Reduction of the ADM1 model

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  - considers the degradation of the organic matter from its most complex, i.e. the composite organics to its simplest forms, i.e. the gaseous degradation products: CO_2, CH_4, H_2.
  - is a high order nonlinear model, not suited for control design.
- Model reduction techniques appear as appropriate tools for obtaining reduced order linear or nonlinear models.

Simplification of the kinetics equations

- Monod growth rate expression:

$$\mu(S) = \mu_{\max} \frac{S}{S + K_S}$$

$\mu_{\max}$ → The maximum growth rate (1/day)
K_S → The half-saturation constant (gCOD/L)
S → The substrate (gCOD/L)
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- Blackman growth rate expression:

$$\mu(S) = \begin{cases} \frac{\mu_{\max}}{K_S} S \text{ if } S \leq K_S \\ \mu_{\max} \text{ if } S > K_S \end{cases}$$

Simplification of components

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- We assume that it is enough to suit our objectives by neglecting all chemical or physicochemical phenomena.
- We remove the state variables whose dynamics are uncoupled from other components:
  - Solube inert component S_I
  - Particulate inert component X_I

The Homotopy method

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- Implies starting with a system in which the relation eigenvalues-states is obvious such as a decoupled system or a diagonalised matrix A. The transformation is described by the equation:
- $$H(r) = (1-r).A_D + r.A \qquad 0 \leq r \leq 1$$

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- A the matrix of the linearized model
- r a parameter that allows to obtain a linear progression from the decoupled system to the coupled one

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- It is assumed that the system admits at least one equilibrium point locally asymptotically stable
- The homotopy method is applied to a stable equilibrium point of the differential system (11 differential equations for the soluble components, 11 for the particulate ones)

Simulation results

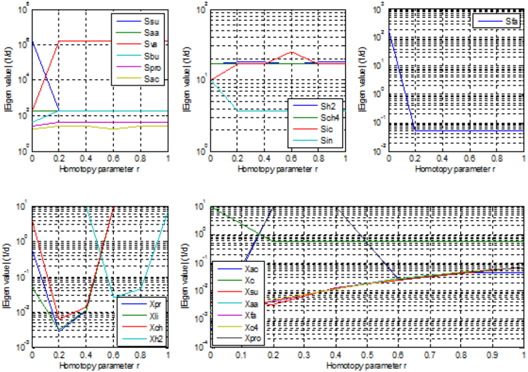

Figure 1 - Results of the homotopy method on the fast and slow variables

State variables	Eigenvalues (1/d)	$\tau(h)$
S_{va}	$-1585389,931$	$0,00002$
S_{aa}	$-200,050$	$0,119$
S_{su}	$-200,049$	$0,119$
S_{bu}	$-196,449$	$0,122$
S_{pro}	$-40,870$	$0,587$
S_{ac}	$-25,162$	$0,953$
S_{h2}	$-17,832$	$1,345$
S_{ch4}	$-16,993$	$1,412$
S_{ic}	$-16,924$	$1,418$
X_{pr}	$-10,050$	$2,388$
X_{li}	$-10,050$	$2,388$
X_{ch}	$-10,049$	$2,388$
X_{h2}	$-7,002$	$3,427$
S_{in}	$-3,636$	$6,600$

State variables	Eigenvalues (1/d)	$\tau(h)$
X_c	$-0,552$	$43,456$
X_{aa}	$-0,070$	$341,491$
X_{fa}	$-0,070$	$342,514$
X_{c4}	$-0,070$	$342,514$
X_{pro}	$-0,069$	$342,906$
X_{su}	$-0,068$	$352,319$
S_{fa}	$-0,050$	$480,000$
X_{ac}	$-0,045$	$527,588$

Table 3 - Eigenvalue and time constant association

- The differential system is reduced to 14 differential equations represented by the following state variables: S_{su}, S_{aa}, S_{va}, S_{bu}, S_{pro}, S_{ac}, S_{h2}, S_{ch4}, S_{ic}, S_{in}, X_{ch}, X_{pr}, X_{li}, X_{h2} other variables being considered as constants.

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- The reduced model keeps the particulate components X_{ch}, X_{pr}, X_{li} which are the most important particulate substrates identified by the ADM1 model (Batstone et al., 2002).

The case study

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- Simulations were performed by using DYMOLA (DYnamic MOdeling LAboratory) to compare the values assumed by the state variables at steady state as predicted by the ADM1 model and by the reduced model

Simulation results

State variables	ADM1 Model	Reduced Model	|Relative error|(%)
Ssu	0,011	0,019	72,72
Saa	0,0053	0,0050	5,66
Sva	0,011	0,009	18,18
Sbu	0,0132	0,0136	3,03
Spro	0,0157	0,0152	3,18
Sac	0,197	0,024	87,81
Sh2	2,359e-7	2,277e-7	3,48
Sch4	0,0549	0,0544	0,91
Sic	0,152	0,147	3,28
Sin	0,130	0,122	6,15
Xch	0,027	0,024	11,11

State variables	ADM1 Model	Reduced Model	|Relative error|(%)
Xpr	0,102	0,099	2,94
Xli	0,029	0,024	17,24
Xh2	0,317	0,296	6,62

Table 4 - Steady state values reached by the two models

State variables	ADM1 Model	Reduced Model	|Relative error|(%)
Sgas CH_4	1,621	1,618	0,18
Sgas CO_2	0,014	0,014	0
q gas CH_4	1709,300	1596,040	6,63
q gas CO_2	952,399	885,347	7,04

Table 5 - Steady state values for CH_4, CO_2 and their flow rate

A disturbance around the steady state conditions

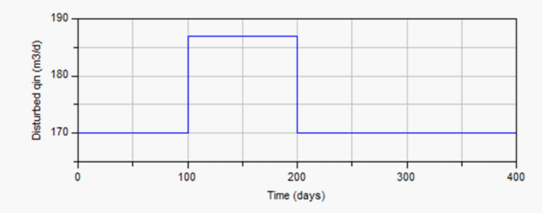
- 

Figure 2 - Step disturbance applied to qin

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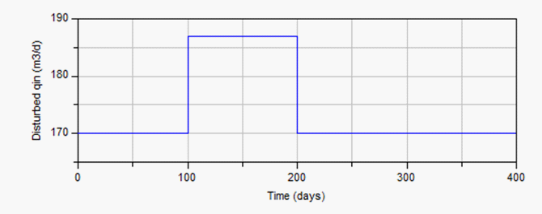
- 

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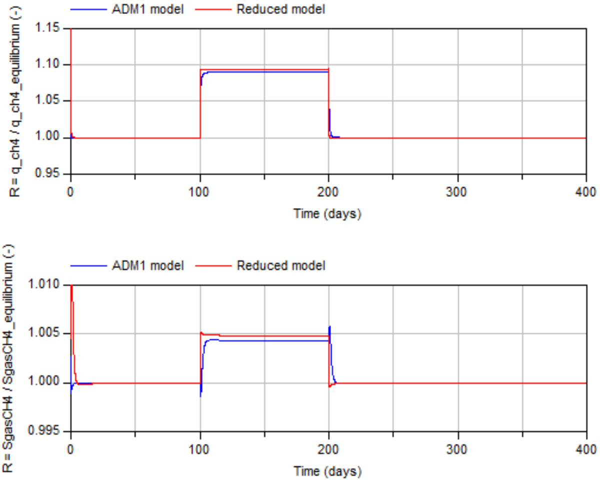
- A-dimensional ratio R:

$$R_{\text{state variable}} = \frac{\text{State variable } (t)}{\text{State variable } (eq)}$$


ADM1 model
Reduced model
R = q_ch4 / q_ch4_equilibrium (-)
1.15
1.10
1.05
1.00
0.95
0
100
200
300
400
Time (days)
ADM1 model
Reduced model
R = SgasCH4 / SgasCH4_equilibrium (-)
1.010
1.005
1.000
0.995
0
100
200
300
400
Time (days)

Sensitivity to the parameters

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- $$\delta_{ij} = \frac{p_i}{y_j(p_i)} . \frac{y_j(p_i + \Delta p_i) - y_j(p_i)}{\Delta p_i} .100$$

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- For each parameter p_i an absolute variation of 20% of the default value was applied

	$k_{hyd,ch}$	$k_{hyd,pr}$	$k_{hyd,li}$	$k_{dec,Xh2}$	$k_{m,su}$	$K_{s,su}$	$k_{m,aa}$	$K_{s,aa}$	$k_{m,c4}$	$K_{s,c4}$	$k_{m,pro}$	$K_{s,pro}$	$k_{m,ac}$	$K_{s,ac}$	$k_{m,h2}$	$K_{s,h2}$
Ssu	1		1		3	3										
Ssu_red	1		1		3	3										
Saa		1					3	3								
Saa_red		1					3	3								
Sva							1	1	3	3						
Sva_red							1	1	3	3						
Sbu					1	1	1	1	3	3						
Sbu_red					1	1	1	1	3	3						
Spro					1	1	1	1	1	1	3	3				
Spro_red					1	1	1	1	1	1	3	3				
Sac					1	1	1	1	1	1	1	1	3	3		
Sac_red					1	1	1	1	1	1	1	1	3	3		
Sh2					1	1	1	1	1	1	1	1			3	3
Sh2_red					1	1	1	1	1	1	1	1			3	3
Sch4													1	1	1	1
Sch4_red													1	1	1	1
Sic	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Sic_red	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Sin				1	1	1	1	1	1	1	1	1	1	1	1	1
Sin_red				1	1	1	1	1	1	1	1	1	1	1	1	1
Xch	3															
Xch_red	3															
Xpr		3														
Xpr_red		3														
Xli			3													
Xli_red			3													
Xh2				2											1	1
Xh2_red				2											1	1

Table 4 - Results of the sensitivity study

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- $2 = 30\% \leq \delta_{ij} \leq 60\%$
- $3 = \delta_{ij} > 60\%$
- The sensitivities of the states with regard to the parameters are preserved in the reduced model

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	ADM1 Model	Reduced Model
State variables	24	14
Biomasses	7	1
Parameters	55	40

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- Sensitivity study proves the similarity and the strong connection between the ADM1 and the reduced model
- To use the reduced model for control synthesis and apply this controller to the ADM1 model

THANK YOU!