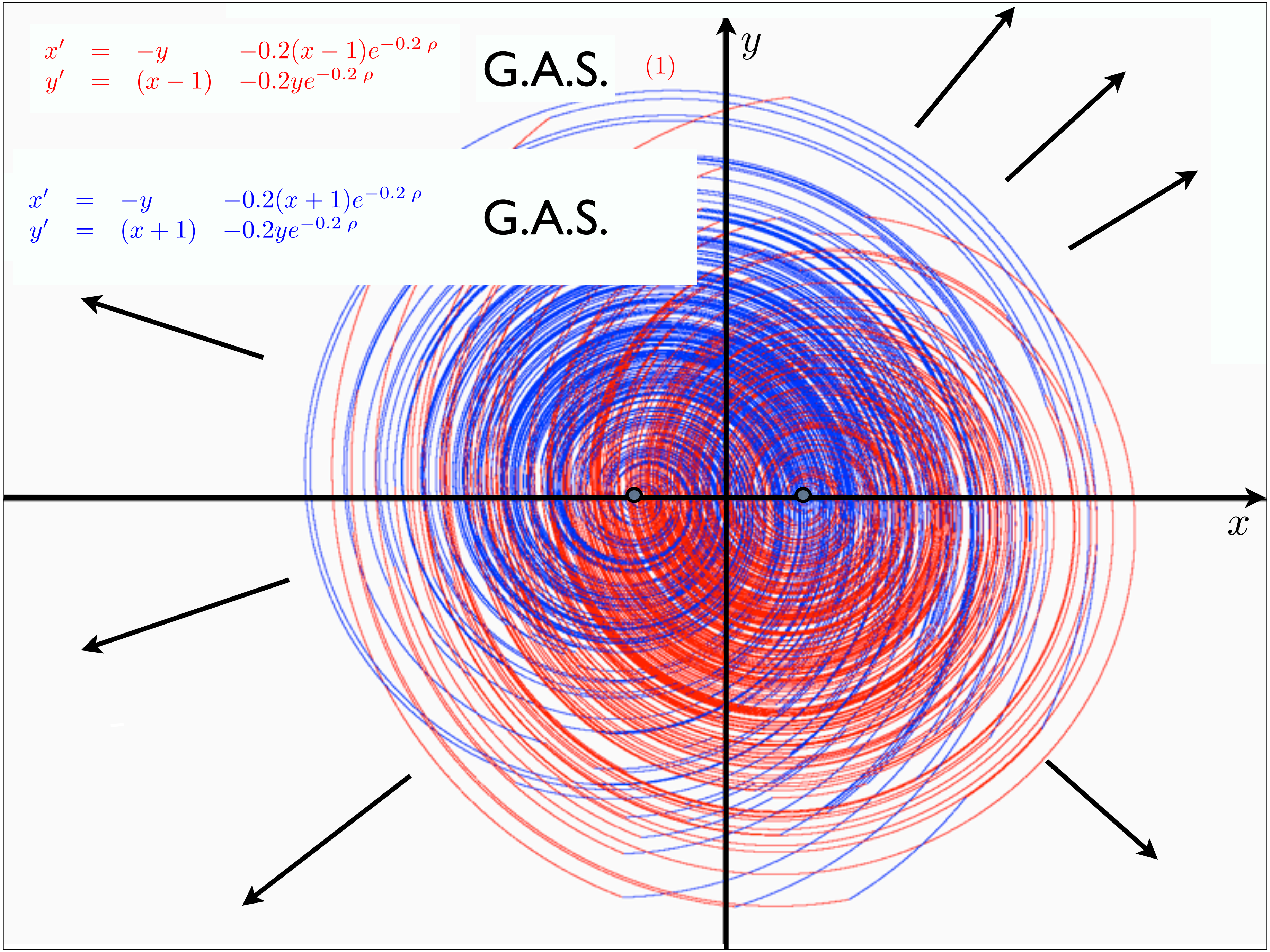


Poincaré first return map



$$
\begin{aligned}
x' &= -y \quad &-0.2(x-1)e^{-0.2\,\rho} \\
y' &= (x-1) \quad &-0.2ye^{-0.2\,\rho}
\end{aligned}
$$

$$
\begin{aligned}
x' &= -y \quad &-0.2(x+1)e^{-0.2\,\rho} \\
y' &= (x+1) \quad &-0.2ye^{-0.2\,\rho}
\end{aligned}
$$

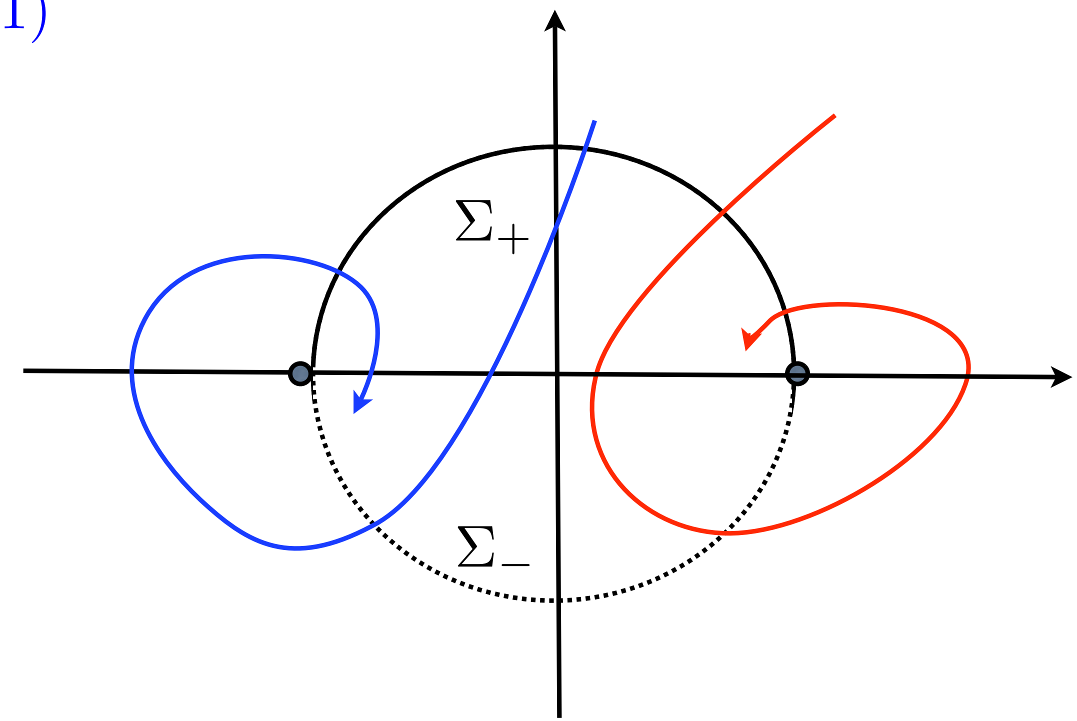


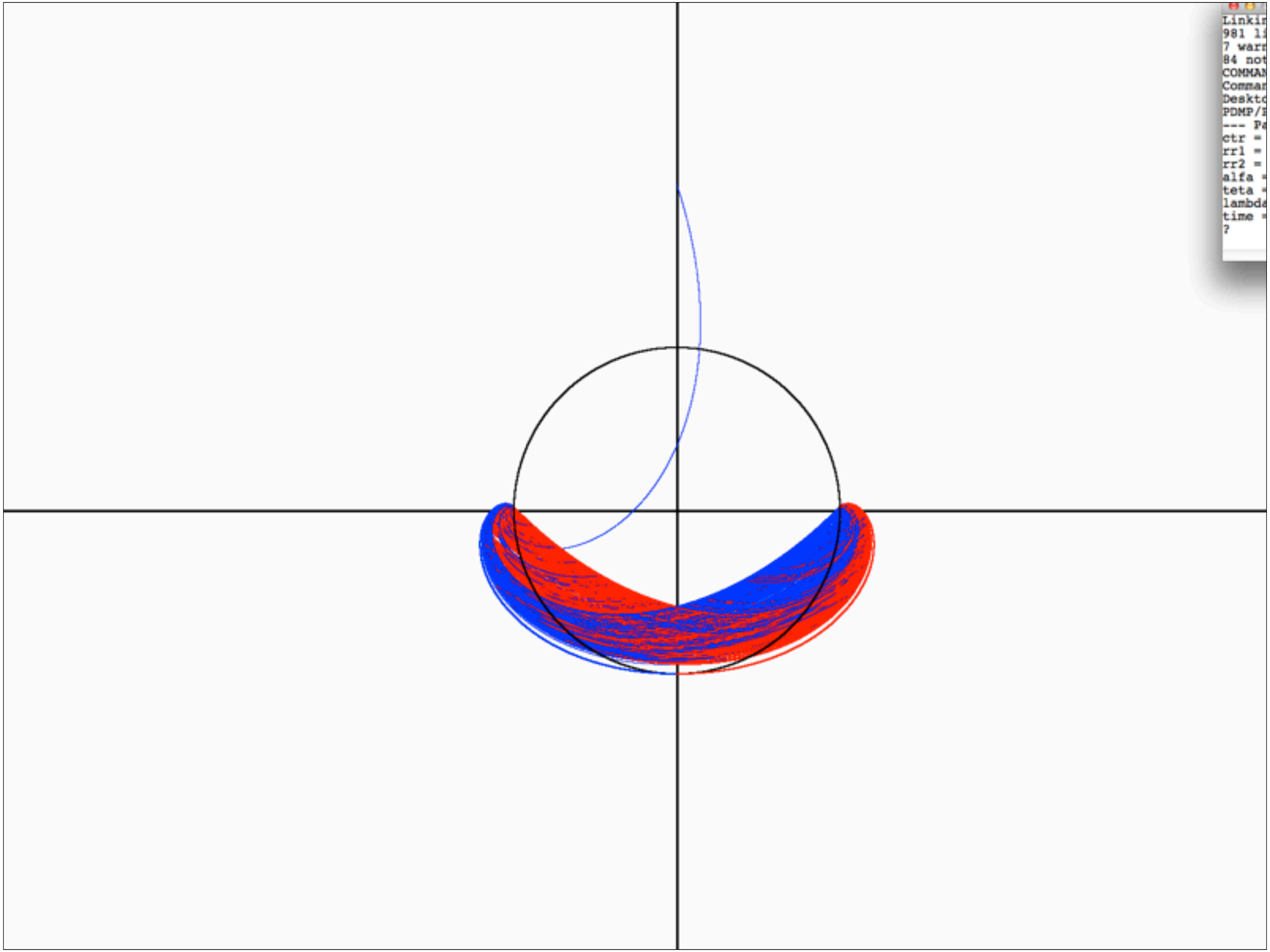
$x' = -y \quad -0.2(x-1)e^{-0.2\,\rho}$
$y' = (x-1) \quad -0.2ye^{-0.2\,\rho}$
G.A.S.
(1)
$x' = -y \quad -0.2(x+1)e^{-0.2\,\rho}$
$y' = (x+1) \quad -0.2ye^{-0.2\,\rho}$
G.A.S.
y
x

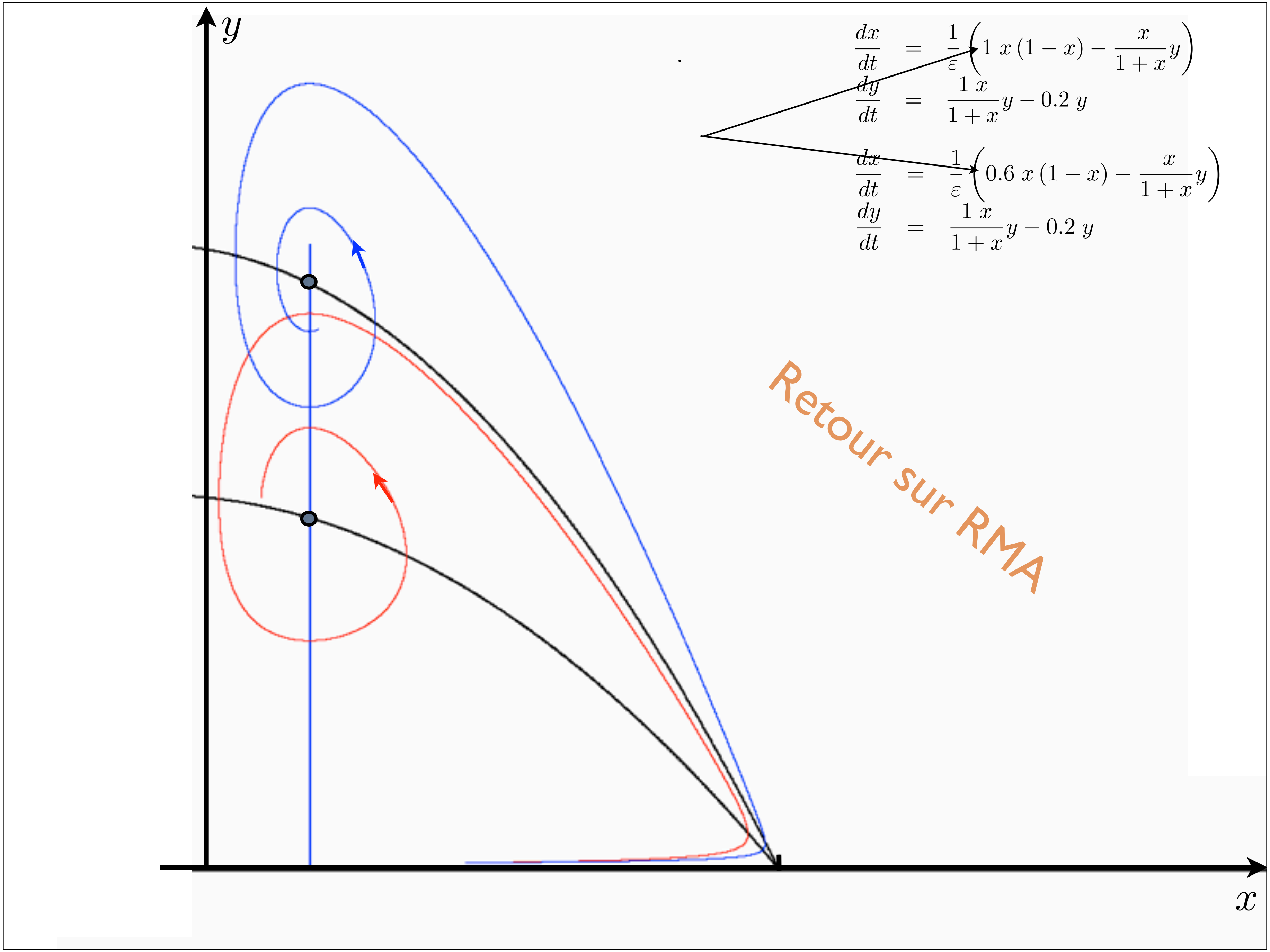
Two G.A.S. focus in the plane

change the direction of one rotation

$$
\begin{aligned}
x' &= -y - (x-1) \\
y' &= (x-1) - y \\
x' &= y - (x+1) \\
y' &= -(x+1) - y
\end{aligned}
$$




ctr =
rr1 =
rr2 =
alfa =
time =
?

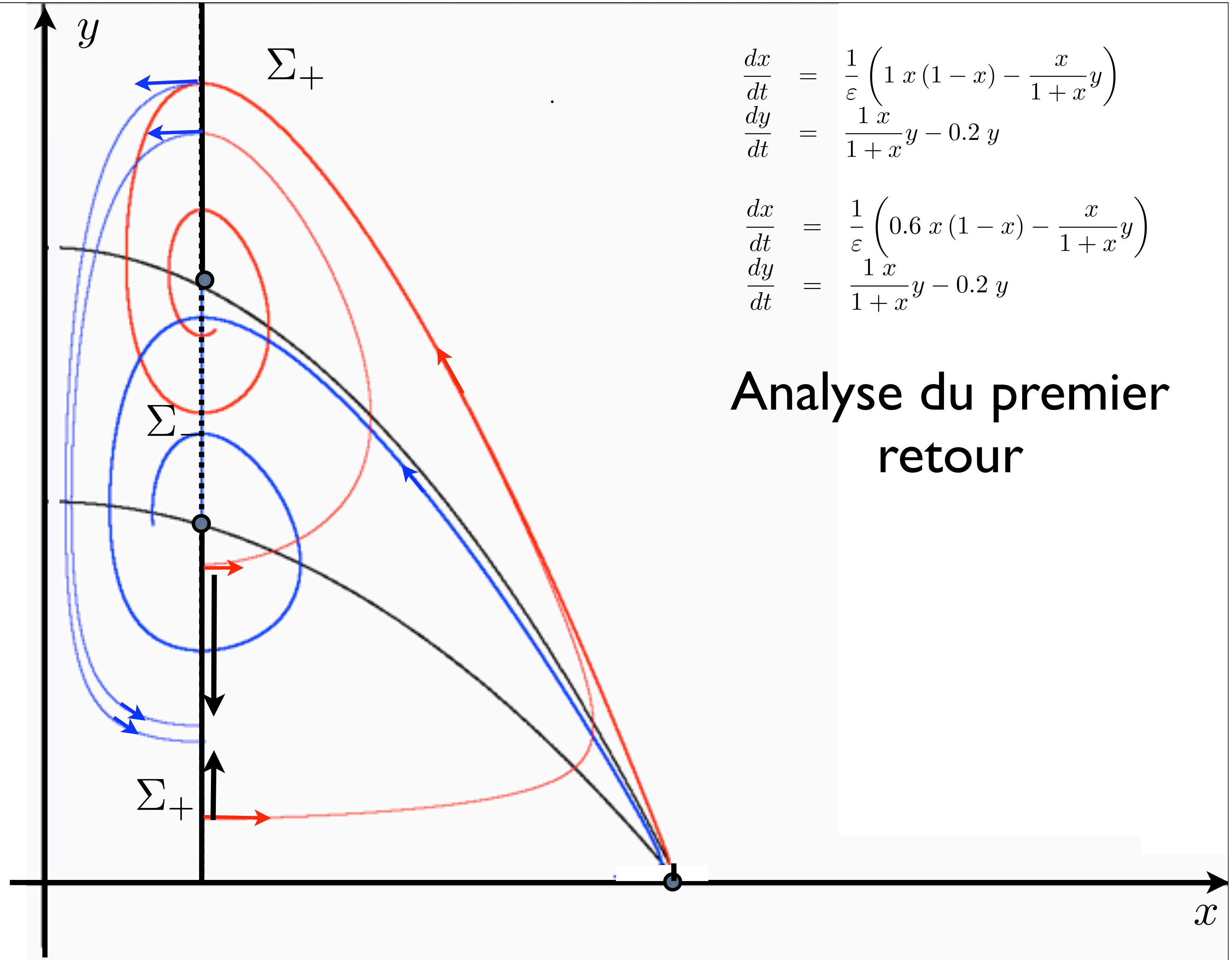


y
x
$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(1\,x\,(1-x) - \frac{x}{1+x}y\right)$
$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$
$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(0.6\,x\,(1-x) - \frac{x}{1+x}y\right)$
$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$
Retour sur RMA

Analyse du premier retour

$$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(1\,x\,(1-x) - \frac{x}{1+x}y\right)$$
$$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$$

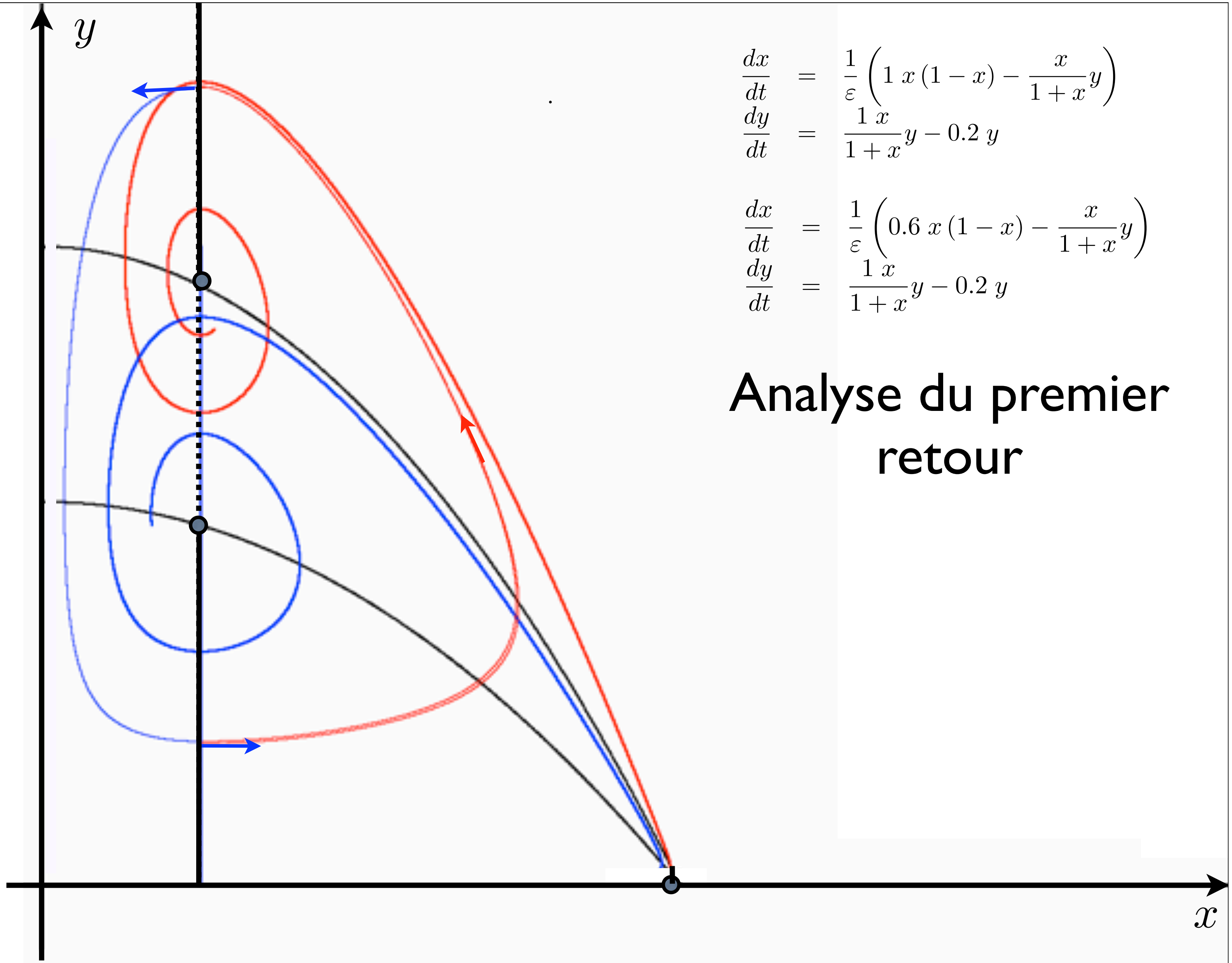
$$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(0.6\,x\,(1-x) - \frac{x}{1+x}y\right)$$
$$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$$

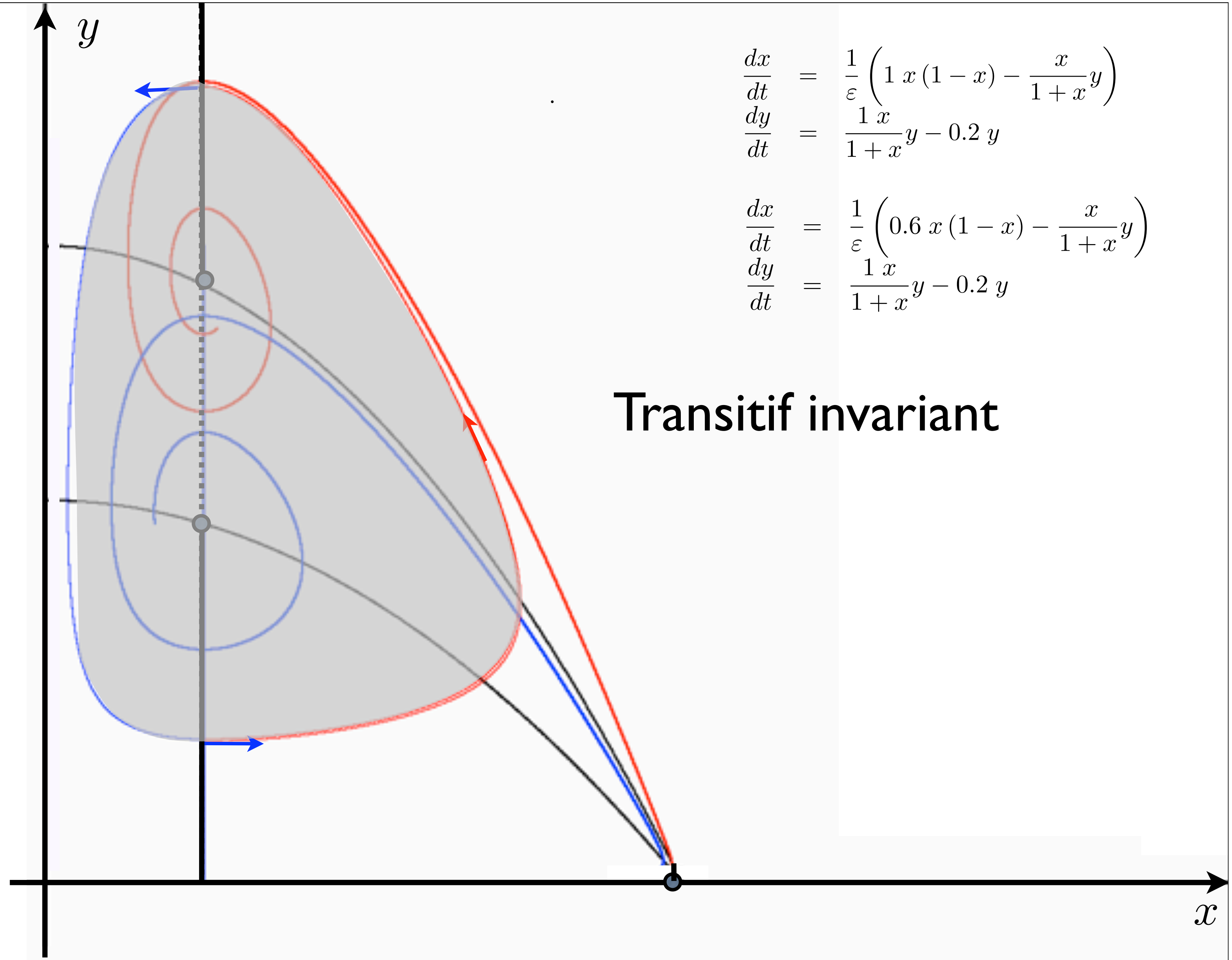


Analyse du premier retour

$$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(1\,x\,(1-x) - \frac{x}{1+x}y\right)$$
$$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$$

$$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(0.6\,x\,(1-x) - \frac{x}{1+x}y\right)$$
$$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$$

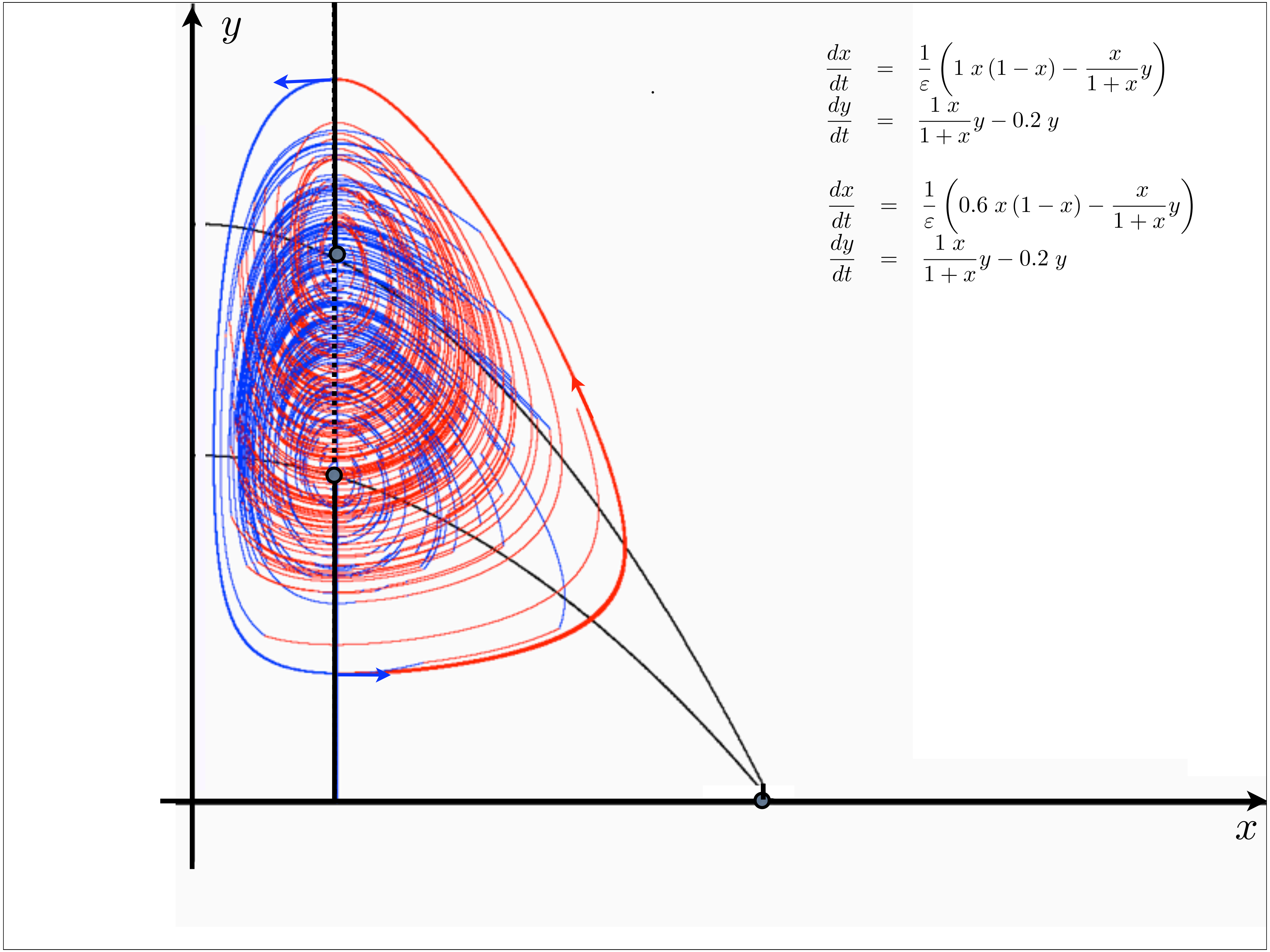




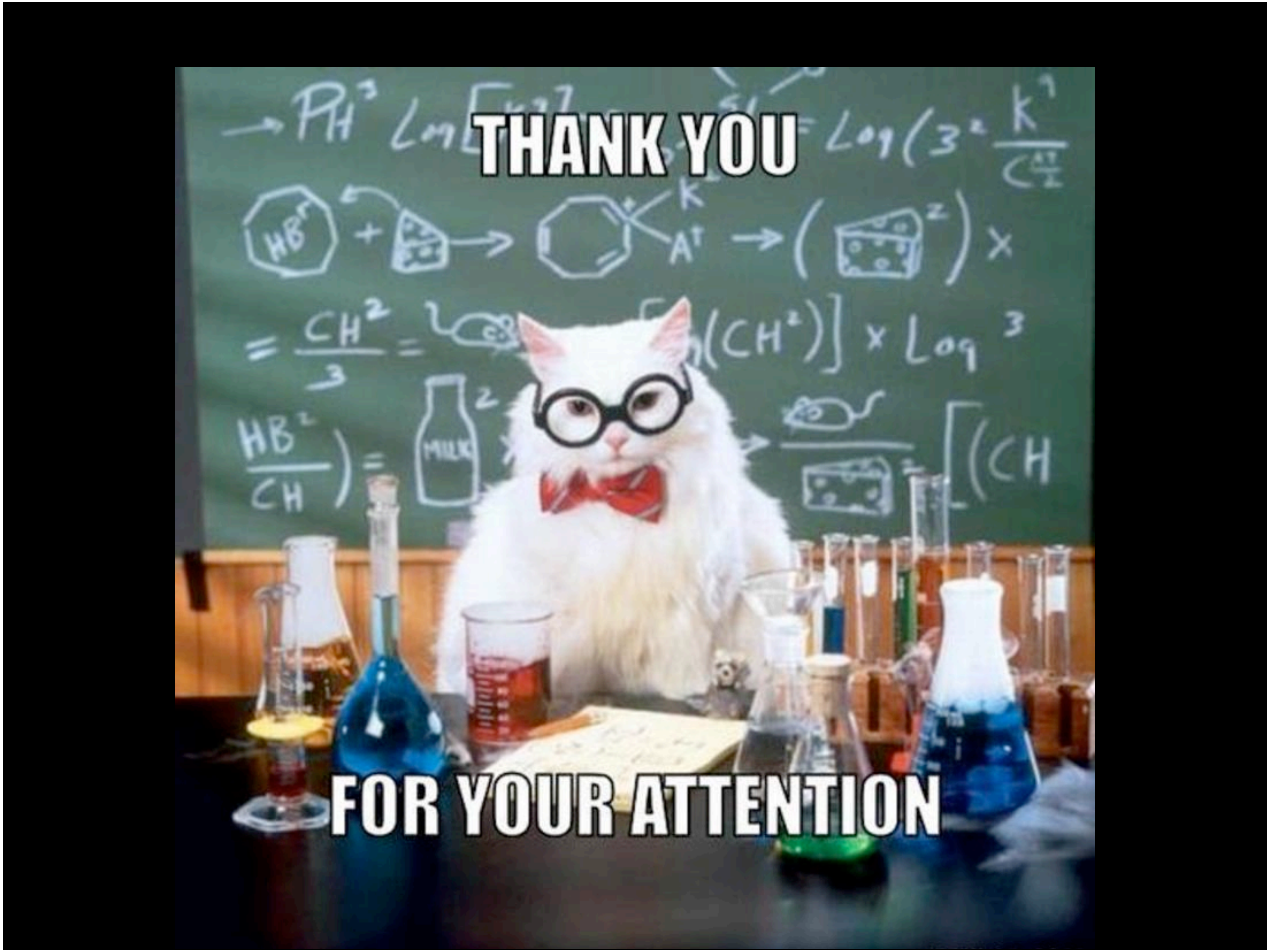
$$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(1\,x\,(1-x) - \frac{x}{1+x}y\right)$$
$$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$$

$$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(0.6\,x\,(1-x) - \frac{x}{1+x}y\right)$$
$$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$$

Transitif invariant



y
$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(1\,x\,(1-x) - \frac{x}{1+x}y\right)$
$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$
$\frac{dx}{dt} = \frac{1}{\varepsilon}\left(0.6\,x\,(1-x) - \frac{x}{1+x}y\right)$
$\frac{dy}{dt} = \frac{1\,x}{1+x}y - 0.2\,y$
x


THANK YOU
FOR YOUR ATTENTION